Stiffness of sky-scrapers

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STIFFNESS OF SKY-SCRAPERS

LA RIGIDITÉ DES GRATTE-CIEL

DIE STEIFIGKEIT DER WOLKENKRATZER

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Psychological perception of sway of sky-scrapers and towers appears to be a function of acceleration as well as maximum deflexion, since stiff towers seem to be more comfortable than flexible ones, though acceleration may be the same.

The purpose of this paper is to obtain discussion of the problem of evaluating a proposed series of measurements of existing towers. A separate investigation, not described in this paper, will be made by psychological tests, to determine the function of amplitude and frequency which defines the equivalent perceptible sway. The study of this question should lead to a formula by which towers of differing amplitudes and frequencies can be located in a statistical diagram, in order of their equivalent sensation of sway. The purpose of this diagram would be to serve as a guide to the judgement in choosing a nominal static deflection for use in SPURR's method of bracing design, something which now must be done by pure guess.

In view of the purpose, for which the results are to be used, it is considered desirable to carry out these studies even though the results cannot have any high degree of accuracy. The towers to be investigated are not braced in accordance with any consistent theory ; and there is no way of estimating the true stiffness of the joints or of the walls. Columns are scattered in irregular lines, beams are eccentric, the fixity of joints and the effect of fireproofing defy mathematical analysis. It is necessary to guess at the probable elastic curve.

The following pages show how a tower of irregular shape and weight can be approximately evaluated with regard to stiffness. The data are the weights and dimensions, the time of vibration readings at various levels to assure that the tower vibrates without nodes. A standard wind is assumed. An approximate elastic curve is located so as to be equivalent to the shape which the building would assume if each square foot of wind acted only on its own element of mass. The top ordinate of the equivalent elastic curve is then taken as the measure of flexibility.

Section I shows a derivation of an elastic curve which can be used as approximate for any building. *Section II* gives the application of the common theory of a vibrating cantilever to an ideal building. *Section III* is an example of a typical building, showing how the deviations from the ideal are adjusted.

Section I. Assume a building designed as follows[1]: a) the deflection of all bents due to change in length of columns, Dc, is the same. b) The deflection due to distortion of panels, Dw, is the same in every panel from top to bottom. c) The stresses in web members due to D_w cause vertical loads in columns such that the floor remains plane.

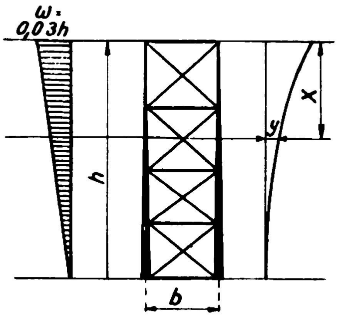

Fig. 1.

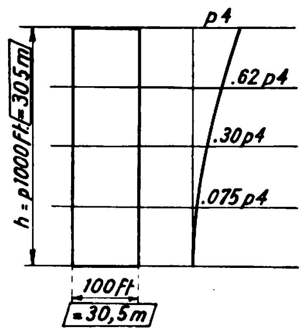

Fig. 2.

1. Curve due to Dc. — In Fig. I is shown a tower of uniform floor area, with columns varying in section from 0 at the top to a maximum at the bottom. The base b is that of an equivalent tower with all the coloumns concentrated into two lines.

J_0 is J of column areas at the base $\qquad$ h is the height

J_x is J at level x, $J_x = \frac{x}{h} J_0$

Mx is Moment of wind load above level x, $Mx = \frac{wx^2}{2} - \frac{wx^3}{6h}$

$$\frac{d^2y}{dx^2} = \frac{M_x}{J_x E} = \frac{\frac{wx^2}{2} - \frac{wx^3}{6\,h}}{\frac{x}{h}\,J_0 E} = \frac{wh}{2J_0E}\,x - \frac{w}{6J_0E}\,x^2 ; \quad \frac{dy}{dx} = \frac{wh}{4J_0E}\,x^2 - \frac{w}{18J_0E}\,x^3 + C$$

$$\text{If } x = h, \quad \frac{dy}{dx} = o, \quad so: C = -\frac{7\,w\,h^3}{36\,J_0\,E}; \quad \frac{dy}{dx} = \frac{w\,h\,x^2}{4\,J_0\,E} - \frac{w\,x^3}{18\,J_0\,E} - \frac{7\,w\,h^3}{36\,J_0\,E}$$

$$y = \frac{w.h}{12\,J_0E}\,x^3 - \frac{w}{72\,J_0E}\,x^4 - \frac{7\,w\,h^3}{36\,J_0\,E}\,x + C ; \text{If } x = h, y = o, so: C = + \frac{w\,h^4}{8J_0E}$$

$$y = \frac{w}{4\,J_0\,E}\left(\frac{h\,x^3}{3} - \frac{x^4}{18} - \frac{7\,h^3x}{9} + \frac{h^4}{2}\right).$$

Wind pressure admitted: $w = \frac{3\,h}{10^2}$ lbs./sq.ft.

$$\text{If } x = o: \quad y = D_c = \frac{3\,h^5}{800\,J_0\,E}$$

1. Modified form of Spurr's Theory (Wind bracing by H. V. Spurr, Mc Graw Hill Co., New York, 1930).

J_o is determined by the fact that in a New York building with the usual loads and construction, if h is 1000 ft., actual base 100 ft., then D_c is 1 ft.

$$D_c = 1 = \frac{3 \times 10^{15}}{800\, J_o E}, \text{ and } J_o = \frac{3 \times 10^{13}}{8\,E} \text{ for } h = 1000.$$

Therefore, for any height, and base 100 ft.:

$$J_o = \frac{h}{1000} \cdot \frac{3.10^{13}}{8E} = \frac{3\,h.10^{10}}{8\,E}$$

$$y = \frac{2}{10^{12}}\left(\frac{h x^3}{3} - \frac{x^4}{18} - \frac{7\,h^3\,x}{9} + \frac{h^4}{2}\right).$$

This formula holds for buildings 100 ft. long, of any height and weight of bracing, provided the bracing is designed to make floors stay plane, and provided the columns are designed for the usual New York office building loads. Let h be any fraction p of 1000 ft.: $h = p \times 10^3$.

Then the shape of the curve will be as follows:

$$y = \frac{p x^3}{15.10^8} - \frac{x^4}{9.10^{12}} - \frac{p^3 x}{640} + p^4$$

When $x = o$ $\quad x = \dfrac{p.10^3}{4} \quad x = \dfrac{p\,10^3}{2} \quad x = \dfrac{3p\,10^3}{4} \quad x = p.10^3$

so $y = D_c = p^4 \quad y = 0.62p^4 \quad y = 0.30p^4 \quad y = 0.075p^4 \quad y = o.$

Fig. 2 shows the shape of the curve, for any height of building. It will be noted that the shape is the same for any height and wathever value of D_c may occur.

2. Curve due to D_w. — The total deflection allowed is assumed to be 0.002h, and the allowable web distortion D_w is 0.002h — D_c. Fig. 3 shows deflections for various heights of building, 100 ft. base. For example at 800 ft., p is 0.8 D_c is p^4 or 0.41 ft., and D total is 1,6 ft. So the allowed D^w is the difference = 1.19 ft.

3. Total curvature of any standard building of height h, base 100. Fig. 4 illustrates the procedure for a building 800 ft. high. D_w as in 2 is 1.19 and the deflection curve due to web distortion is a sloping straight line. For example, at midheight yw is 0. 595 ft. From Fig. 2, at half height, y_o is 0.3 D_c or 0.123 ft. Therefore the total y at midheight is y_w plus y_c or 0.718 ft. As D is 1.6, the middle ordinate 0.718 is 0.45 D.

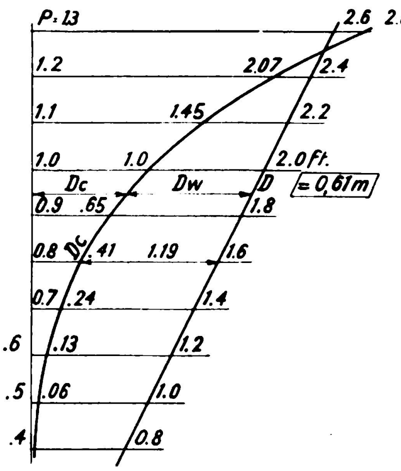

Fig. 3.

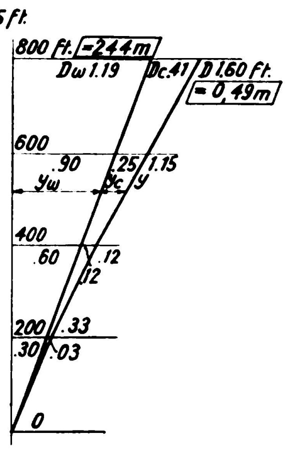

Fig. 4.

In the discussion to follow, it is assumed that if any building is 8 times as high as it is long, and has any particular D at the top as determined by its actual stiffness, then its total elastic curve will be approximately similar to the curve of Fig. 4, that is, its middle ordinate will be about 0.45 D.

By a similar procedure, if the slenderness ratio is 5, the middle ordinate will be 0.49 D ; if it is 10, the middle ordinate will be 0.40 D.

It will be noted that if the building were to deflect entirely as a cantilever, the middle ordinate would be, by Fig. 2 about 0.3D. If the columns were infinitely stiff vertically, and all the deflection were due to the web system, the middle ordinate would be 0.5D. Fig. 5 shows ratios of mid ordinate for various ratios of slenderness. As will be seen these ratios show an increasing departure from the sloping straight line as the buildings become more slender, and cannot be very far from correct for any given building.

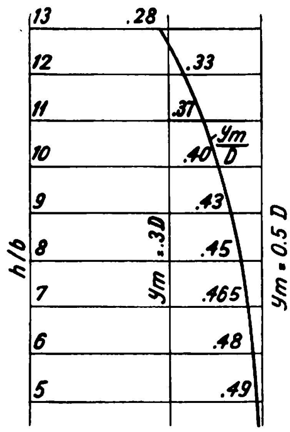

Fig. 5.

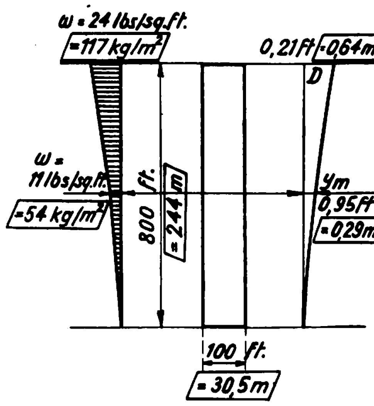

Fig. 6.

Section II. Fig. 6 shows an ideal building 100 ft. square and 800 ft. high, with a wind load of 0.03 h, or 24 lbs., at the top, and a wind pressure diagram similar to the elastic curve, with a wind load per square foot at mid height of 0.45×24, or 11 lbs.

The building vibrates 15 times per minute, T is 2 sec. The weight of one vertical foot of the building is 150000 lbs., and the weight acted on by 1 square foot of wind is 1500 lbs.

For harmonic vibration with period T the maximum acceleration with amplitude q is $\frac{\pi^2}{T^2} \cdot q$, and the force of the spring for a static deflection a is $\frac{w}{g} \frac{\pi^2}{T^2} \cdot r$. Therefore the static deflection for 1 lb. pressure is $\frac{g}{w} \frac{T^2}{\pi^2}$

In fig. 6, the static deflection at the top is

$$\frac{24 . gT^2}{w . \pi^2} = \frac{24 . 32 . 2^2}{1500 . \pi^2} = 0.21 \text{ ft.}$$

The static deflection at mid height is

$$\frac{11 . gT^2}{w \pi^2} = 0.095 \text{ ft., or } 0.45 \times 0.21 \text{ ft.}$$

It will be noted that for each element of the tower, the vibration amplitude, wind load, acceleration, and static deflection are all proportional to the ordinate of the elastic curve. In a actual building the deflection can be approximated by evaluating the differences from the ideal.

Section III. Deflection of an actual building. The height is 8 times the base, and the value of T is 4 sec. Fig. 7 is a diagram of the weight per foot high, obtained from the column loads including estimated live load. Fig 8 shows : a) the assumed wind pressure, 0.03 *h* lbs. per square foot. b) the width of building across the wind. c) is the product of a) and b), giving the total wind force on a horizontal slice 1 foot high.

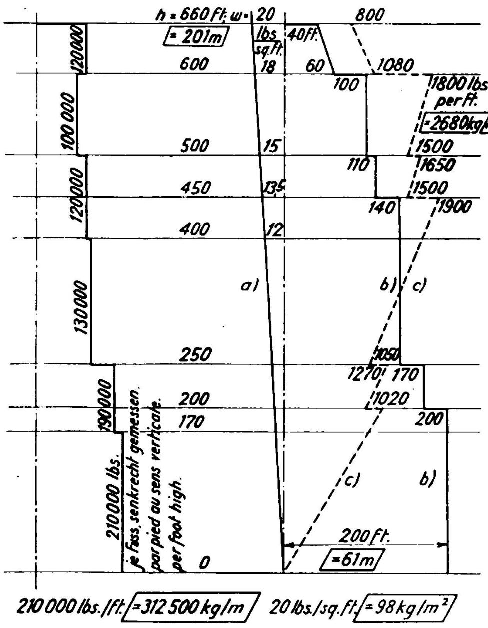

Fig. 7. Fig. 8.

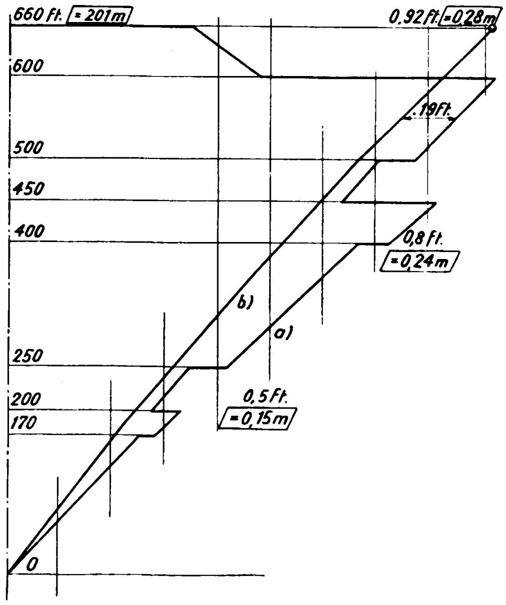

Fig. 9.

Fig. 9 shows the nominal deflection *a*) which is the wind force (fig. 8 c), divided by the weight (fig. 7), and multiplied by $\frac{gT^2}{\pi^2} = 53$.

Curve *b*) is an elastic curve, with middle ordinate 0.45D, located by trial so that it will be equivalent to a) by the following criterion : Consider, for example, the section between the 500 and 600 ft. levels. The average discrepancy between a) and b) is 0.19 ft., which multiplied by 100 000 and divided by 53 as above, corresponds to 360 lbs. per foot high, on Fig. 8 c. The overturning moment of this differential on the section, 100 ft. high, is 360 $\times$ 100 $\times$ 550, or 19 800 000 ft. lbs. If the summation of all these overtuning moments is 0 then the curves a) and b) may be said to be equivalent. On curve b) the top ordinate D is 0.92 ft. This indicates that with the assumed wind the top of this building will deflect about 0.92 ft. The amplitude of the harmonic vibration is, of course, much smaller than the static deflection.

This method gives the static deflection for any existing tower, but not a coefficient of stiffness for comparing two towers, unless they happen to have the same T. For practical purposes, in order to compare two towers, it will be necessary to find by experimental psychology what function of T and D will give approximately a mesure of the sensory intensity of motion. It is apparently not the acceleration.

500 et 600 pieds (soit 152 et 183 mètres, c'est-à-dire un écart de 100 pieds. ou 30,50 mètres). L'écart entre *a* et *b* est de 0,058 mètre, ce qui, multiplié par 100.000 et divisé par 16, comme précédemment, correspond à 675 kg. par mètre, ainsi que l'indique la figure 8 *c*. Le moment de renversement de cet élément de section de 100 pieds de hauteur (soit 30,50 mètres) est de 2.740.000 kilogrammètres. Si la somme de tous ces moments de renversement est nulle, on peut dire que les courbes *a* et *b* sont équivalentes. Sur la courbe *b*, l'ordonnée au sommet est de 0,27 m. Ceci montre qu'avec la force supposée pour le vent, le sommet de l'édifice subira une déviation d'environ 0,27 mètre. L'amplitude de l'oscillation est naturellement beaucoup plus faible que la valeur de cette déviation statique.

Cette méthode permet de déterminer la déviation d'ordre statique pour toute construction en forme de tour déjà existante mais ne donne pas un coefficient de rigidité permettant de faire la comparaison entre deux tours, à moins qu'elles ne possèdent la même période. En pratique, pour comparer deux tours, il faudra déterminer, par des essais physiologiques, quelle fonction de T et de D peut donner une mesure approximative de l'intensité avec laquelle le mouvement est perçu. Il ne semble pas que ce soit uniquement l'accélération.

Summary.

Skyscrapers and other tower-like buildings are set perceptibly in vibration by wind pressure. The author endeavours to calculate these vibrations by investigating the elastic properties of the towers. and an approximate method is given for determining their stiffness. The method developed is also applicable to buildings of irregular shape and with irregular distribution of weight.

Résumé.

Les gratte-ciel et autres édifices dont la construction affecte la forme d'une tour, sont soumis, par suite de l'influence de la pression du vent, à des oscillations dont l'importance n'est pas négligeable. L'auteur traite ces oscillations en cherchant à déterminer les caractéristiques élastiques propres des tours et il donne une méthode approximative pour la détermination de leur rigidité. La méthode développée peut être également appliquée à des constructions de formes non régulières et dans lesquelles la répartition des masses est irrégulière.

Zusammenfassung.

Wolkenkratzer und andere turmähnliche Gebäude werden unter dem Einfluss des Winddruckes in fühlbare Schwingungen versetzt. Der Verfasser trachtet diese Schwingungen durch Untersuchung der elastischen Eigenschaften der Türme zu berechnen und gibt ein Näherungsverfahren zur Bestimmung ihrer Steifigkeit. Die entwickelte Methode ist auch bei unregelmässiger Gebäudeform und unregelmässiger Gewichtsverteilung anwendbar.