Analysis of the Nielsen System Bridge by digital computer

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Analysis of the Nielsen System Bridge by Digital Computer

Calcul des ponts Nielsen ou de type analogue à l'aide d'une calculatrice

Berechnung von Brückenträgern nach dem Nielsen-System mit Hilfe von digitalen Rechengeräten

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Introduction

It has been said that in the tied arch (Langer girder) bridge with inclined hangers, the truss action of the inclined hangers reduces the bending moment of the arch (girder), and that such bridges are more economical than the usual types of bridge with vertical hangers. Many Nielsen System bridges have been erected in Sweden, and there are papers dealing with the analysis of the system by the force method. However, no bridge on this system has yet been constructed in Japan and little work has been done in connection with the system in that country. The recent construction of the Fehmarnsundbrücke in Germany induced the authors to initiate analytical work on the Nielsen System bridge by the displacement method. This paper describes the analytical solution, its programming and application, and the model test. The main reasons why the displacement method was used are as follows:

1. It is simpler than the force method for purposes of analysis.
2. The mechanical tabulation of the stiffness matrix is possible and is more convenient for use with a digital computer.
3. It is possible to use the same analytical procedure not only for Nielsen System bridges, but also for similar bridges with vertical hangers.

Part I. Analysis by the Displacement Method

The fundamental equation in the displacement method for the member ij of a plane frame is expressed by Eq. (1) in Cartesian coordinates (Fig. 1),

$$\begin{aligned}\mathfrak{P}_{ij} = &-\frac{12\,E\,I_{ij}}{l_{ij}^3}\frac{(y_j-y_i)}{l_{ij}}\left\{\frac{(y_j-y_i)}{l_{ij}}(u_j-u_i)-\frac{(x_j-x_i)}{l_{ij}}(v_j-v_i)\right\}\\ &-\frac{6\,E\,I_{ij}}{l_{ij}^2}\frac{(y_j-y_i)}{l_{ij}}(\theta_j+\theta_i)\\ &-\frac{E\,A_{ij}}{l_{ij}}\frac{(x_j-x_i)}{l_{ij}}\left\{\frac{(x_j-x_i)}{l_{ij}}(u_j-u_i)+\frac{(y_j-y_i)}{l_{ij}}(v_j-v_i)\right\},\end{aligned} \tag{1}$$

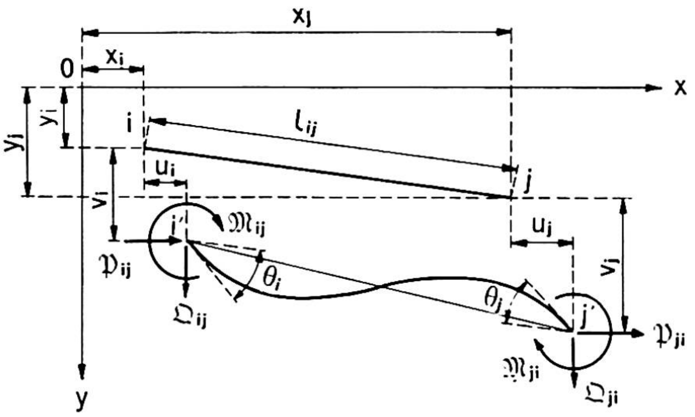

Fig. 1.

$$
\begin{aligned}
\mathfrak{Q}_{ij} = \; & \frac{12\,E\,I_{ij}}{l_{ij}^3}\frac{(x_j-x_i)}{l_{ij}}\left\{\frac{(y_j-y_i)}{l_{ij}}(u_j-u_i)-\frac{(x_j-x_i)}{l_{ij}}(v_j-v_i)\right\} \\
& +\frac{6\,E\,I_{ij}}{l_{ij}^2}\frac{(x_j-x_i)}{l_{ij}}(\theta_j+\theta_i) \\
& -\frac{E\,A_{ij}}{l_{ij}}\frac{(y_j-y_i)}{l_{ij}}\left\{\frac{(x_j-x_i)}{l_{ij}}(u_j-u_i)+\frac{(y_j-y_i)}{l_{ij}}(v_j-v_i)\right\}, \\
\mathfrak{M}_{ij} = \; & \frac{6\,E\,I_{ij}}{l_{ij}^2}\left\{\frac{(y_j-y_i)}{l_{ij}}(u_j-u_i)-\frac{(x_j-x_i)}{l_{ij}}(v_j-v_i)\right\}+\frac{2\,E\,I_{ij}}{l_{ij}}(\theta_j+2\,\theta_i).
\end{aligned} \tag{1}
$$

where,

$\mathfrak{P}_{ij}$, $\mathfrak{Q}_{ij}$, and $\mathfrak{M}_{ij}$: are the components of the force acting on the end i of the member ij in the direction of the x-axis and the y-axis, and the moment at the same point, respectively;

u_i, v_i and θ_i: are the displacement of the end i of the member ij in the direction of the x- and y-axes, and the rotation of the tangent at the end i of the elastic curve of the member ij, respectively,

$E\,I_{ij}$: flexural rigidity of the member ij,

$E\,A_{ij}$: extensional rigidity of the member ij,

l_{ij}: length of the member ij.

Substituting Eq. (1) in the equilibrium equations at the panel point i we have the following Eq. (2):

$$
\begin{aligned}
& [\{\ \sum(a_{ij})\}\,u_i-\sum(a_{ij})\,(u_j)]+[\{\sum(b_{ij})\}\,v_i-\sum(b_{ij})\,(v_j)] \\
& \qquad +[\{-\sum(c_{ij})\}\,\theta_i-\sum(c_{ij})\,(\theta_j)] = \overline{P}_i, \\
& [\{\ \sum(b_{ij})\}\,u_i-\sum(b_{ij})\,(u_j)]+[\{\sum(\bar{a}_{ij})\}\,v_i-\sum(\bar{a}_{ij})\,(v_j)] \\
& \qquad +[\{\ \sum(\bar{c}_{ij})\}\,\theta_i+\sum(\bar{c}_{ij})\,(\theta_j)] = \overline{Q}_i, \\
& [\{-\sum(c_{ij})\}\,u_i+\sum(c_{ij})\,(u_j)]+[\{\sum(\bar{c}_{ij})\}\,v_i-\sum(\bar{c}_{ij})\,(v_j)] \\
& \qquad +[\{\ 2\sum(d_{ij})\}\,\theta_i+\sum(d_{ij})\,(\theta_j)] = \overline{M}_i.
\end{aligned} \tag{2}
$$

where, $\overline{P}_i$, $\overline{Q}_i$ and $\overline{M}_i$: are the components of the external force applied to the panel point i in the direction of the x- and y-axes, and the external moment applied to the panel point i, respectively

$a_{ij} \sim d_{ij}$: the coefficients calculated by the following Eq. (3):

$$
\begin{aligned}
a_{ij} &= \frac{12\,E\,I_{ij}}{l_{ij}^3}\,\frac{(y_j-y_i)^2}{l_{ij}^2} + \frac{E\,A_{ij}}{l_{ij}}\,\frac{(x_j-x_i)^2}{l_{ij}^2}, \\
\bar{a}_{ij} &= \frac{12\,E\,I_{ij}}{l_{ij}^3}\,\frac{(x_j-x_i)^2}{l_{ij}^2} + \frac{E\,A_{ij}}{l_{ij}}\,\frac{(y_j-y_i)^2}{l_{ij}^2}, \\
b_{ij} &= \left(\frac{E\,A_{ij}}{l_{ij}} - \frac{12\,E\,I_{ij}}{l_{ij}^3}\right)\frac{(x_j-x_i)}{l_{ij}}\,\frac{(y_j-y_i)}{l_{ij}}, \qquad c_{ij} = \frac{6\,E\,I_{ij}}{l_{ij}^2}\,\frac{(y_j-y_i)}{l_{ij}}, \\
\bar{c}_{ij} &= \frac{6\,E\,I_{ij}}{l_{ij}^2}\,\frac{(x_j-x_i)}{l_{ij}}, \qquad d_{ij} = \frac{2\,E\,I_{ij}}{l_{ij}}.
\end{aligned}
\tag{3}
$$

If Eq. (2) is formulated at each panel point of the frame, the stiffness matrix is obtained. The arrangement of the element can be tabulated mechanically according to the following procedures:

1. Number the panel points of the given frame from left to right.
2. Calculate the six coefficients for each member from the given data.
3. Prepare the space in which the element of the stiffness matrix will be written, and write the same numbers as the panel points in the outer side of the row and column of submatrices 1, 2, 3, 4 and 7.
4. Write the coefficients a, b and c of the unknown terms u, v and θ of Eq. (2.1) in submatrices 1, 2 and 3.
5. Write the coefficients b, $\bar{a}$ and $\bar{c}$ of the terms u, v and θ of Eq. (2.2) in submatrices 4, 5 and 6.
6. Write the coefficients c, $\bar{c}$ and d of the terms u, v and θ of Eq. (2.3) in submatrices 7, 8 and 9 (Table 1).

At steps 4, 5 and 6, each element must be written according to the rule shown in Table 1, observing the panel points in the order of the numbers.

Up to step 6, no consideration is paid to the conditions of the supports and hinged points. In the following step 7 these must be considered in order to complete the stiffness matrix.

7. When the conditions of the supports are taken into consideration, the unnecessary rows and columns are eliminated. In this case, $u=v=\theta=0$, $u=v=0$ and $v=0$ are obtained at the fixed support, hinged support and horizontally movable support, respectively. The unnecessary columns corresponding to the support numbers are first eliminated, and thereafter the rows corresponding to these columns are eliminated.
8. From the conditions of the hinged points, the columns of θ corresponding to the number of hinged points and the rows of submatrices 7, 8 and 9 of the same number must be eliminated. In this case, where the system contains the member hinged to the other member at both ends, the coefficients $a \sim d$ must be calculated in advance on the assumption $I=0$ for the member.

In the stiffness matrix thus completed, the elements are symmetrical about the main diagonal and at the same time about the subdiagonal in each submatrix except the signs.

Table 1. Rule for Arrangement of Element of Stiffness Matrix

Equilibrium eq.	Unknown Terms						Right Hand Terms
	u		v		θ		
	coef. of u_i diagonal element	coef. of u_j non-diagonal element	coef. of v_i diagonal element	coef. of v_j non-diagonal element	coef. of θ_i diagonal element	coef. of θ_j non-diagonal element	
	sub matrix 1		sub matrix 2		sub matrix 3		
$\sum H = 0$	$\sum a_{ij}$	$-a_{ij}$	$\sum b_{ij}$	$-b_{ij}$	$-\sum c_{ij}$	$-c_{ij}$	$\bar{P}_i$
	sub matrix 4		sub matrix 5		sub matrix 6		
$\sum V = 0$	$\sum b_{ij}$	$-b_{ij}$	$\sum \bar{a}_{ij}$	$-\bar{a}_{ij}$	$\sum \bar{c}_{ij}$	$\bar{c}_{ij}$	$\bar{Q}_i$
	sub matrix 7		sub matrix 8		sub matrix 9		
$\sum M = 0$	$-\sum c_{ij}$	c_{ij}	$\sum \bar{c}_{ij}$	$-\bar{c}_{ij}$	$2 \sum d_{ij}$	d_{ij}	$\bar{M}_i$

The matrix inversion of the above stiffness matrix leads to the influence coefficients of the displacement and rotation of each panel point due to unit panel point loads. In order to calculate the sectional forces of each member, the following Eq. (4) is used, after obtaining $\mathfrak{P}$, $\mathfrak{Q}$ and $\mathfrak{M}$ in Eq. (1):

$$\begin{aligned} M_i &= \mathfrak{M}_{ij}\frac{(x_j-x_j)}{|x_j-x_i|}, \qquad N_i = -\left\{\mathfrak{P}_{ij}\frac{(x_j-x_i)}{l_{ij}}+\mathfrak{Q}_{ij}\frac{(y_j-y_i)}{l_{ij}}\right\}, \\ Q_i &= \mathfrak{P}_{ij}\frac{(y_j-y_i)}{l_{ij}}-\mathfrak{Q}_{ij}\frac{(x_j-x_i)}{l_{ij}}. \end{aligned} \tag{4}$$

The signs of the sectional forces are the same as those used in ordinary structural analysis.

Part II. Programming for the NEAC 2203 Computer

All the steps of the analytical method described in Part I were programmed for the electronic digital computer NEAC 2203. The block diagram is shown in Fig. 2. The memory of the NEAC 2203 comprises a total of 12,000 words and the calculable maximum number of the panel is 14 for all types of the system, because magnetic tape is not used.

Each step of the programme is almost same as those described in Part I, and only the following steps are different; that is to say, in the computation of the stiffness matrix, steps 7 and 8 are calculated in advance in the internal magnetic drum for every row of the stiffness matrix, and its elements are transferred to be stored in the external magnetic drum.

The following data are prepared as input data:

1. Length of the member projected to the x- and y-coordinates, (x_j-x_i) and (y_j-y_i).
2. Sectional area A_{ij} and moment of inertia I_{ij} of each member.
3. Total number of members connected to each panel point and the point number of the other end of the member.
4. Type of loads and their position of application.
5. Total number of members.
6. Total number of panel points including supports.
7. Total number of hinged panel points except supports.
8. Minimum point number of the hinged panel points.

The data are arranged from lower numbers to higher numbers with regard to the number of the panel point and also in the same order with regard to the number of the other end of the member connected to a certain panel point. From these data, the computer can determine automatically whether the bridge to be analysed is a Lohse girder, Langer girder, or tied arch bridge, and can adopt the calculation corresponding to each system. The result of the computer calculation is printed in the form of the influence coefficients of the displace-

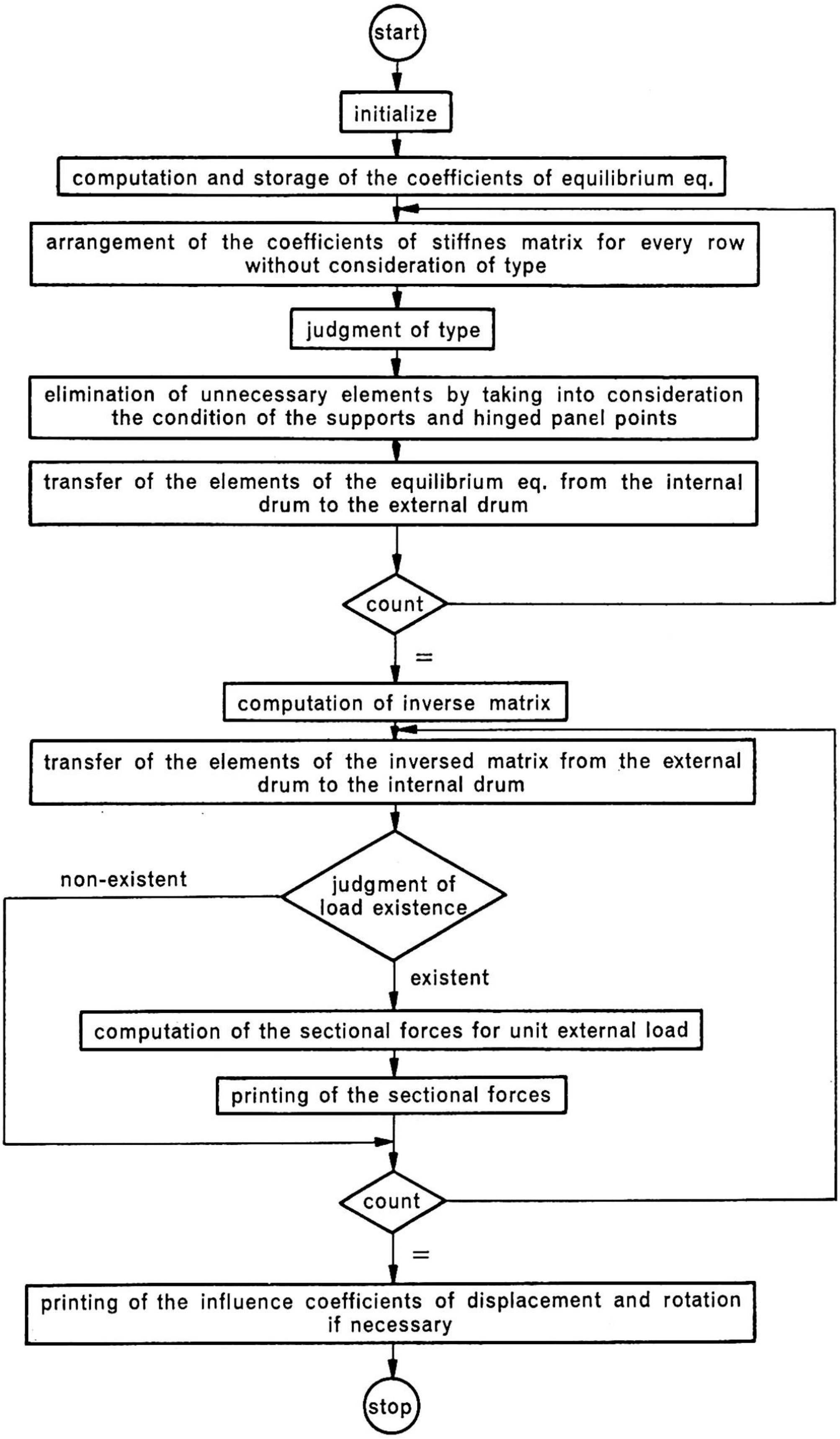

Fig. 2. Block Diagram.

ment and rotation of each panel point and the sectional forces of each member. According to this programme, not only Nielsen System bridges, but also ordinary tied arch, Langer girder and Lohse girder bridges with vertical hangers can be calculated.

Part III. Example of Calculation

The Langer girder bridge with nine panels shown in Fig. 3, will be calculated. The assumed values for A and I are as follows:

for member $\overline{01}$, $A=137\ \mathrm{cm}^2$, for member $\overline{13}, \overline{35}, \overline{57}, \overline{79}$, $A=123\ \mathrm{cm}^2$, for member $\overline{12}$, $A=51.2\ \mathrm{cm}^2$, for member $\overline{34}, \overline{45}, \overline{56}, \overline{67}, \overline{78}, \overline{89}$, $A=40.15\ \mathrm{cm}^2$, for member $\overline{02}, \overline{24}, \overline{46}, \overline{68}, \overline{8\text{—}10}$, $A=223.7\ \mathrm{cm}^2$ and $I=834865\ \mathrm{cm}^4$.

In addition to above data, the necessary input data described in Part II are (5) = 33, (6) = 19, (7) = 9 and (8) = 1.

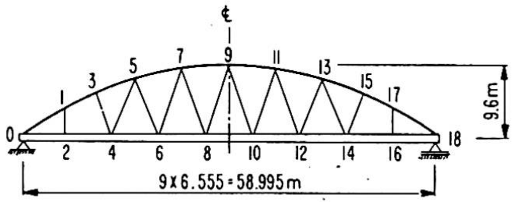

Fig. 3. Skelton Diagram of Example 1.

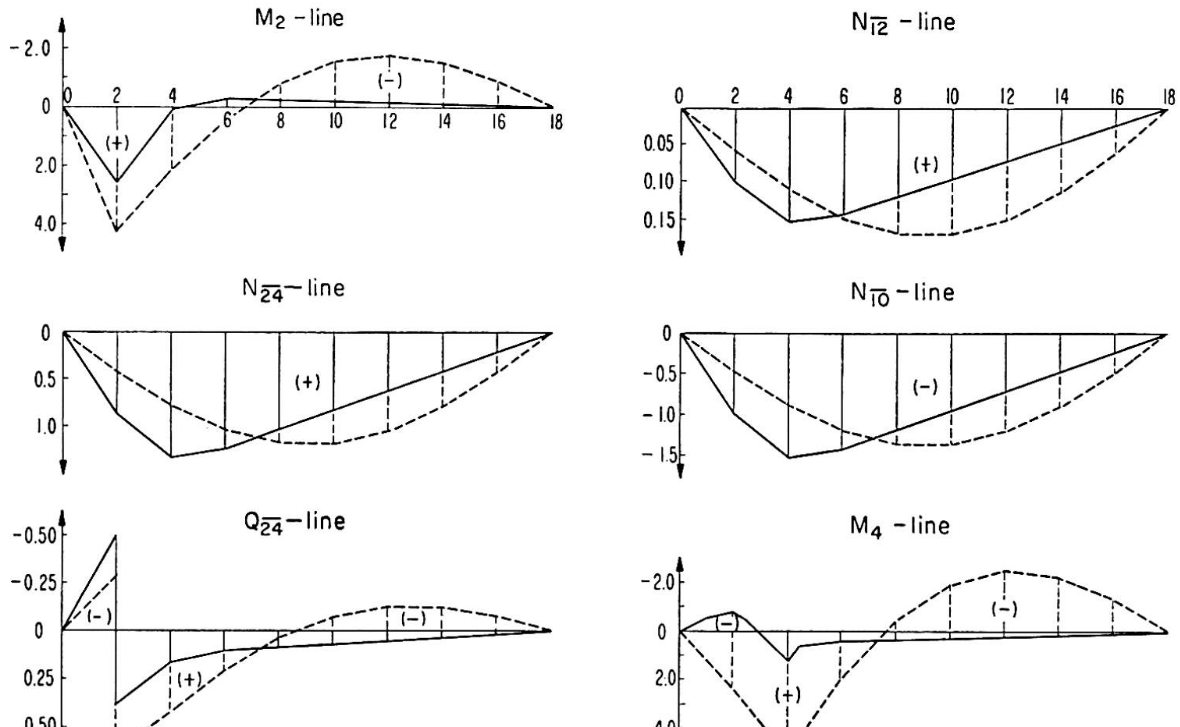

Fig. 4. Influence Lines of Sectional Force of Example 1.

The machine time was 50 min. for calculating the influence coefficients of the bending moment, normal and shearing forces of all members for each panel point load, including printing time. The element of stiffness matrix was 45×45.

The influence coefficients of the sectional forces of several members $\overline{24}$, $\overline{12}$, $\overline{10}$ and $\overline{46}$ due to unit vertical load are shown in Fig. 4. In Fig. 4, the full line shows the influence line for the system having inclined hangers and the broken line gives the influence line for the system having vertical hangers at even point numbers in Fig. 3 instead of inclined hangers. As may be understood from Fig. 4, the bending moment decreases remarkably in the case of inclined hangers, compared with the case of vertical hangers, whereas the normal and shearing forces do not vary significantly.

The total weight of steel was 86.4 tons for a bridge of span = 58.995 m, effective width = 6.0 m and carrying Class I Design Load in accordance with the Japanese Standard Specification for Steel Highway Bridges (provisional, June 12, 1962). This shows a steel saving of about 10% compared with an ordinary Langer girder bridge with vertical hangers.

Part IV. Model Test

As an experimental verification of the theoretical analysis, a model test was performed for the tied arch shown in Fig. 5. The model material was

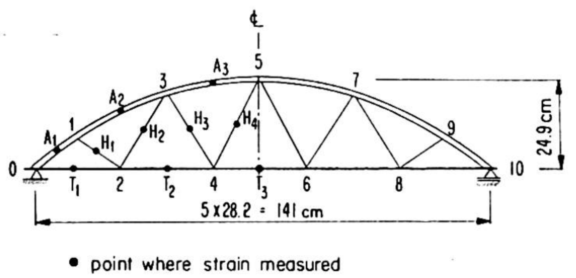

Fig. 5. Model Tied Arch.

polymethylmethacrylate. The sectional area and moment of inertia of the upper chord, and the sectional area of the ties and inclined hangers are 3.0 cm^2, 5.0625 cm^4, 1.0 cm^2, and 0.2 cm^2, respectively. The compressive force which acts in the inclined hangers due to the truss action can be eliminated by the tensile force due to the dead load. In this test, preloads were applied to all panel points of the lower chord, and then a concentrated load was applied. The result is shown in Fig. 6. In Fig. 6, the full line shows the theoretical values, the chain line the mean of several observed values, and the broken line the theoretical values for an ordinary tied arch bridge with vertical hangers.

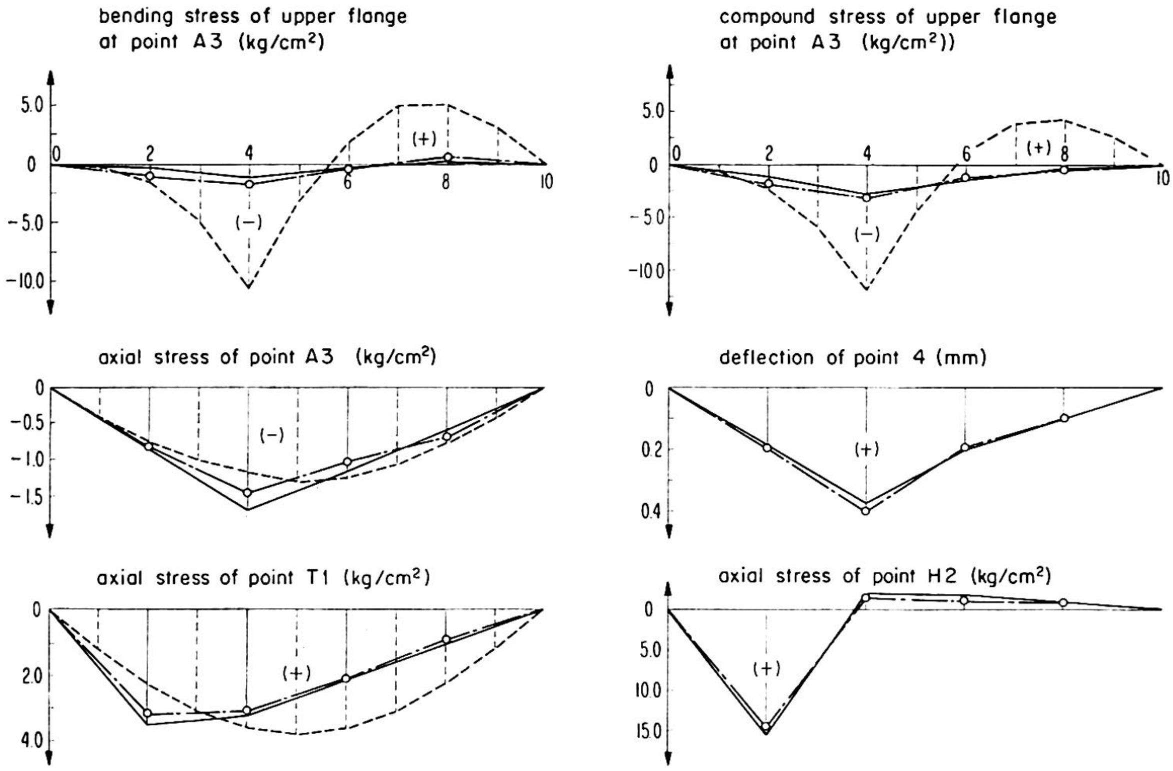

Fig. 6. Result of Model Test (point load $P = 3.5\,\mathrm{kg}$).

Conclusion

The analytical solution of Nielsen System bridges by the displacement method and especially the formulation of the stiffness matrix have been described. It was programmed for the NEAC 2203 computer for the purpose of automatic calculation. According to this programme, not only Nielsen System bridges, but also any pin- and rigid-jointed plane frame which is simply supported can be analysed. This paper shows only one example, but, as can be understood from Parts III and IV, this system is advantageous as compared with a similar system with vertical hangers. Finally, a model test showed that the solution proposed by the authors is useful.

Taking advantage of the programme, the authors are now studying three types of Nielsen System bridge and the general characteristics will be published in the near future.

Summary

This paper describes a theoretical analysis which is applicable to all types of Nielsen System bridge with arbitrarily inclined hangers and its programming for calculation by computer. It consists of 4 parts:

Part 1: Theoretical analysis by the displacement method.
Part 2: Programming for the NEAC 2203 computer.
Part 3: Calculation for a Langer girder bridge with inclined hangers.
Part 4: Model test.

This method of analysis can be applied not only to Nielsen System bridges, but also to similar bridges with vertical hangers such as tied arch, Langer girder and Lohse girder bridges.

Résumé

Les auteurs présentent une méthode de calcul de tout pont de type Nielsen avec suspentes d'inclinaison arbitraire et exposent l'établissement du programme de la calculatrice. On trouvera quatre parties principales:

1. Calcul par la méthode des déformations.
2. Etablissement du programme de la calculatrice NEAC 2203.
3. Calcul d'un pont bow-string à suspentes inclinées.
4. Essais sur modèle.

Outre les poutres Nielsen, cette méthode de calcul est applicable à d'autres ponts similaires à suspentes verticales tels que l'arc à tirant, les ponts à poutres bow-string du type Langer ou de Lohse.

Zusammenfassung

Die Autoren beschreiben eine theoretische Untersuchungsmethode, welche sich auf alle Arten von Nielsen-Trägern mit beliebig geneigten Hängestangen anwenden läßt. Ferner wird die Programmierung für die elektronische Berechnung erläutert. Die Arbeit besteht aus vier Teilen:

1. Teil: Theoretische Untersuchung mit Hilfe der Deformationsmethode.
2. Teil: Programmierung für den NEAC 2203-Rechner.
3. Teil: Berechnung für einen Langerschen Brückenträger mit geneigten Hängestangen.
4. Teil: Modelluntersuchung.

Diese Berechnungsmethode kann nicht nur auf Nielsen-Träger, sondern auch auf ähnliche Brückenträger mit vertikalen Hängestangen, z. B. Bogen mit Zugband, Langer- und Lohse-Brückenträger angewendet werden.