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Dynamic increments in an elementary case

Les influences dynamiques considérées dans un cas élémentaire

Dynamische Zuschläge in einem einfachen Fall

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In the Preliminary Publication to the Congress in Liège in 1948, the author presented the first results of an investigation of dynamic influences of moving loads on girders. This work was carried out at the Institution of Structural Engineering and Bridge Building at the Royal Institute of Technology, Stockholm, Sweden, under the supervision of Professor G. Wästlund. The final results of the investigation were published in 1951 in a treatise,* which also describes the theoretical and experimental methods used. A summary of the practical results will be given here.

The case that has been studied is that of a single load moving smoothly at a constant speed along a simply supported girder. The girder has been supposed to be of uniform section and to be straight under dead load. The following factors have been taken into account:

the mass of the girder,
the mass of the load,
the velocity of the load,
spring-mounting of the load,
viscous damping of the girder (internal and external),
dry friction in the load-carrying spring.

These factors have been given a dimensionless form by introducing the notations:

$$\nu=\frac{\text{mass of load}}{\text{mass of girder}}$$

$$\alpha=\frac{\text{velocity of load}}{2\times\text{length of girder}\times\text{frequency of girder}}$$

$$\mu=\frac{\text{frequency of load}}{\text{frequency of girder}}$$

$$\theta=\frac{\text{spring friction force}}{\text{weight of load}}$$

$e^{-\frac{2\pi\beta}{\sqrt{1-\beta^2}}}$ = ratio of two consecutive amplitudes in the same direction of the free vibration of the girder.

* *Dynamic Influences of Smoothly Running Loads on Simply Supported Girders.*

In the above notations, the frequency of the girder is the fundamental frequency of the undamped girder at no load.

Two different values are used for the constant β. One of them, denoted only by β, refers to an external damping force, while the other, denoted by β_1, refers to an internal damping force.

In the investigation, a distinction was made between two cases, viz. spring-borne and non-spring-borne loads, but, as the former is of much greater practical importance, only the results relating to spring-borne loads will be given here.

A dynamic increment in a quantity is defined by:

$$\epsilon = \frac{\text{dynamical value}}{\text{static value}} - 1$$

To make the definition strict, it is also necessary to know what kind of quantity is measured and what dynamical and static values are to be used. This is indicated

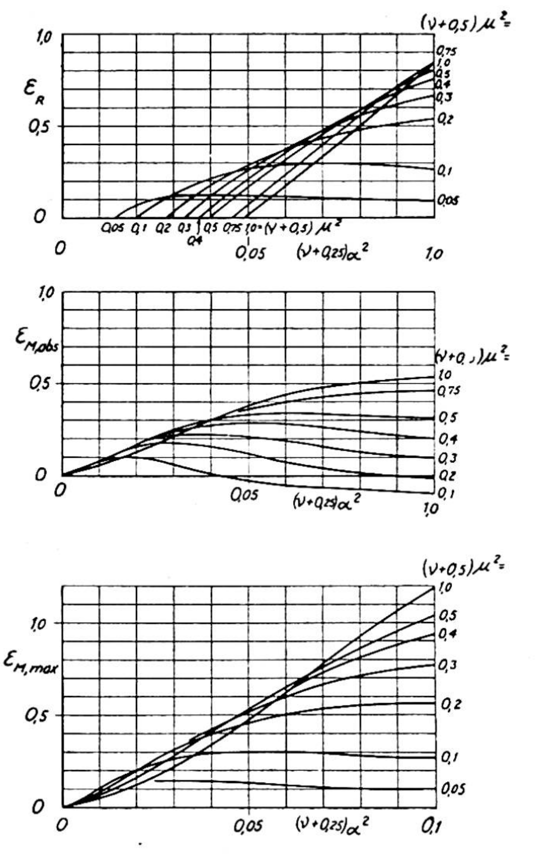

Fig. 1

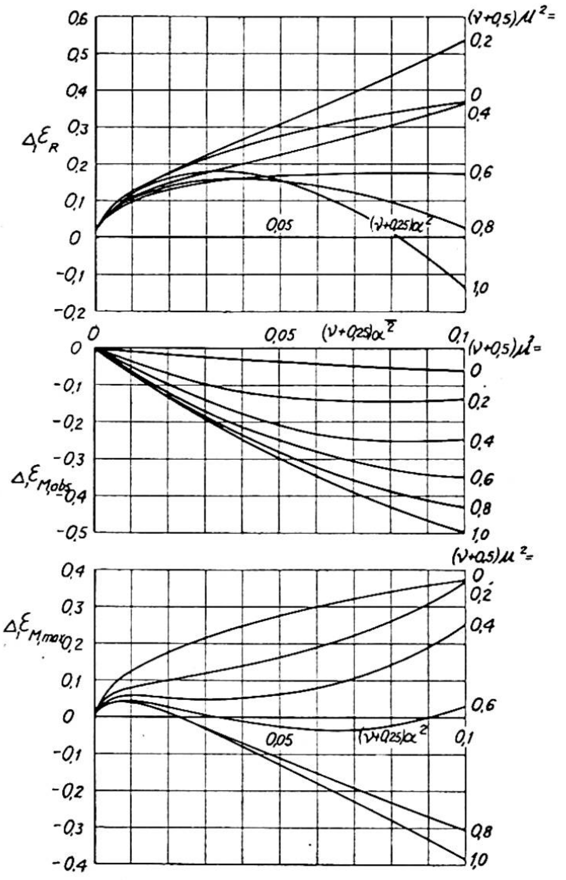

Fig. 2

by subscripts as follows: M for moments, Q for shearing forces, and R for reaction forces. The following definitions show what values are to be taken:

$$\epsilon_{abs} = \frac{\text{greatest dynamical value for the girder}}{\text{greatest static value for the girder}} - 1$$

$$\epsilon_{max} = \text{maximum of } \frac{\text{greatest dynamical value at any point}}{\text{greatest static value at the same point}} - 1$$

The value ϵ_{abs} (the absolute increment) expresses the greatest influence of a given kind (for instance, the greatest moment) on the girder, and is therefore the most interesting value in dealing with girders of uniform strength. The value ϵ_{max} gives the greatest dynamic increment at any section of the girder. This value is of great interest in studying girders of non-uniform strength (for instance, reinforced-concrete girders).

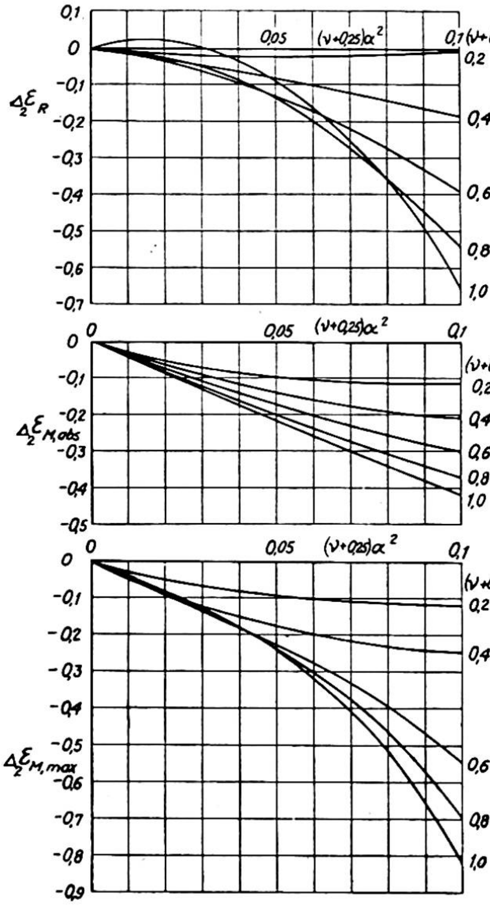

Fig. 3

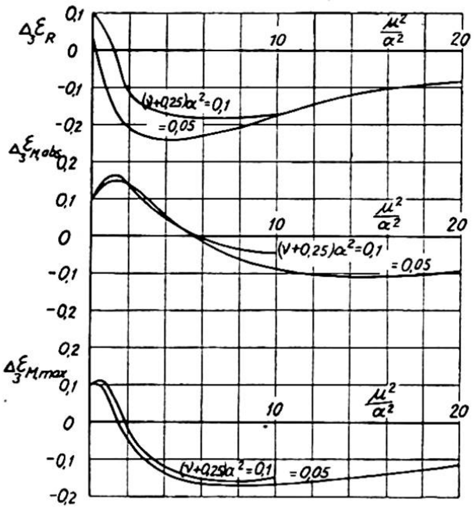

Fig. 4

The most interesting dynamic increments are $\epsilon_{M,\,max}$, $\epsilon_{M,\,abs}$, $\epsilon_{Q,\,max}$, $\epsilon_{Q,\,abs}$, and ϵ_R. The latter has only one subscript, as the gauge point must be at the support, and the definition of ϵ_R is:

$$\epsilon_R = \frac{\text{greatest dynamical reaction force}}{\text{greatest static reaction force}} - 1$$

It can be shown that:

$$\epsilon_{Q,\,abs} = \epsilon_R$$

Further, it has been shown that $\epsilon_{Q,\,max}$ may with sufficient accuracy be put equal to $\epsilon_{M,\,max}$ in this case. It is therefore sufficient to plot diagrams for the dynamic increments $\epsilon_{M,\,max}$, $\epsilon_{M,\,abs}$, and ϵ_R. Such diagrams are shown in figs. 1 to 4, from which the dynamic increments for any arbitrary values of ν, α, μ, β, β_1, and θ (within practical limits) can be calculated by means of the formula:

$$\epsilon = \epsilon_o + \frac{\beta}{\sqrt{\nu + 0\cdot 5}} \cdot \Delta_1\epsilon + \frac{\beta_1}{\sqrt{\nu + 0\cdot 5}} \cdot \Delta_2\epsilon + \frac{\theta}{0\cdot 1} \cdot \Delta_3\epsilon$$

In this formula ϵ is the value taken from fig. 1, and the three $\Delta\epsilon$-values are taken from figs. 2 to 4.

The values of ϵ which are given by this formula are approximate, as it has been constructed in the way that is described below, but it seems always to give sufficiently accurate values.

For studying the dynamic increments, use can be made of the theoretical methods described in the above-mentioned treatise. In the general case, however, the calculations are so intricate that it takes about two days to carry them out for a single case. If complete calculations including four values of each of the six variables were to be made, the number of calculations would be $4^6=4096$, and the time required would be about twenty-five years. This is obviously impracticable, and some other method must be found in order to limit the work, even if the results will be less accurate.

For plotting the diagrams in figs. 1 to 4 the following method has been used. To begin with, the case $\nu=\infty$ has been studied, that is, the case where the mass of the

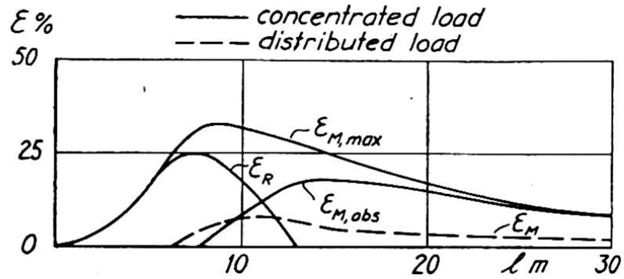

Fig. 5

girder is neglected in comparison with that of the load. In this case the calculations are so simple that they can be carried out almost completely. When studying the results of these calculations, trials have been made to find simple approximate relations between ϵ and the variables. It was then found that the above formula gave sufficiently accurate results in this case. This formula and the corresponding diagrams have thus first been made for the case $\nu=\infty$, in which the numbers 0·25 and 0·5 added to ν are without significance. It is to be noted that, in this case, the values of $\nu\alpha^2$, $\nu\mu^2$, and $\frac{\beta}{\sqrt{\nu}}$ are finite.

After the case $\nu=\infty$ had been studied theoretically, a very complete series of tests comprising ν-values between 0·75 and 5 was made. The test values were then compared with the theoretical values for $\nu=\infty$, and it was found that if ν was increased by the values given in the formula and the diagrams, the agreement was sufficiently close for all test values.

In order to give an idea of the order of magnitude of the dynamic increments caused by the influence studied in this investigation, the diagram in fig. 5 has been plotted on the following assumptions:

(1) The deflection under live load is 1/1250 of the span length.
(2) The velocity is 30 m./sec (=108 km./hour).
(3) The mass of the girder is neglected (this gives too small values of ϵ).
(4) The damping is neglected.
(5) The frequency of the load is 3 cycles per second.

For comparison, a curve for a distributed load is also shown in fig. 5. The assumptions on which this curve is based are such that it gives only a lower limit for the increments.

The investigation has shown that dynamic influences of moving loads of nearly any kind on simply supported girders can be calculated theoretically, but in complicated cases the calculations are very laborious. This difficulty is still more pronounced when the girder is supported in a more intricate manner, for instance when it is continuous, although the calculations are possible in principle. On the other hand, the investigation has also shown that a comparatively simple test set-up can give reliable test values with a small amount of work. It therefore seems advisable that future investigations of this subject should mostly be based on model tests, especially in relatively complicated cases. Theoretical studies are of course of great value for the right understanding of the dynamical problems, but the number of numerical calculations should be limited.

In addition to such studies of elementary cases, it is of course also valuable to make tests on real bridges under real loads. However, these tests must be carried out and treated in a scientific and methodical way, and not at random. Thanks to the development of measurement engineering, we are today much better equipped for making such tests than we were only ten years ago. Resistant wire strain gauges and oscillographic recorders have made it possible to get accurate records of strains in any points of the load-carrying structures without much work and at small costs.

It seems to the author that the conditions are now favourable for acquiring a much better knowledge of the dynamical problems in bridge building if they are attacked methodically.

Summary

The practical results of an investigation of dynamic problems are summarised. A complete report on the investigation was published in 1951 in a book entitled *Dynamic Influences of Smoothly Running Loads on Simply Supported Girders.*

It is pointed out that the conditions are now favourable for acquiring a better knowledge of the dynamic problems if they are attacked methodically.

Résumé

L'auteur expose sommairement les résultats pratiques d'une étude relative aux problèmes dynamiques. Un rapport complet sur cette étude a été publié en 1951 dans un livre intitulé *Dynamic Influences of Smoothly Running Loads on Simply Supported Girders* (Influences dynamiques des charges roulantes à allure uniforme sur les poutres à appuis simples).

L'auteur fait remarquer que les conditions actuelles sont favorables à l'approfondissement de nos connaissances des problèmes dynamiques, si l'on aborde ces problèmes d'une manière méthodique.

Zusammenfassung

Der vorliegende Bericht enthält eine Zusammenfassung der praktischen Ergebnisse einer Untersuchung dynamischer Probleme. Ein vollständiger Bericht über diese Untersuchung wurde 1951 in einem Buch unter dem Titel *Dynamic Influences of Smoothly Running Loads on Simply Supported Girders* (Dynamische Einflüsse gleichmässig beweglicher Lasten auf einfach unterstüzten Trägern) veröffentlicht.

Der Verfasser weist darauf hin, dass die gegenwärtigen Verhältnisse für eine Vertiefung unserer Kenntnisse der dynamischen Probleme günstig sind, wenn diese Probleme methodisch in Angriff genommen werden.

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