

# **CI1: Composition of concrete; influence of the preparation, transport and placing on the design of structures**

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# **CI 1**

## **A rational method of proportioning concrete in India and its economic importance**

## **Une méthode rationnelle pour l'élaboration du béton et ses avantages économiques**

## **Eine rationelle Methode der Betonmischung und ihre wirtschaftliche Bedeutung**

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### **ECONOMIC IMPORTANCE OF RATIONAL PROPORTIONING**

The object of proportioning the ingredients, namely, cement, water and aggregates, of which concrete is made is to obtain durable concrete of strength to suit the requirements and at the most economical rate. Although strength is all-important, consideration has to be given to other important requirements such as durability, economy of the resulting concrete and workability of the mixture. Strength is essential to enable a structure or its components to fulfil its functions. Durability is required so that concrete does not deteriorate with time or when exposed to the atmosphere. The concrete must also be thoroughly sound, of uniform quality and with the voids reduced to a minimum.

As little cement as possible, consistent with the required strength of the resulting concrete, should be used for purposes of economy. The size and proportion of coarse aggregate should be as large as possible for effecting the greatest possible economy, but the quantity will depend on the workability of the mixture. Economy is effected due to the larger aggregate having less surface to be covered with cement paste than the smaller aggregate. The quantity of fine aggregate should be adequate to fill the voids in the coarse aggregate, otherwise cement paste will have to be wasted in filling the voids. The quantity of fine aggregate therefore depends on the voids in the coarse aggregate.

The proportioning of concrete is, to a great extent, done by what is termed the "mix method." In this method the proportions of cement, fine aggregate and coarse aggregate are specified. Although this method has been satisfactory so far, it is not an economical method; there is always an element of uncertainty as to strength and it is not suited for concrete of high strength. A far more logical and satisfactory

method is to decide on the strength of concrete required and then to determine the proportions of components which will give the most economical results.

The method of proportioning concrete should be rational, namely, such that it results in concrete of required strength at the least possible cost.

It may be assumed that with rational proportioning, the extent of saving in cement would be at least 3% and this may not be considered an extravagant assumption. The annual saving in India with annual consumption of over 3 million tons would, on this assumption, be about 90,000 tons, equivalent to 9 million rupees. The potential saving is of such magnitude that rational proportioning of concrete might with advantage be made obligatory to Indian engineers.

#### FUNDAMENTAL LAWS FOR RATIONAL PROPORTIONING

The fundamental laws of the water-cement ratio, consistency, and grading may be briefly stated as follows.

The strength and other desirable properties of concrete depend upon:

- (a) The quantity of water which is mixed with a given quantity of cement. A decrease in the water content increases the strength. Other properties of concrete are also improved. This law applies to a workable mixture only, namely, a mixture which can be placed satisfactorily in the forms.
- (b) The consistency of the mixture, which should be plastic and workable. The mixture should be thoroughly compacted, and for thorough compaction the consistency of the mixture should be plastic and workable.
- (c) For a given consistency, the most economic results are obtained when coarse and fine aggregates are well graded in size so that the mixed aggregates have the maximum density.

#### WATER-CEMENT RATIO AND DURABILITY OF CONCRETE

Although considerable experimental data with regard to the water-cement ratio are available in other countries, such data may not be applicable to India in view of the fact that almost all the experiments have been carried out in temperate climates.

Fortunately tests were carried out by an Indian railway a few years ago and data were collected from over 1,000 tests. In these tests the compressive strengths of concrete mixes, varying from 1:1½:3 to 1:3:6, were obtained by testing to destruction standard concrete cylinders. The results of these tests are given in fig. 1.

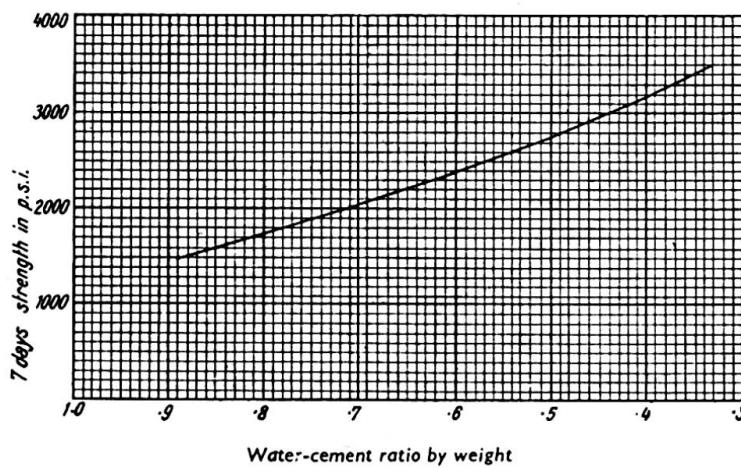


Fig. 1

It will be noted that the average strength at 7 days, which is 3,500 lb./in.<sup>2</sup> with a water-cement ratio of 0·33, falls to 1,500 lb./in.<sup>2</sup> when the water-cement ratio is increased to 0·89. This bears out forcibly the very great necessity for controlling the quantity of water in a concrete mix.

It may be added that as a result of these tests the original specification for concrete in a large construction which was given as "concrete of 1:2:4 mix" was altered to "concrete with a cylinder crushing strength averaging 2,000 lb./in.<sup>2</sup> at 7 days." The mix required for this specification was found to be much leaner than 1:2:4, and not only was a considerable saving in cement effected but concrete of a specified strength was guaranteed. The application of rational methods of proportioning concrete with its guaranteed strength and resulting economy has therefore been proved to be possible in India and there is no reason why this method should not be made the standard method for all concrete works of any size in India.

For purposes of comparison, the standard American figures of probable minimum compressive strength at 28 days for various water-cement ratios are given in Table I. Indian values are compared with American values in fig. 2. For this purpose, Indian figures of strength at 7 days have been corrected to 28-day strength by assuming the 28-day strength to be 150% of the 7-day strength. This assumption is close to

TABLE I

Water-cement ratio	0·45	0·49	0·54	0·58	0·63	0·67	0·71	0·76	0·80	0·85	0·89
Weight of water in lb./cwt. (1 bag) of cement	50	55	60	65	70	75	80	85	90	95	100
Probable minimum compressive strength at 28 days in lb./in. <sup>2</sup>	4,900	4,450	4,000	3,650	3,300	3,000	2,800	2,650	2,500	2,350	2,200

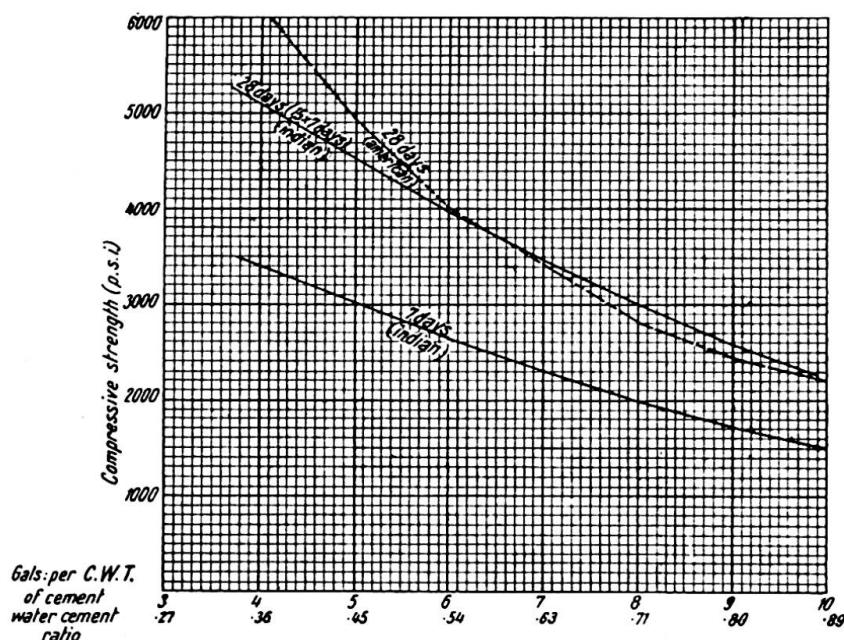


Fig. 2

the relation established in America between 7-day and 28-day strengths, as  $S_{28} = 1.51S_7 + 49$ .

As already indicated, the water-cement ratio governs not only the strength of the resulting concrete but many of its other properties, one of the most important being durability or resistance to exposure. For different types of exposures, American investigations indicate different water-cement ratios. Unfortunately investigations on this aspect have not so far been carried out in India.

### WORKABILITY

The term workability in a concrete mix is a relative term and normally indicates the ease with which the concrete can be placed in the forms or moulds and the degree to which it resists segregation. Workability has varying values depending on several factors, namely, size and grading of the aggregate, placing of the reinforcement, the size of the structural members and the methods of compacting.

Workability of concrete is normally ascertained through the slump test, in which the partial collapse of fresh concrete moulded to a particular shape is the measure of the workability of concrete.

Workability, as already indicated, depends on several factors, including the size of the aggregates, and definite limits of slump cannot therefore be fixed, but the values in Table II form a suitable guide.

TABLE II  
Slumps for various types of construction

Type of work	Slump (in.)	
	Without vibration	With vibration
(1) Normal R.C. sections, e.g. slabs, beams, columns, walls, etc.	2 to 6	1 to 3
(2) Foundations, footings, substructures, walls, etc.	1 to 5	1 to 2½
(3) Mass concrete	1 to 3	1

In making slump tests all aggregates above 2 in. should be removed.

It should be remembered that the vibration of concrete has an appreciable effect on workability. This is apparent from columns 2 and 3 of Table II. As workability is increased by vibration, the water-cement ratio can be reduced, resulting in increased strength. Other effects of vibration are an increase in the modulus of elasticity and a reduction in plastic flow and shrinkage on account of the reduction in water-cement ratio.

### SIZE AND GRADING OF AGGREGATES AND FINENESS MODULUS

The chief advantage of using as large a size of coarse aggregate as possible is the increased yield for a given water-cement ratio. This is due to the proportion of coarse aggregate to fine aggregate becoming greater for the same workability with an increase in the size of coarse aggregate. The water content and therefore the cement content are therefore reduced. The size, however, is limited by the section of the concrete member, spacing of reinforcement, and in the case of large size aggregates by the type of mixer available. Although a maximum size of 9 in. has been used, the normal limit may be considered as 6 in.

Suitable grades of aggregates, both coarse and fine, result in a dense concrete and also in the required workability being obtained with the minimum quantity of water. The lower the water content, the smaller is the cement content as well, and therefore grading of aggregates leads to the most economical concrete of the required strength. Other advantages of grading are uniformity in the quality of the resulting concrete and reduced shrinkage cracks due to reduced water content.

It is possible to obtain concrete of the required strength without suitable grading of the aggregates by using excess fine aggregate, cement and water, but it is not the most economical mixture.

The grading of aggregate is ascertained by passing it through a series of sieves. The sizes of sieves selected are such that the linear dimensions of the mesh of sieve A are double those of sieve B, the next in the series. The quantities retained on each sieve are either (a) tabulated or plotted, or (b) totalled, and a factor known as "fineness modulus" obtained. It is equally correct to use the quantities passing through the sieve. The sieves used for grading aggregates are: Nos. 100—52—25—14—7— $\frac{3}{16}$  in.— $\frac{3}{8}$  in.— $\frac{3}{4}$  in.— $1\frac{1}{2}$  in.—3 in. and 6 in.

"Fineness modulus" is an empirical factor obtained by totalling the cumulative percentages of aggregates retained on each sieve of the series given above, and dividing by 100.

As a result of research and judgment, certain values of fineness modulus for mixed aggregates and varying with the maximum size of aggregate have been accepted as giving the required workability and the most economical mix. It is emphasised that there cannot be a definite fineness modulus for each maximum aggregate size, but values within a suitable range (Table III) are likely to give the best results. A series

TABLE III  
Fineness moduli of mixed aggregates for concrete of maximum strength

Maximum size of coarse aggregate (in.)	Fineness moduli		Remarks
	Minimum	Maximum	
$\frac{3}{8}$	3.3	3.7	
$\frac{1}{2}$	4.5	5.0	
$\frac{3}{4}$	4.8	5.3	
1	5.0	5.5	
$1\frac{1}{2}$	5.1	5.7	
$1\frac{1}{2}$	5.4	6.0	
3	5.9	6.5	
6	6.5	7.0	An increase in value above the maximum figures will result in a "harsh" mix. A decrease in value below the minimum figures will give an uneconomical mix.

of experiments carried out in India has shown that for a range of mixes varying from 1:1 $\frac{1}{2}$ :3 to 1:3:6 no appreciable alteration is necessary and that the value of the fineness modulus depends chiefly on the maximum size and shape and on the quality of grading of the aggregates. Reasonable ranges of fineness moduli for fine aggregates and coarse aggregates are given in Table IV. If the fineness modulus of the aggregate tested does not fall within the given range it does not mean that the aggregate should be discarded; it only indicates that the particular aggregate is not likely to give the most economical results. It may also be noted that the coarser the aggregate, the greater is the value of the fineness modulus.

The proportion of fine aggregate to coarse aggregate can be determined from their fineness moduli (F.M.).

TABLE IV  
Recommended limits of fineness modulus of fine and coarse aggregates

Maximum size of aggregate (in.)	Fineness modulus limits	
	Minimum	Maximum
Fine aggregate up to $\frac{3}{16}$	2.34	3.61
Coarse aggregate $\frac{3}{4}$	6.30	6.90
" " $1\frac{1}{2}$	6.95	7.60
" " 3	7.60	8.20
" " 6	8.00	8.60

If  $F_c$  and  $F_f$  are the fineness moduli and if  $V_c$ ,  $V_f$  and  $V_m$  are the percentage of coarse, fine and mixed aggregates respectively, and if  $F_m$  is a suitable fineness modulus for mixed aggregate, then:

$$\frac{V_f}{V_m} = \frac{F_c - F_m}{F_c - F_f} \times 100$$

In order to obtain the fineness moduli of aggregates, sieving may be started with the finest of the sieves, i.e. No. 100, or with the largest of the sieves. If sieving is commenced with the finest sieve, the quantity retained on each sieve is to be simply totalled and divided by 100. From practical considerations, sieving has to be started with the largest sieve. The value of F.M. is obtained either by tabulation or with the help of a formula. In the example given in Table V the quantities retained on each sieve are tabulated in column 3 and the corresponding percentages worked out in column 4. The values in column 4 are multiplied by the sieve factor in column 2 to obtain the cumulative percentage in column 5. The total of column 5 divided by 100 gives the F.M.

TABLE V

Sieve size (1)	Sieve factor (2)	Quantity retained on each sieve (3)	Percentage retained on each sieve (4)	Cumulative percentage on each sieve (5)
$\frac{3}{16}$ in.	6	0.10	5.0	30.0
No. 7	5	0.23	11.5	57.5
No. 14	4	0.30	15.0	60.0
No. 25	3	0.83	41.5	124.5
No. 52	2	0.35	17.5	35.0
		2 lb.	100	316.5 (F.M.=3.17)

The same results may be obtained without tabulation from the following formula:

$$F.M. = \frac{n x_1 + (n-1) x_2 + (n-2) x_3 + E_c \dots + x_n}{w} \quad . . . . \quad (1)$$

where  $n$  is the number of sieves used (six if the size of aggregate is  $3/16$  in., nine for  $1\frac{1}{2}$  in. aggregate, etc.).  $x_1, x_2, \dots, x_n$  are the quantities retained on each sieve,  $x_1$  being the quantity retained on the largest sieve and  $x_n$  that on the smallest sieve, i.e. No. 100, and  $w$  is the total weight of the sample.

The ranges of F.M. in Table IV give the approximate limits of suitable grading of

various aggregates. For more accurate results, the percentage of aggregate retained on each sieve is indicated. In fig. 3 are given the recommended grading ranges for fine aggregate and for coarse aggregates of maximum size  $1\frac{1}{2}$  in. and  $\frac{3}{4}$  in. If the sieve analysis of a given aggregate falls within these ranges, the aggregate is suitable. If the grading is coarser than that given by the lower limits of fig. 3, the mix is likely to be too harsh for good workability; and if it is finer than that indicated by the upper limits, the mix will be uneconomical.

A large quantity of coarse aggregate results in an economic mix, whilst an increased quantity of fine aggregate produces a more workable mix. An economic and at the same time workable range of the mixed aggregate falls within narrow limits. These limits are indicated approximately by the fineness modulus of mixed aggregate and more accurately by grading curves. The average grading curves for mixed aggregates with the maximum size of aggregates varying from  $\frac{3}{8}$  in. to 6 in. are shown in fig. 4. Equivalent fineness moduli are also indicated. The aim in deciding the proportion of fine to coarse aggregate should be to obtain a grading curve as near to the average curve given in fig. 4 for the particular size of coarse aggregate.

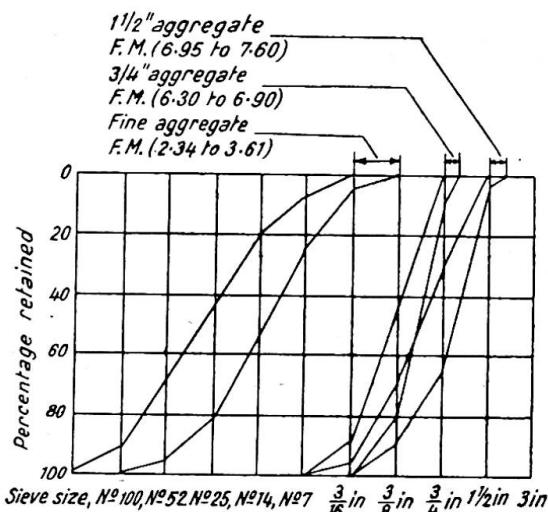


Fig. 3

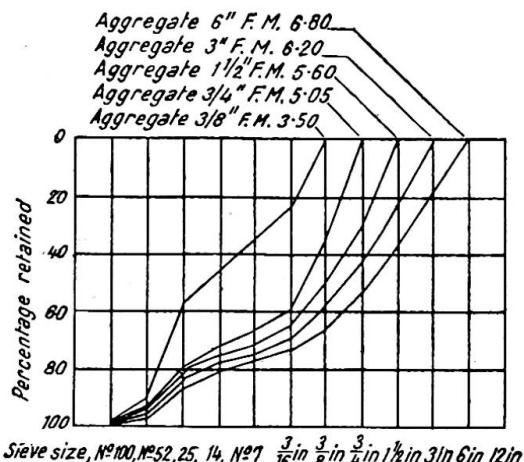


Fig. 4

The simplest method of obtaining the proportion of fine to coarse aggregate is by formula (1). The proportion of fine to coarse aggregate has no effect on the richness or otherwise of the concrete, but satisfactory grading is more important with lean mixes, namely, those with a comparatively small proportion of cement.

With a larger proportion of coarse aggregate, volume change as well as the heat of hydration is reduced. The proportion also depends on the shape of the individual pieces, the proportion of coarse aggregate being reduced with flat elongated pieces. In terms of absolute volume, as defined later in the paper, the quantity of coarse aggregate of  $1\frac{1}{2}$  in. size in a concrete mix with a suitable grading and proportioning, ranges between 35% and 45% for broken or crushed stone and between 40% and 50% for gravel.

#### PROPORTION OF CEMENT TO MIXED AGGREGATE

The relation between the water-cement ratio and the quantities of cement and mixed aggregate is necessary in order to determine the quantity of mixed aggregate per cwt. of cement for a required strength. Experiments have been carried out in

India in which various proportions of coarse and fine aggregates have been used to ascertain the water-cement ratio in various mixes of concrete and of mortar. The results are summarised as follows.

With the consistency of concrete suitable for mass concrete work (see Table II) the quantity of mixing water required is:

- (a) 7.5 lb. per cubic foot of dry compacted mixed aggregate, the proportions being based on the fineness modulus;
- (b) 24 lb. per hundredweight of cement.

In carrying out these experiments, the varying factors have been eliminated by drying the coarse and fine aggregates before use and compacting with a steel rod before measuring any volumes. When dry fine and coarse aggregates are measured separately after compacting and again after mixing the two aggregates and compacting, the volume of the mixed aggregate is less than the sum of the separate volumes. The mixed aggregate factor is obviously equal to :

$$\frac{\text{Volume of dry compacted mixed aggregate}}{\text{Volume of dry compacted fine aggregate} + \text{volume of dry compacted coarse aggregate}}$$

The author suggests the following formula on the strength of the above results:

$$V_m = 1.33 \left( \frac{w}{ab} - 2C \right) \dots \dots \dots \dots \quad (2)$$

where  $w$  is the quantity of water in gallons per cubic foot of cement,

$C$  is the cement in cubic feet,

$V_m$  is the volume of dry compacted mixed aggregate,  
and  $a$  and  $b$  are factors.

Factor  $a$  depends on the quality of both coarse and fine aggregates and the values may be taken as shown in Table VI.

TABLE VI

Value of factor $a$	Conditions
1.2	For crushed-stone aggregate and poorly graded fine aggregate
1.1	For crushed-stone aggregate
1.1	For poorly graded fine aggregate
1.0	For shingle aggregate and well-graded fine aggregate

Factor  $b$  depends on the required consistency and the values recommended are given in Table VII.

TABLE VII

Value of factor $b$	Conditions
1.2	For slump up to 6 in.
1.1	" " " " 4 in.
1.0	" " " " 2 in.

For 1 cubic foot of cement formula (2) becomes:

$$V_m = 1.33 \frac{w}{ab} - 2.67 \dots \dots \dots \dots \quad (3)$$

The quantity of water in gallons per cubic foot of concrete of the required strength having been determined from fig. 1, values of factors  $a$  and  $b$  are selected and the volume of mixed aggregate in cubic feet per cubic foot of cement is obtained from formula (3). The quantities of coarse and fine aggregates are then obtained from the percentages of these materials worked out from the fineness modulus.

#### MOISTURE IN AGGREGATES, BULKING AND YIELD

The percentage of water in the water-cement ratio is affected (a) by water absorbed by the aggregate if the aggregate is very dry, (b) by surface water if the aggregate is wet, and (c) by evaporation if concrete is mixed in hot dry weather. Corrections have therefore to be made for these factors, the quantity of water being increased for (a) and (c), and reduced for (b).

Experience in India indicates a fairly constant percentage for absorption for broken "trap" stone aggregate.

When water is added to dry sand, it covers the particles of sand. These films of water push the sand particles further apart and the volume increases. This is known as bulking. With increase in water, the volume continues to increase until the sand is completely saturated, when the volume of sand is reduced to its original dry volume.

The finer the sand, the greater is the bulking, and this may be as much as 45%. Average figures may be taken as 30% for fine and 20% for coarse sand. Tests carried out in India gave a bulking percentage of about 32% with sand of fineness modulus 2.2. Saturation point was reached when the quantity of water was about 5 lb. per cubic foot of sand.

Bulking also takes place in coarse aggregate but to a smaller extent. Indian experiments show that wet coarse aggregate (broken stone and shingle) when loosely filled bulks about 10% to 12%.

The method of measuring the percentage of bulking is either by volume or by weight. The sand is loosely filled in a volume measure. It is then removed and thoroughly dried, filled again in the volume measure and consolidated by prodding with a rod. The difference in volume gives the bulking percentage. A more accurate measure is by weight. The weights of (a) wet and loosely filled sand, (b) dry and consolidated sand, and (c) water content are obtained. The bulking percentage is then:

$$\frac{M_d - (M_c - M_w)}{(M_c - M_w)} \times 100 \quad \dots \dots \dots \quad (4)$$

where  $M_d$  is weight of dry sand,

$M_c$  is weight of sand combined with water,

$M_w$  is weight of water.

For uniformity and accuracy, all measurements are based on consolidated aggregates.

When certain quantities of cement and coarse and fine aggregates are mixed together, the resultant volume of concrete is much less than the sum of volume of the dry unmixed materials. The reduction in volume depends upon several factors such as the quality of the aggregates, grading, proportions, cement and water content and the extent and type of consolidation. Three methods of ascertaining the yield are given, the choice depending on the degree of accuracy required:

- (i) Very approximate figures of the quantities of ingredients of concrete may be obtained by considering the volume of ingredients as 1.5 times that of the resultant

concrete. For example, the quantities of sand, broken stone and cement required for 100 ft.<sup>3</sup> of 1:2:4 concrete are very approximately:

$$1.5 \times 100 \times \frac{2}{7} = 43 \text{ ft.}^3$$

$$1.5 \times 100 \times \frac{4}{7} = 86 \text{ ft.}^3$$

$$1.5 \times 100 \times \frac{1}{7} \times \frac{90}{112} = 17.2 \text{ bags}$$

(ii) Accurate values are obtained by adopting the absolute volume method. The absolute volume of a material is the volume of the solid particles in it, i.e. it is the total volume minus voids. The quantity of concrete resulting from a mixture of its ingredients is the sum of the absolute volumes of its ingredients. Absolute volume is obtained by multiplying the absolute specific gravity of the material by the weight of a cubic foot of water (62.4 lb.). The absolute specific gravity is the unit weight of the solid material divided by the unit weight of water. In the absence of tests of the materials used, the absolute specific gravity of cement and fine and coarse aggregates may be taken as 3.15, 2.65 and 2.55 respectively.

(iii) Another fairly accurate method of determining the yield is to make use of Table VIII based on the results of a large number of experiments carried out in India.

TABLE VIII

Proportion of cement to mixed aggregate .	1:2	1:3	1:3.5	1:4	1:4.5	1:5	1:5.5	1:6
Cwt. of cement per 100 ft. <sup>3</sup> of concrete .	33	23.5	21	18.5	16.8	15	14	12.8
Proportion of cement to mixed aggregate .	1:6.5	1:7	1:7.5	1:8	1:8.5	1:9	1:9.5	1:10
Cwt. of cement per 100 ft. <sup>3</sup> of concrete .	12	11	10.5	9.8	9.3	8.8	8.3	7.8

In this table the proportion is one cubic foot of cement to the volume of dry compacted mixed aggregate and the table is applicable when formula (3) is used.

#### PROCEDURE FOR PROPORTIONING OF CONCRETE

It will be seen that the concrete mix is economically designed by first selecting the water-cement ratio which will give concrete of required strength and durability, determining a suitable consistency and finding the proportion of coarse and fine aggregates with the available grading to give adequate workability when mixed with the cement paste in definite proportions of water and cement. The steps are therefore as follows:

- Determine a water-cement ratio which will give the required strength (see fig. 1).
- Ascertain whether the ratio selected will be suitable for the type of exposure to which it will be subjected.
- Determine the slump which is required depending on the type of structure or part of structure (see Table II).

- (d) Determine the maximum size of coarse aggregate.
- (e) Ascertain the fineness moduli of the available coarse and fine aggregates (compare with Table IV), select a suitable fineness modulus for the mixed aggregate (see Table III) and work out the proportions of coarse and fine aggregates.
- (f) Determine the mixed aggregate factor.
- (g) Determine the bulking percentage (see formula (4)).
- (h) Determine the quantity of dry compacted mixed aggregate per cubic foot of cement (see formula (3)).
- (i) Determine the field mix and the quantities of aggregates per bag of cement.
- (j) Deduct the moisture content in the aggregates and determine the actual quantity of mixing water to be used.
- (k) Determine the yield of concrete per cwt. of cement (see Table VIII).

An example is given below.

The proportions given in the chart (fig. 5) may be used as a guide for obtaining, without calculations, approximate quantities of the ingredients of concrete for the required compressive strength and consistency. It is emphasised that the figures are approximate and tests with available aggregates may result in figures slightly different from those obtained from the chart.

#### EXAMPLE

Concrete with an ultimate compressive strength of 1,800 lb./in.<sup>2</sup> is required for the interior 9 in. × 9 in. columns with heavy reinforcement. The properties of available sand and shingle aggregate are as follows:

	Fineness modulus	Moisture content lb./ft. <sup>3</sup>	Weight without drying and compacting lb./ft. <sup>3</sup>	Weight after drying and compacting lb./ft. <sup>3</sup>
Sand . . . . .	2.7	3.5	90	101
"Trap" stone . . . . .	6.6	0.5	109	112

A field mix is required for the most economical unvibrated concrete of requisite strength and also the quantities of materials for 100 ft.<sup>3</sup> of concrete:

- (a) The water-cement ratio for 1,800 lb./in.<sup>2</sup> is 0.77 (or 6.95 gal./ft.<sup>3</sup>).
- (b) The limiting water-cement ratios for exposure do not apply, as the columns are in the interior of a building.
- (c) The average slump as per Table II is 4 in.
- (d) The maximum size of aggregate is 1½ in., but ¾ in. size will be used due to heavy reinforcement.
- (e) A suitable fineness modulus for mixed aggregate with ¾ in. maximum size is 5.0 (see Table III). The percentage of sand to be mixed with coarse aggregate is therefore:

$$\frac{F_c - F_m}{F_c - F_f} \times 100 = \frac{6.6 - 5.0}{6.6 - 2.7} \times 100 = 41\%$$

The percentage of coarse aggregate = 100 - 41 = 59%.

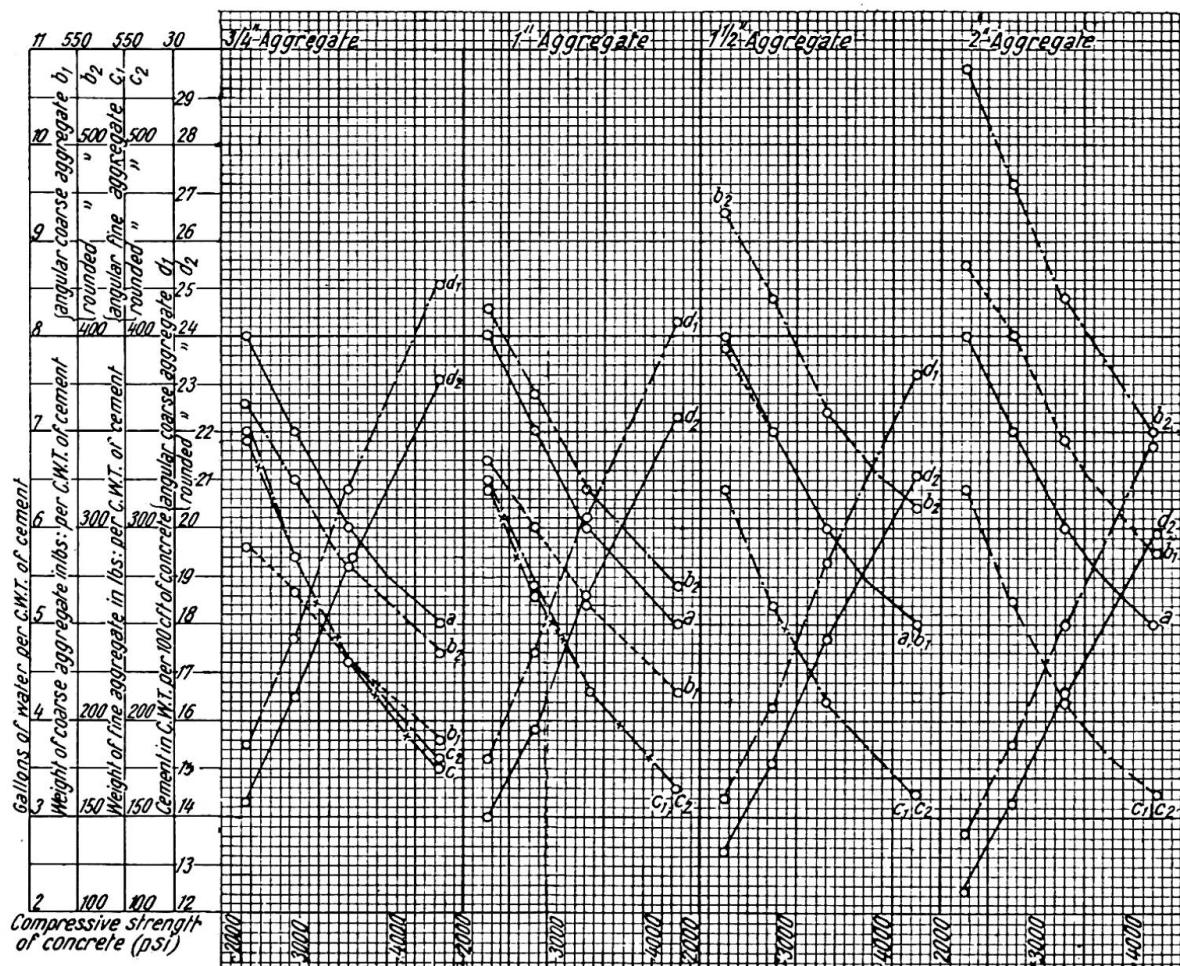


Fig. 5. Proportions for concrete of medium consistency (3 in. slump)

1. (a) For each increase of 1 in. in slump:

- (i) Increase the water by 3.7 gallons/100 ft.<sup>3</sup> of concrete;
- (ii) Increase the cement as follows:

Water-cement ratio	Additional cement for 100 ft. <sup>3</sup> of concrete		
5 gallons/cwt. cement	..	..	9.5 lb.
6 gallons/cwt. cement	..	..	12.5 lb.
7 gallons/cwt. cement	..	..	15.8 lb.
8 gallons/cwt. cement	..	..	19.0 lb.

(iii) Decrease the coarse aggregate by 24 lb./cwt. of cement;

(b) For each decrease of 1 in. slump reverse the modifications shown above.

2. The proportions given in the table are for average aggregates. If very fine aggregate is used the quantity of fine aggregate should be reduced by about 10 lb./cwt. of cement and coarse aggregate increased by the same quantity. For fine aggregates of coarse quality the quantity should be increased and a corresponding reduction made in the coarse aggregate.

3. The approximate weights of fine and coarse aggregates are based on a specific gravity of 2.65 for saturated surface dry condition.

- (f) Quantities of dry compacted sand and coarse aggregate are mixed in proportion of 1:1.44 (i.e. 41:59) and compacted. The volume of the mixed aggregate is found to be 2.24. The compacting factor is therefore:

$$\frac{2.24}{1+1.44} = 0.92$$

- (g) The bulking percentage of sand is:

$$\frac{101-(90-3.5)}{(90-3.5)} \times 100 = 16.8\%$$

The bulking percentage of coarse aggregate is:

$$\frac{112-(109-0.5)}{(109-0.5)} \times 100 = 3.2\%$$

- (h) The quantity of mixed dry aggregate per cubic foot of cement is obtained from:

$$V_m = 1.33 \frac{w}{ab} - 2.67 = \left( 1.33 \times \frac{6.95}{1 \times 1.1} \right) - 2.67 = 5.73$$

- (i) The quantity of sand (in the available condition) required per cubic foot of cement is:

$$\left( \frac{41}{100} \times 5.73 \right) \times \frac{1}{0.92} \times \frac{116.8}{100} = 2.98 \text{ ft.}^3$$

The quantity of coarse aggregate (in the available condition) is:

$$\left( \frac{59}{100} \times 5.73 \right) \times \frac{1}{0.92} \times \frac{103.2}{100} = 3.78 \text{ ft.}^3$$

The field mix is therefore 1:2.98:3.78.

The quantities of sand and coarse aggregate required per bag (1 cwt.) of cement are  $1.2 \times 2.98 = 3.58 \text{ ft.}^3$  and  $1.2 \times 3.78 = 4.54 \text{ ft.}^3$  respectively.

- (j) As the moisture contents of sand and coarse aggregate are 3.5 ft.<sup>3</sup> and 0.5 ft.<sup>3</sup> respectively, the reduction in mixing water per cubic foot of cement is:

$$(3.5 \times 2.98) + (0.5 \times 3.78) = 12.32 \text{ lb.} = 1.23 \text{ gallons}$$

Hence water to be used per cubic foot of cement =  $6.95 - 1.23 = 5.72 \text{ gallons}$  and per bag of cement =  $1.2 \times 5.72 = 6.86 \text{ gallons}$ .

- (k) The quantity of cement (in bags) required per 100 ft.<sup>3</sup> of concrete with the proportion of 1:5.73 is 13.5 bags (see Table VIII). The quantities of sand and coarse aggregates per 100 ft.<sup>3</sup> of concrete are  $13.5 \times 3.58 = 48.3 \text{ ft.}^3$  and  $13.5 \times 4.54 = 61.3 \text{ ft.}^3$  respectively.

Hence with the sand and coarse aggregate available, the field mix will be 1:2.98:3.78 and the following quantities of materials will be required for 100 ft.<sup>3</sup> of concrete: 13.5 bags cement, 92.6 gallons of water, 48.3 ft.<sup>3</sup> of fine and 61.3 ft.<sup>3</sup> of coarse aggregates.

## CONCLUSIONS

Results of Indian research have been used to a very large extent in this paper in explaining rational procedure for proportioning concrete. The experiments so far conducted cannot, however, be considered exhaustive and there is not only considerable scope but also great necessity for undertaking further research on a large scale.

Such research should deal with the various types of aggregates available all over India and also for varying climatic conditions, seasons and even for large temperature variations during the day.

The procedure for proportioning concrete given in the paper may be considered as tentative and further research may necessitate modification.

Preparation of charts and tables covering all the varying conditions from which proportions of the constituent materials of concrete may be read off would be invaluable. Such charts and tables should, however, be prepared only as a result of numerous tests carried out in different parts of the country. Such charts and tables would make rational proportioning as simple as the present unsatisfactory mix method and assist not only in producing concrete of guaranteed strength but also in effecting a large annual saving.

#### Summary

By a rational method of proportioning concrete is meant a method which results in concrete of any required strength at the most economical rate.

The rational proportioning of concrete has considerable bearing on Indian economy as it is estimated that with an anticipated consumption of over three million tons of cement in 1950–51 the saving could be as much as nine million rupees, assuming that 3% cement is saved in proportioning concrete by the rational method as compared with the present approximate method.

The paper indicates briefly the present approximate method of proportioning and describes in detail a method of scientific proportioning based on Indian data. The method is simple and requires little equipment and an example has been given to illustrate the method.

Results of Indian research have been used to a large extent in the paper, but data from other nations, particularly the U.S.A., have been included where Indian data are inadequate. Experiments so far conducted in India cannot be considered as exhaustive and there is considerable scope and necessity for further research in India on this subject.

#### Résumé

Une méthode rationnelle pour l'élaboration des mélanges de béton est une méthode, qui permet d'obtenir des bétons présentant les caractéristiques de résistance voulues, dans les rapports de mélange les plus économiques.

L'élaboration rationnelle des mélanges de béton présente une importance considérable pour l'économie indienne. On estime en effet que sur une consommation prévue de plus de trois millions de tonnes de ciment, pour les années 1950 et 1951, on a pu réaliser une économie de neuf millions de roupies, grâce à l'application de la "méthode rationnelle," qui permet d'économiser en moyenne 3% de ciment par rapport aux autres méthodes actuelles d'approximation.

L'auteur expose brièvement les méthodes actuelles d'approximation et décrit d'une manière détaillée une méthode scientifique de dosage des mélanges, basée sur les conditions économiques particulières à l'Inde. Cette méthode est simple et exige peu de moyens auxiliaires. Elle est illustrée par un exemple.

L'auteur fait très largement appel aux résultats des recherches indiennes et même, lorsqu'ils ne suffisent pas, à ceux des recherches étrangères et tout particulièrement américaines. Les essais qui ont été jusqu'à maintenant effectués en Inde ne doivent nullement être considérés comme achevés; de nombreuses possibilités s'offrent encore et de larges investigations sont encore nécessaires.

**Zusammenfassung**

Unter einer rationellen Methode der Betonmischung versteht man eine Methode, nach welcher Beton von einer gewünschten Festigkeit mit dem wirtschaftlichsten Mischungsverhältnis erzeugt werden kann.

Die rationelle Mischung von Beton hat einen beträchtlichen Einfluss auf die indische Wirtschaft, wenn man abschätzt, dass bei einem voranschlagten Verbrauch von mehr als drei Millionen Tonnen Zement in den Jahren 1950/51 die Ersparnis von neun Millionen Rupees erzielt werden könnte, wenn durch Anwendung der "Rationalen Methode" anstelle der gegenwärtigen Näherungsmethoden durchschnittlich 3% Zement gespart würden.

Die Arbeit erläutert kurz die gegenwärtige Näherungsmethode für die Mischung und beschreibt im Einzelnen eine wissenschaftliche Methode der Dosierung, basierend auf den indischen Verhältnissen. Diese Methode ist einfach und erfordert wenig Hilfsmittel. Zur Erläuterung ist ein Beispiel beigefügt.

Die Ergebnisse der indischen Forschung sind in dieser Arbeit in grossem Umfang benutzt worden doch wo diese nicht genügen, wurden auch Angaben aus anderen Ländern, besonders aus den U.S.A. mit herangezogen. Die bis jetzt in Indien durchgeführten Versuche können keineswegs als erschöpfend betrachtet werden; es bestehen noch beträchtliche Möglichkeiten und weitere Forschungen auf diesem Gebiete sind in Indien noch notwendig.

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# CI 1

## La composition du béton

## The composition of concrete

## Die Zusammensetzung des Betons

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### AVERTISSEMENT

Des milliers d'ouvrages ont été publiés au cours des soixantes dernières années, dans les divers pays du monde, sur la composition du béton.

La multiplicité des théories, des courbes granulométriques et des méthodes proposées montre assez combien le sujet est complexe. Notre propos n'est donc pas de présenter une mise au point définitive de ce problème, mais plutôt un de ses aspects.

Le présent mémoire fait partie d'une étude d'ensemble *théorique et expérimentale* qui doit être publiée dans les Annales de l'Institut Technique du Bâtiment et des Travaux Publics en 1952. Il nous a été impossible de la résumer efficacement et nous avons préféré en extraire un des principes théoriques essentiels.

### L'EFFET DE PAROI ET LA THÉORIE DE M. A. CAQUOT

M. Caquot a exposé dans les Mémoires de la Société des Ingénieurs Civils de France (juillet-août, 1937) une théorie qui a jeté une vive lumière sur les problèmes de la composition du béton. Nous allons la résumer brièvement.

Si  $\beta$  est la compacité d'un agrégat de dimension uniforme  $d$  (pour un serrage donné) une surface fictive  $S$  coupe les grains en milieu indéfini suivant une aire  $\beta S$ . Si les grains sont dans un moule, sa surface intérieure les rencontre suivant une aire nulle. Tout se passe donc comme si le moule était réduit d'un volume  $KSd$  (pour une paroi indéfinie). M. Caquot adopte pour  $\beta$  la valeur 0,56 en moyenne.

### Granulométrie discontinue

Considérons un récipient de volume unité rempli de deux sortes de grains de dimensions respective  $d_1$  et  $d_2$ , très petites par rapport à lui. Soient  $v_1$  et  $v_2$  leurs volumes apparents et posons  $\alpha = d_2/d_1$ . Portons  $v_1$  en ordonnées et  $v_2$  en abscisses

(fig. 1). Si le récipient est rempli de grains de dimension  $d_1$ , le point figuratif est A.  
Ajoutons des grains de dimension  $d_2$ :

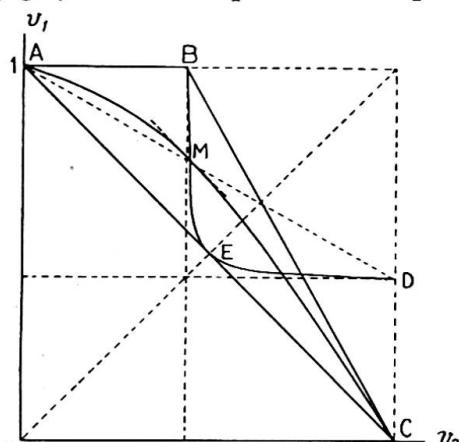


Fig. 1

Si  $\alpha=1$ , le point figuratif décrit le segment AC

Si  $\alpha=0$ , le point figuratif décrit les segments ABC

Si  $\alpha=\infty$ , le point figuratif décrit les segments ADC

Si  $\alpha$  est quelconque, le point figuratif décrit une courbe AMC.

La compacité du mélange est maximum en un point M où la tangente est parallèle à AC.

L'expérience montre que le lieu géométrique du point M est une hyperbole BED d'équations paramétriques:

$$\left. \begin{array}{l} v_1 = a + \frac{b}{\mu} \\ v_2 = a + b\mu \end{array} \right\} \text{avec } \mu = \left[ \frac{\alpha + \mu_0^{1/3}}{1 + \mu_0^{1/3}\alpha} \right]^{1/3}$$

Les constantes  $a$ ,  $b$  et  $\mu_0$  se déterminent aux points B et E. On en déduit facilement le Tableau I.

TABLEAU I

$\alpha =$	1/2	1/4	1/8	1/16	1/64	1/256	1/512	1/1024	1/4096
$v_1$	0,566	0,667	0,776	0,862	0,965	0,990	0,996	0,998	0,999
$v_2$	0,467	0,452	0,445	0,443	0,441	0,440	0,440	0,440	0,440
$v_1 + v_2 - 0,44$	0,593	0,679	0,781	0,865	0,966	0,990	0,996	0,998	0,999

On voit qu'en première approximation, on peut prendre:

$$\left\{ \begin{array}{l} v_2 = 0,44 = 1 - \beta \text{ (ce qui se voit bien sur la figure 1)} \\ v_1 = v_1 + v_2 - 0,44 \end{array} \right.$$

tout en conservant la véritable valeur de la somme  $v_1 + v_2$ , ce qui est l'essentiel.

Ainsi, dans le cas de la compacité maximum les deux sortes de grains supportent un effet de paroi mutuel, mais tout se passe comme si cet effet ne portait *que sur les gros grains* dont le volume est réduit par le facteur  $v_1 + v_2 - 0,44$ . Par exemple, pour  $\alpha = 1/4$  on a pour les *volumes absolus* des deux sortes de grains:

$$\begin{aligned} \sigma_2 &= 0,44 \times 0,56 = 0,246 \\ \sigma_1 &= 0,679 \times 0,56 = 0,380 \end{aligned}$$

$$\underline{\sigma_1 + \sigma_2 =} \quad \underline{0,626} \quad \text{vide} = 0,374$$

Pour trois sortes, on a:

$$\sigma_3 = 0,374 \times 0,56 = 0,209$$

$$\sigma_2 = 0,679 \times 0,246 = 0,167$$

$$\sigma_1 = 0,865 \times 0,380 = 0,329$$

$$\underline{\sigma_1 + \sigma_2 + \sigma_3 =} \quad \underline{0,705} \quad \text{vide} = 0,295, \text{ etc.}$$

On peut ainsi tracer les courbes représentatives des vides en fonction des dimensions extrêmes des grains pour différentes valeurs de  $\alpha$  (fig. 2). On voit que toutes ces courbes admettent sensiblement pour enveloppe inférieure la courbe d'équation  $0,47 (D/d)^{-1/5}$ .

#### Granulométrie continue et indéfinie

L'expérience montre que le mélange de plusieurs ensembles de granulométrie quelconque aboutit à un ensemble dont le volume des vides est au plus égal à la somme des vides des ensembles constituants.

Si l'on sépare les grains d'un ensemble continu par une série de passoires dont les ouvertures sont en progression géométrique, par exemple de rayon  $100,3 \approx 2$ , et si on leur affecte les indices 1, 2, 3, 4, 5 . . . , on peut dire que cet ensemble est un mélange de quatre ensembles discontinus par exemple:

$$\begin{aligned} n, n+4, n+8, \dots \\ n+1, n+5, n+9, \dots \\ n+2, n+6, n+10, \dots \\ n+3, n+7, n+11, \dots \end{aligned}$$

dont les rapports de dimension sont  $\alpha = 1/16$ .

Le vide minimum vers lequel tend naturellement l'ensemble est proportionnel à  $(D/d)^{-1/5}$ .

Il en résulte que si  $V_n$  est un volume très grand contenant des grains jusqu'à l'indice  $n$  inclusivement, et  $V_{n+1}$  le volume contenant des grains jusqu'à l'indice  $n+1$  et ayant le même vide que  $V_n$ , on a:

$$\frac{\text{Vide de } V_n}{V_n} = \frac{\text{Vide de } V_{n+1}}{V_{n+1}} \times 2^{1/5}$$

D'où les expressions suivantes:

$$\begin{aligned} V_{n+1} &= V_n \times 2^{1/5} = V_n \times 1,149 \\ V_{n+2} &= V_n \times 2^{2/5} = V_n \times 1,320 \\ V_{n+3} &= V_n \times 2^{3/5} = V_n \times 1,516 \\ V_{n+4} &= V_n \times 2^{4/5} = V_n \times 1,741 \\ V_{n+5} &= V_n \times 2^{5/5} = V_n \times 2 \\ V_{n+6} &= V_n \times 2^{6/5} = V_n \times 2,297, \text{ etc.} \end{aligned}$$

D'où l'on tire:

$$\text{Volume absolu des grains d'indice } n+1 = 0,149 V_n$$

$$\begin{array}{llllll} " & " & " & " & " & n+2 = 2^{1/5} \times 0,149 V_n \\ " & " & " & " & " & n+3 = 2^{2/5} \times 0,149 V_n \\ " & " & " & " & " & n+4 = 2^{3/5} \times 0,149 V_n \\ " & " & " & " & " & n+5 = 2^{4/5} \times 0,149 V_n \\ " & " & " & " & " & n+6 = 2 \times 0,149 V_n \end{array}$$

#### Granulométrie continue et limitée—Correction pour les plus gros grains

##### (a) Béton en masse indéfinie

Prenons  $V_n = 1$  et soit  $\rho$  le rayon moyen des grains d'indice  $n$ , c'est-à-dire que:

$$\rho = \frac{\text{volume des grains d'indice } n}{\text{surface des grains d'indice } n}$$

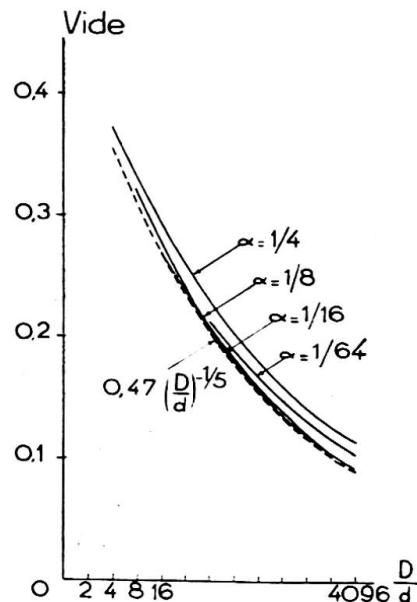


Fig. 2

Pour une granulométrie indéfinie, le volume des grains d'indice  $n+1$  serait 0,149 et leur rayon moyen serait  $2\rho$ .

La surface des grains d'indice  $n+1$  serait donc  $\frac{0,149}{2\rho}$

La surface des grains d'indice  $n+2$  serait  $\frac{2^{1/5} \times 0,149}{2^2 \rho}$

La surface des grains d'indice  $n+3$  serait  $\frac{2^{2/5} \times 0,149}{2^3 \rho}$ , etc.

La surface totale des grains d'indice  $>n$  (somme d'une progression géométrique illimitée) serait :

$$\frac{\frac{0,149}{2\rho}}{1 - \frac{2^{1/5}}{2}} = \frac{1}{5,7\rho}$$

et le rayon moyen de leurs intervalles (de volume unité puisque  $V_n=1$ ) serait donc par définition :  $5,7\rho$ .

Supposons maintenant que  $n$  soit l'indice de la dernière sorte de grains (les plus gros) de l'ensemble. Cette dernière sorte de grains a pour volume absolu  $V_n - V_{n-1}$ .

Posons

$$V_n - V_{n-1} = xV_{n-1}$$

La surface de la dernière sorte de grains est par définition  $(xV_{n-1})/\rho$ . Et le rayon moyen des intervalles de cette dernière sorte est par définition  $\rho/x$ .

Or le rayon moyen de l'avant dernière sorte de grains est  $\rho/2$ . Il faut donc que  $\rho/x = 5,7 \times \rho/2$ . D'où  $x=0,35$  au lieu de 0,149 pour une granulométrie indéfinie. On voit donc que la proportion de la dernière sorte de grains doit être plus forte que pour la granulométrie indéfinie.

### (b) Effet de paroi des coffrages

Posons encore

$$V_n - V_{n-1} = xV_{n-1}$$

La surface des intervalles formés par la dernière sorte de grains est  $(xV_{n-1})/\rho$ .

La surface des coffrages est  $\frac{V_n}{R} = \frac{V_{n-1}(1+x)}{R}$

On doit donc avoir :

$$\frac{xV_{n-1}}{\rho} + \frac{V_{n-1}(1+x)}{R} = \frac{0,35 V_{n-1}}{\rho}$$

D'où

$$x = \frac{0,35 - \rho/R}{1 + \rho/R}$$

Cette formule permet de calculer la proportion de la dernière sorte de grains pour un rayon moyen donné des coffrages.

Comme  $r$ , rayon moyen des grains les plus gros  $= \rho\sqrt{2}$ , on en déduit facilement le Tableau II.

M. Caquot considère que pour un vide 100, la limite de la zone du liant ressort à 0,4 mm. correspondant à un volume 200.

TABLEAU II

$\rho/R$	0,175	0,140	0,105	0,070	0,035	0
$r/R$	0,25	0,20	0,15	0,10	0,05	0
$x$	0,149	0,184	0,222	0,262	0,304	0,35

Le tracé de ses courbes granulométriques est donc représenté par la figure 3.  
Exemple: Moule cubique de 140 mm. d'arête; agrégat roulé  $D \approx 12,5$  mm.

$$R = \frac{(140)^3}{6 \times (140)^2} = 23,3 \text{ mm.}$$

$$r \approx \frac{\pi(12,5)^3}{\pi \times 12,5^2} = 2,1 \text{ mm.}$$

$$\frac{r}{R} = \frac{2,08}{23,3} = 0,09.$$

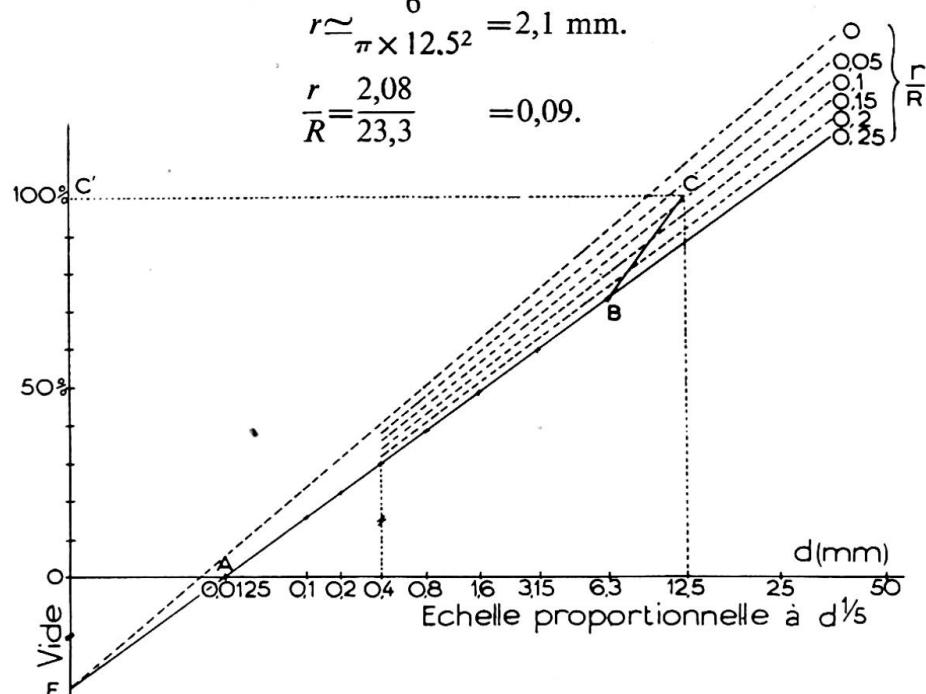


Fig. 3. Volume absolu de matériaux solides passant à travers les passoires

La courbe granulométrique est formée par les deux segments de droite ABC. On en déduit facilement les proportions des diverses classes de grains:

		%
0-0,1 mm.	.	16
0,1-0,2 mm.	.	7
0,2-0,4 mm.	.	7
0,4-0,8 mm.	.	8
0,8-1,6 mm.	.	11
1,6-3,15 mm.	.	11
3,15-6,3 mm.	.	13
6,3-12,5 mm.	.	27
		<hr/> 100

On a aussi le dosage de l'eau:  $OE/EC' = 227 \text{ litres/m}^3$

## PRINCIPE DE LA COMPACITÉ MAXIMUM

Pour obtenir une courbe granulométrique de référence, M. Caquot a été conduit à adopter un nombre fixant la compacité d'un ensemble de grains solides de même grosseur. Il a choisi 0,56 qui est la compacité moyenne qu'on obtient habituellement, mais on trouve expérimentalement suivant les formes de grains et aussi suivant le serrage adopté des compacités sensiblement différentes: La compacité maximum de billes sphériques identiques est  $\frac{\pi}{3\sqrt{2}}=0,74$  et pratiquement des grains d'agrégat de même grosseur atteignent des compacités de 0,65 grâce à certaines vibrations sous pression.

Par contre, la compacité de certains ensembles de grosseur uniforme peut être assez faible, dans le cas de serrage par coulage par exemple, surtout si la forme des grains est défectueuse, s'ils comportent des plaquettes ou des aiguilles, ou s'ils sont petits.

Il convient donc d'envisager les conséquences des écarts de compacité sur l'échelle des abscisses qui doit être adoptée dans chaque cas pour que la courbe granulométrique reste linéaire. Pour cela, nous étudierons quelques valeurs de la compacité  $\beta$  d'un ensemble de grains de même grosseur.

1er cas  $\beta=0,52$

L'hyperbole est évidemment toujours la courbe la plus simple qu'on puisse adopter comme lieu du point M (fig. 1) correspondant au maximum de compacité.

Posons comme l'a fait M. Caquot:

$$\left. \begin{array}{l} v_1 = a + \frac{b}{\mu} \\ v_2 = a + b\mu \end{array} \right\} \text{avec } \mu = \left[ \frac{\alpha + \mu_0^{1/3}}{1 + \mu_0^{1/3}\alpha} \right]^3$$

Il est clair que la forme de cette fonction  $\mu$  n'interviendra pas dans le résultat final; nous cherchons en effet l'enveloppe de toutes les courbes telles que celles de la figure 2, c'est-à-dire des courbes correspondant à toutes les valeurs de  $\mu$ , ou à toutes les valeurs de  $\alpha$ . La fonction  $\mu$  n'intervient que dans la répartition de ces courbes, mais l'enveloppe reste la même.

On détermine facilement les constantes  $a$ ,  $b$ ,  $\mu_0$  aux points B et E (fig. 1):  $a=0,479$ ;  $b=0,021$ ;  $\mu_0=0,04$ . Et l'on en déduit le Tableau III:

TABLEAU III

$\alpha =$	1/2	1/4	1/8	1/16	1/32	1/64	1/128	1/256	1/512	1/1024	1/2048	1/4096
$v_1$	0,535	0,608	0,711	0,815	0,896	0,942	0,970	0,984	0,993	0,996	0,998	1,000
$v_2$	0,487	0,482	0,481	0,480	0,480	0,480	0,480	0,480	0,480	0,480	0,480	0,480
$v_1 + v_2 - 0,48$	0,542	0,610	0,712	0,815	0,896	0,942	0,970	0,984	0,993	0,996	0,998	1,000

En prenant comme précédemment:

$$v_2 = 0,48 \text{ quel que soit } \alpha$$

et  $v_1 = v_2 - 0,48$

on peut calculer les vides d'ensembles discontinus pour différentes valeurs de  $\alpha$ , comme il a été indiqué précédemment; on obtient le Tableau IV:

TABLEAU IV

$\alpha$	Nombres de sortes de grains						
	1	2	3	4	5	6	7
1	0,48	0,48	0,48	0,48	0,48	0,48	0,48
1/2	0,48	0,468	0,449	0,430	0,411	0,394	0,377
1/4	0,48	0,433	0,364	0,307	0,256	0,215	0,179
1/8	0,48	0,380	0,276	0,205	0,156		
1/16	0,48	0,326	0,208	0,134			
1/32	0,48	0,284	0,162	0,083			
1/64	0,48	0,260	0,140				
1/128	0,48	0,243	0,123				
1/256	0,48	0,237					

On peut ainsi construire les courbes représentatives des vides *en fonction des dimensions extrêmes* des grains pour les diverses valeurs de  $\alpha$  (fig. 4).

On trouve que toutes ces courbes admettent pour enveloppe:

$$0,504(D/d)^{-1/6} = \frac{0,504}{(D/d)^{0,167}}$$

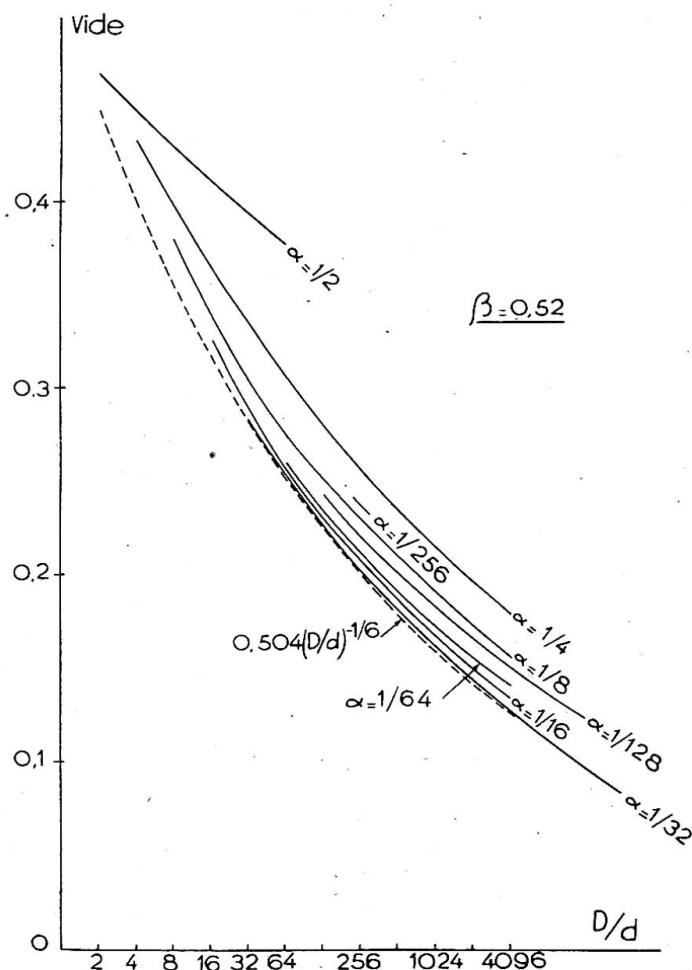


Fig. 4

2ème cas  $\beta=0,56$

On trouve:  $a=0,432$ ;  $b=0,0682$ ;  $\mu_0=0,12$ .

Les valeurs de  $v_1$  et  $v_2$  en fonction de  $\alpha$  ont été données précédemment.

On en déduit les vides d'ensembles discontinus pour différentes valeurs de  $\alpha$  du Tableau V.

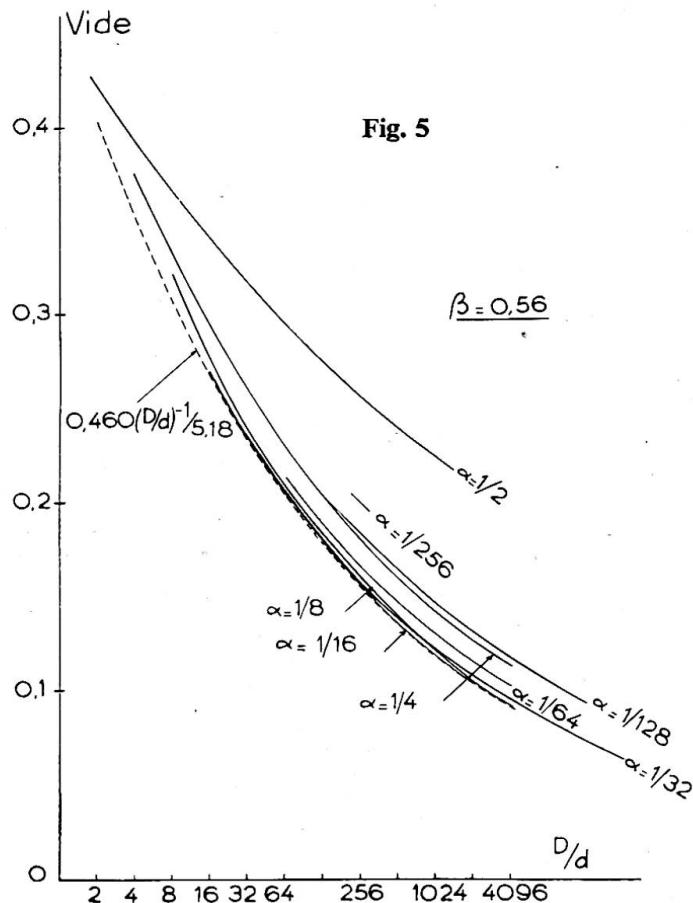
TABLEAU V

$\alpha$	Nombres de sortes de grains						
	1	2	3	4	5	6	7
1	0,44	0,44	0,44	0,44	0,44	0,44	0,44
1/2	0,44	0,422	0,392	0,364	0,341	0,317	0,295
1/4	0,44	0,374	0,295	0,230	0,181	0,142	0,114
1/8	0,44	0,322	0,208	0,139	0,092		
1/16	0,44	0,270	0,157	0,092			
1/32	0,44	0,235	0,122	0,064			
1/64	0,44	0,213	0,103				
1/128	0,44	0,204	0,094				
1/256	0,44	0,200					

On peut ainsi tracer les courbes représentatives de ces vides en fonction des dimensions extrêmes des grains (fig. 5).

On trouve que toutes ces courbes admettent pour enveloppe:

$$0,460 (D/d)^{-1/5,18} = \frac{0,460}{(D/d)^{0,193}}$$



3<sup>ème</sup> cas  $\beta=0,60$

On trouve:  $a=0,375$ ;  $b=0,125$ ;  $\mu_0=0,2$ . On en déduit les valeurs de  $v_1$  et  $v_2$  (Tableau VI).

TABLEAU VI

$\alpha =$	1/2	1/4	1/8	1/16	1/32	1/64	1/128	1/256	1/512	1/1024	1/2048	1/4096
$v_1$	0,586	0,697	0,806	0,885	0,940	0,967	0,985	0,991	0,994	0,997	1,000	1,000
$v_2$	0,449	0,423	0,411	0,406	0,403	0,401	0,401	0,400	0,400	0,400	0,400	0,400
$v_1 + v_2 - 0,40$	0,635	0,720	0,817	0,891	0,943	0,968	0,986	0,991	0,994	0,997	1,000	1,000

En prenant:

$$\begin{aligned}v_2 &= 0,40 \\ \text{et } v_1 &= v_1 + v_2 - 0,40\end{aligned}$$

on peut calculer les vides d'ensembles discontinus pour différentes valeurs de  $\alpha$  du Tableau VII.

TABLEAU VII

$\alpha$	Nombres de sortes de grains						
	1	2	3	4	5	6	7
1	0,400	0,400	0,400	0,400	0,400	0,400	0,400
1/2	0,400	0,379	0,347	0,315	0,286	0,260	0,238
1/4	0,400	0,328	0,246	0,184	0,139	0,105	0,078
1/8	0,400	0,270	0,168	0,106	0,065		
1/16	0,400	0,226	0,121	0,065			
1/32	0,400	0,194	0,094	0,045			
1/64	0,400	0,179	0,080				
1/128	0,400	0,168	0,070				
1/256	0,400	0,165					

On peut ainsi tracer les courbes représentatives de ces vides en fonction des dimensions extrêmes des grains (fig. 6).

On trouve que toutes ces courbes admettent pour enveloppe:

$$0,416 (D/d)^{-1/4,5} = \frac{0,416}{(D/d)^{0,222}}$$

La récapitulation des calculs précédents fournit le Tableau VIII:

TABLEAU VIII

Compacité $\beta$ d'un ensemble de grains de même grosseur	0,52	0,56	0,60
Equation de la courbe représentative du vide en fonction du rapport des dimensions extrêmes des grains	$0,504 (D/d)^{0,167}$	$0,460 (D/d)^{0,193}$	$0,416 (D/d)^{0,222}$

On vérifie que pour une granulométrie uniforme, la somme  $\beta + \text{vide}$  est indépendante de  $\beta$  et voisine de 1 :

Compacité $\beta$	:	0,52	0,56	0,60
$\beta + \text{vide}$	:	1,024	1,020	1,016

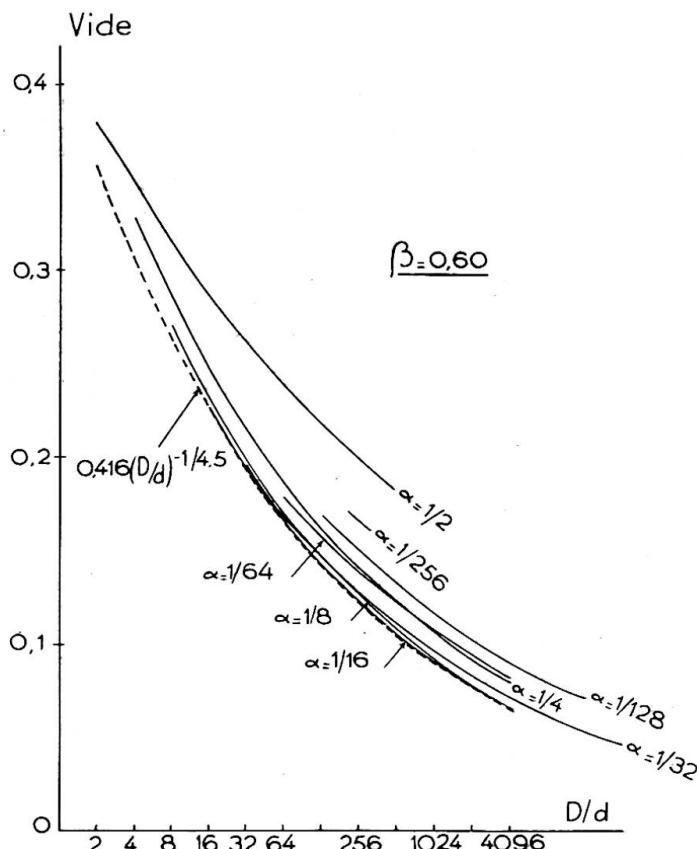


Fig. 6

Si l'on porte sur un graphique (fig. 7) en abscisse  $\beta$  et en ordonnée l'exposant de  $D/d$  figurant dans l'équation de la courbe représentative du vide en fonction du rapport  $D/d$ , on voit que les points obtenus définissent une courbe régulière. Ainsi, la courbe granulométrique idéale de l'agrégat est une droite dans un graphique où les abscisses sont proportionnelles à  $d^m$  (fig. 8).

Cet exposant  $m$  dépend de la compacité  $\beta$  à laquelle se tasse un agrégat de granulométrie uniforme en milieu indéfini.

#### *Correction pour les plus gros grains*

##### (a) Béton en masse indéfinie

Le principe du calcul qu'a fait M. Caquot pour des abscisses proportionnelles à  $d^{1/5}$  est toujours valable. Si  $V_n=1$  et si  $\rho$  est le rayon moyen des grains d'indice  $n$ , la surface totale des grains d'indice  $>n$  serait encore :

$$\frac{\frac{2^m - 1}{2\rho}}{1 - \frac{2^m}{2}} = \frac{2^m - 1}{(2 - 2^m)\rho}$$

et le rayon moyen de leurs intervalles:  $\frac{2-2^m}{2^m-1} \rho$ .

Si  $n$  est l'indice de la dernière sorte de grains (les plus gros) cette dernière sorte a pour volume absolu:

$$V_n - V_{n-1} = x V_{n-1}$$

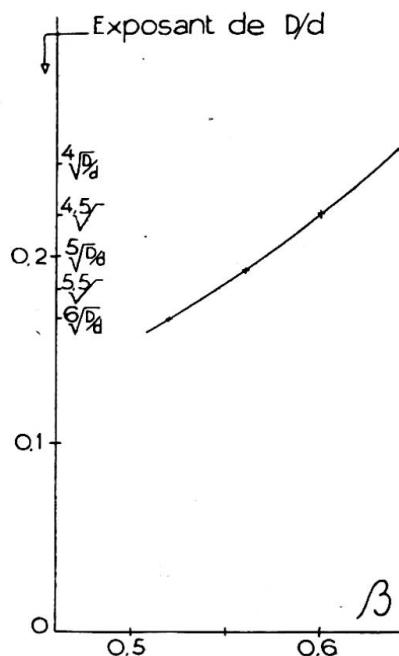


Fig. 7

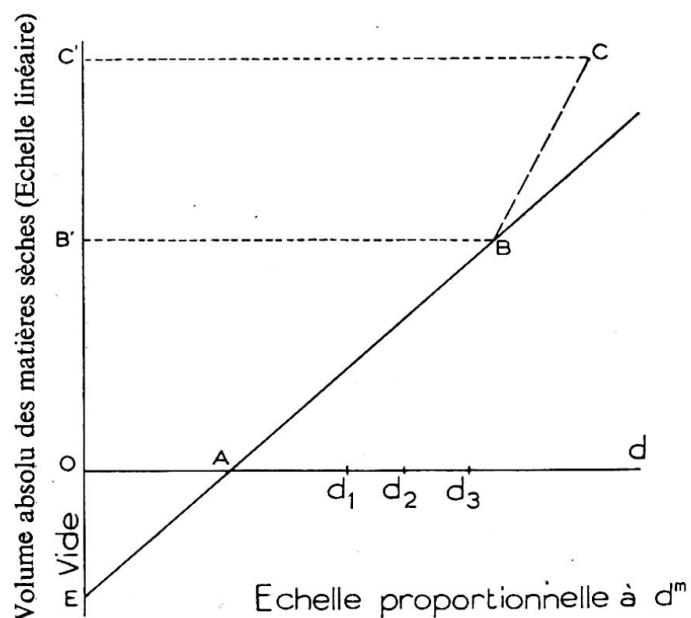


Fig. 8

La surface de la dernière sorte de grains est par définition  $(x V_{n-1})/\rho$ . Et le rayon moyen des intervalles de cette dernière sorte est par définition  $\rho/x$ .

Or le rayon moyen de l'avant-dernière sorte de grains est  $\rho/2$ .

Il faut donc que

$$\frac{\rho}{x} = \frac{2-2^m}{2^m-1} \frac{\rho}{2}$$

D'où

$$x = \frac{B'C'}{EB'} = \frac{2^m-1}{1-2^{m-1}}$$

On obtient donc  $x$  en fonction de  $m$ :

$m$	1/6	1/5	1/4
$x$	0,278	0,350	0,464

### (b) Effet de paroi des coffrages

Comme précédemment, on a:

$$x = \frac{B'C'}{EB'} = \frac{\frac{2^m-1}{1-2^{m-1}} - \frac{\rho}{R}}{1 + \frac{\rho}{R}}$$

Comme  $r$  (rayon moyen des grains les plus gros) =  $\rho\sqrt{2}$ , on en déduit facilement le Tableau IX des valeurs de  $x$  qui fixent la proportion de la dernière sorte de grains:

TABLEAU IX

$\rho/R$	0,175	0,140	0,105	0,070	0,035	0
$r/R$	0,25	0,20	0,15	0,10	0,05	0
$x$	$m=1/6$ 0,088	$m=1/5$ 0,149	$m=1/4$ 0,246	0,121 0,184 0,284	0,157 0,222 0,325	0,194 0,262 0,368

La figure 9 représente les valeurs de  $x$  pour les valeurs de  $r/R$  de 0 à 0,25, en fonction de l'exposant  $m$ , ou de la compacité  $\beta$  d'un agrégat de granulométrie uniforme.

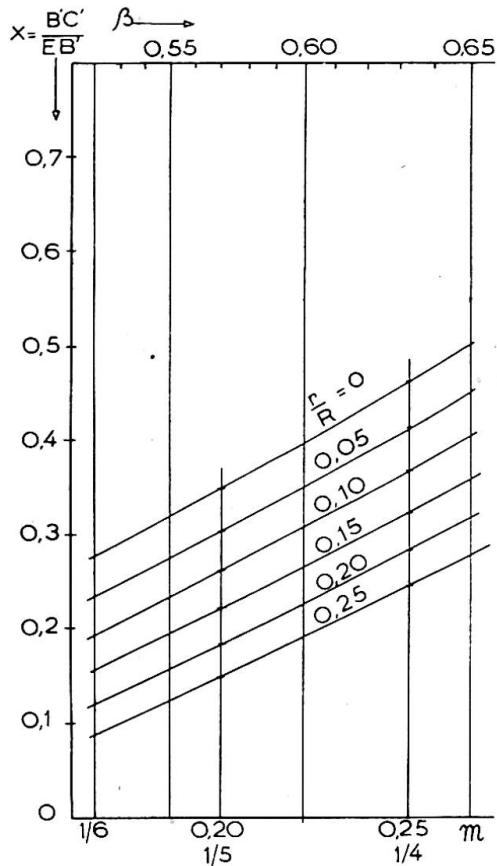


Fig. 9

l'exposant  $m_p, p+1$  doit être diminué et l'échelle de l'intervalle ( $d_p, d_{p+1}$ ) augmentée. Il en résulte que la proportion de cet agrégat doit aussi être augmentée puisqu'elle est toujours prise sur des ordonnées linéaires.

#### Résumé

La partie moyenne de la courbe granulométrique d'un béton correspondant à la compacité maximum est une droite dans un graphique où les ordonnées sont normales, et où les abscisses sont proportionnelles à  $d^m$  si  $d$  est l'ouverture des passoires.

L'exposant  $m$  est de l'ordre de 1/5 comme l'a prévu M. Caquot; il peut varier de 1/6 à 1/4 environ en fonction de la forme des grains et du mode de serrage.

La proportion des plus gros grains doit être calculée en fonction du coffrage.

Si le point de rencontre A de la courbe granulométrique et de l'axe des abscisses (fig.8) est fixé, le dosage de l'eau (représenté par le rapport  $OE/EC'$  en volume par mètre cube de béton) est d'autant plus faible que l'échelle est plus grande, c'est-à-dire que  $m$  est plus grand, ou que  $\beta$  est plus grand ce qui était facile à prévoir. On peut donc diminuer le dosage de l'eau en choisissant un agrégat qui se tasse facilement (grains bien arrondis) et un moyen de serrage puissant qui favorise l'orientation des grains (vibration, pression, etc.).

#### CONCLUSION

Si l'on cherche la compacité maximum d'un béton, et si l'agrégat est un mélange de diverses classes dont chacune a une forme et une facilité de serrage propre, chacune doit correspondre à une échelle particulière: si ces agrégats sont séparés par  $d_1, d_2, d_3$ , etc. (fig. 8), à l'agrégat  $d_1/d_2$  doit correspondre une échelle d'exposant  $m_{1, 2}$ ; à l'agrégat  $d_2/d_3$  une échelle d'exposant  $m_{2, 3}$ , etc.

Si la vibration a pour effet d'augmenter surtout la compacité de l'agrégat  $d_p/d_{p+1}$ ,

**Summary**

The middle part of the granulometric curve corresponding to a concrete of maximum density is graphically represented by a straight line if the abscissa  $d^m$  is chosen as the vertical ordinate,  $d$  representing the size of the sieve openings.

The order of magnitude of the exponent  $m$  is  $1/5$ , as M. Caquot has foreseen. It can vary between  $1/6$  and  $1/4$ , depending on the shape of grain and the manner of compaction.

The proportion of the largest particles must be calculated having regard to the shutting.

**Zusammenfassung**

Der mittlere Teil der Kornzusammensetzung-Kurve eines Betons mit grösster Dichte ist in der graphischen Darstellung eine Gerade, wenn bei senkrechten Ordinaten die Abszisse  $d^m$  gewählt wird, wobei  $d$  die Grösse der Sieböffnungen bedeutet.

Der Exponent  $m$  ist von der Größenordnung  $1/5$ , wie es M. Caquot vorgesehen hatte. Er kann zwischen  $1/6$  und  $1/4$  variiieren, je nach Kornform und Verdichtungsart.

Das Verhältnis der grössten Kornkomponente muss in Zusammenhang mit der Schalung berechnet werden.

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