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# I d 1

## Matrix Analysis

*Analyse matricielle*

*Matrizenrechnung*

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The present discussion is concerned with two subjects.

I. The advantages of the matrix formulation of structural analysis problems as compared with other formulations. This follows the suggestion of the general reporter of subject I — General Questions.

The matrix methods, from the strictly structural point of view, were introduced in the last decade by the pioneering work of FALKENHEINER [1], LANGEFORS [2], and ARGYRIS [3, 4]. Formulations from the topological point of view were proposed by KRON [5], LANGEFORS [6], SAMUELSON [7], and FENVES [8].

The introduction of these formulations and the commencement of the generalised use of digital computers combined to open up new perspectives for the analysis and design of structures.

Some points which appear to constitute definite advantages of the matrix formulation are mentioned below.

1. A unified formulation of the processes of structural analysis (improperly termed "methods") of the first order. To examplify briefly, an equation of the type

$$a^t\, r\, a\, X = R$$

is used indifferently in the force or in the displacement process. It is only necessary to interpret, according to the process used, the matrices that enter into the equation.

2. A rapid and compact presentation of the complete theory of hyperstatic structures.

3. The problem of the modification of structural elements is treated simply as an operation performed with matrices previously obtained, and matrices that define the modification.

4. Application to any structure idealised as an assemblage of a finite number of structural elements. Trusses, rigid frames, aeronautical structures, composed of stringers and shear panels, are all capable of being analysed by the matrix formulation. In [9] examples are given of a rigid frame and an aeronautical structure analysed by the matrix formulation in the IBM-1620 computer of the Instituto Tecnológico de Aeronáutica.

More recently the matrix formulation has been applied to the study of continuous systems such as plane stress systems, plates, shells, and three-dimensional stress systems [10, 11, 12]. In this approach these systems are divided by a mesh into a certain number of structural elements; the assemblage of the resulting structural elements constitutes the idealisation of the continuous system.

5. It is specially suitable for the use in the routines of matrix algebra which exist for the various types of computers [13, 14].

The advantage of other formulations over the matrix formulation is that the former, when applied to particular structural systems, can be more efficient with regard to the preparation of input data, the use of the core memory, and the time of computation.

II. Paper by MICHALOS and GROSSFIELD, I d 1, Analysis of Interconnected Space Frames.

The approach used is simply the displacement method applied to spatially rigid frames. Since the computational procedure depends markedly on the inversion (Eq. (6)) of the stiffness matrix $K$ (Eq. (4)) a study of the conditioning of $K$ would be desirable.

For an arbitrary space frame there is interaction only between the degrees of freedom corresponding to each joint and the degrees of freedom of the joint adjacent to that joint.
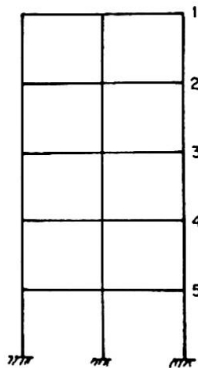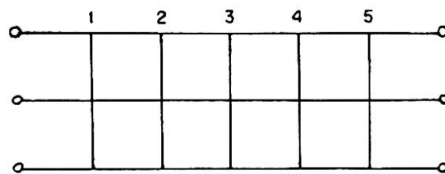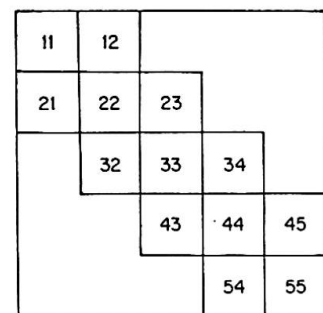


Fig. 1 a.                    Fig. 1 b.                    Fig. 2.

For structures such as plane or spatially rigid frames, Fig. 1 a, and plane grids, Fig. 1 b, there is interaction only between the degrees of freedom of one storey and of the two adjacent storeys, Fig. 1 a, and between the degrees of freedom of one cross-beam and of the two adjacent cross-beams, Fig. 1 b. This leads to a stiffness matrix $K$ which is a tri-diagonal band matrix. Such a matrix is represented schematically in Fig. 2.

The inversion of a tri-diagonal band matrix is efficiently performed through a process of recurrence and back-substitution [15]. In each step of this process the order of the matrix to be inverted is equal to the number of unknowns of the storey considered, and the data that must be available in the core

memory of the computer are only those of that storey and of the two adjacent storeys. By means of this approach CLOUGH [15], and TEZCAN [16] have analysed systems with thousands of degrees of freedom.

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# Summary

A general survey of the main features of the matrix formulation of structural analysis problems is presented.

The conditioning of the stiffness matrix of spatially rigid frames is dis-

cussed in connection with the paper by MICHALOS and GROSSFIELD, I d 1,
Analysis of Interconnected Space Frames.

## Résumé

L'auteur présente les caractéristiques principales de l'application des ma-
trices aux calculs statiques. En rapport à l'article «Calcul des systèmes hyper-
statiques tridimensionnels», I d 1, de MM. MICHALOS et GROSSFIELD, il discute
la forme générale de la matrice de rigidité des cadres spatiaux.

## Zusammenfassung

Es wird eine allgemeine Übersicht über die Darstellung der Rechenabläufe
der Tragwerksstatik in Matrizenform gegeben. Die Beschaffenheit der Stei-
figkeitsmatrizen räumlicher Rahmensysteme wird diskutiert im Zusammen-
hang mit der Arbeit von MICHALOS und GROSSFIELD, I d 1, «Berechnung
von räumlichen Netzwerken».

# Structural Analysis of Space Frames Supporting Solid Parabolic Reflectors

*Le calcul des ossatures spatiales supportant des réflecteurs paraboliques pleins*

*Untersuchungen an räumlichen Tragkonstruktionen für Radioteleskope*

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Steerable aerials comprising solid steel parabolic reflectors supported and stiffened by complex frameworks are becoming more common throughout the world, and with the recent developments in satellite communications, for which an international agreement was concluded last year, there is greater interest in the techniques for analysing these types of structure.

The principal structural requirement for a steerable radio aerial involves the production of a solid parabolic reflecting surface which has to keep its shape within fine limits, whatever its position in azimuth or elevation, and in many cases under severe loading from high winds.

The precise operational conditions can vary according to the type of instrument, and the tolerances within which a reflector must keep its shape are dependent on the frequency of the radio waves to be either received or transmitted by the instrument.

An aerial used for radio astronomy at a research station can perhaps be put out of action during periods of high winds because it is not usually a vital matter if this type of research activity is interrupted, but on the other hand in the case of an instrument used for satellite communications, the ground station must be fully operational in all kinds of weather if it is to be of any use.

Thus, on the one hand a radio telescope might be designed to keep within the specified deflection limits in winds up to, say, 20 m. p. h., whereas an aerial for tracking satellites might be required to maintain its shape in 80 m. p. h. force winds.

For satellite communications high frequency transmissions are involved, and these require strict specifications for profile accuracy. The permissible tolerance in the shape of a reflector is a function of wavelength, and the shorter the wavelength, the more nearly must the reflecting surface approach the condition of an optical mirror.

The analysis of these structures is almost entirely concerned with calculating

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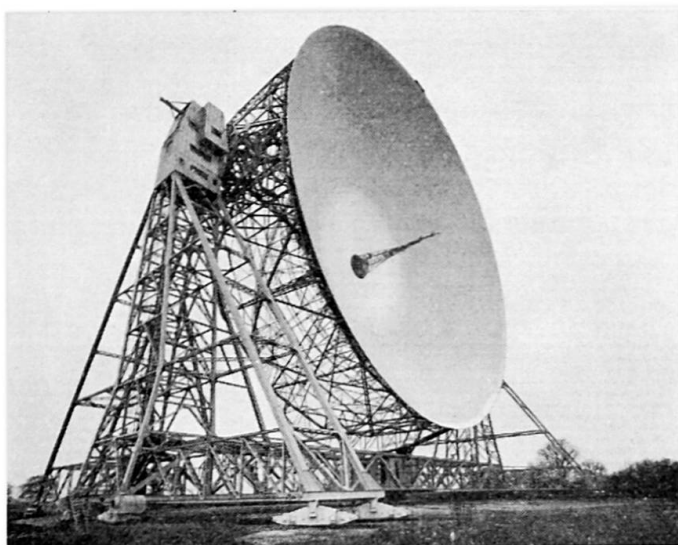[1] Partner, Husband & Co., Consulting Engineers.

Fig. 1. 250-ft. diameter radio telescope at Jodrell Bank.
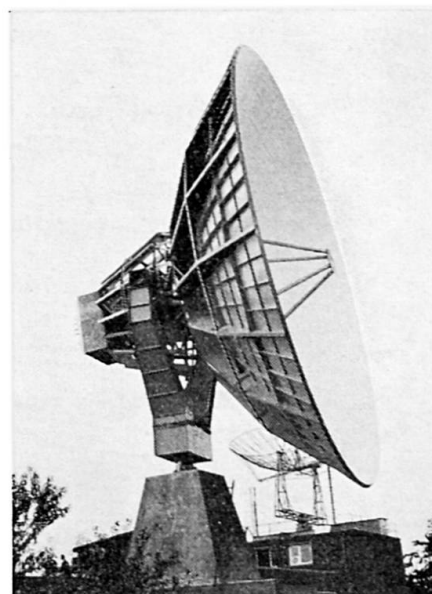


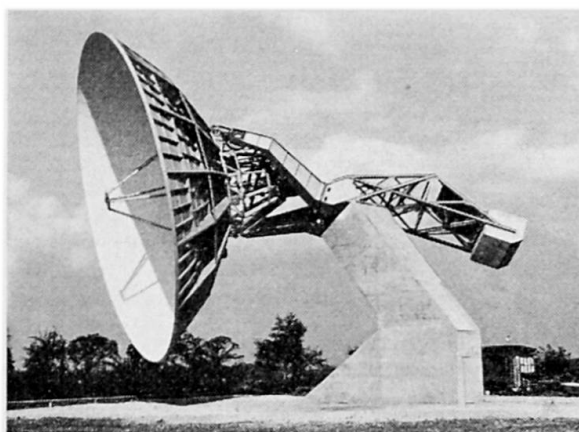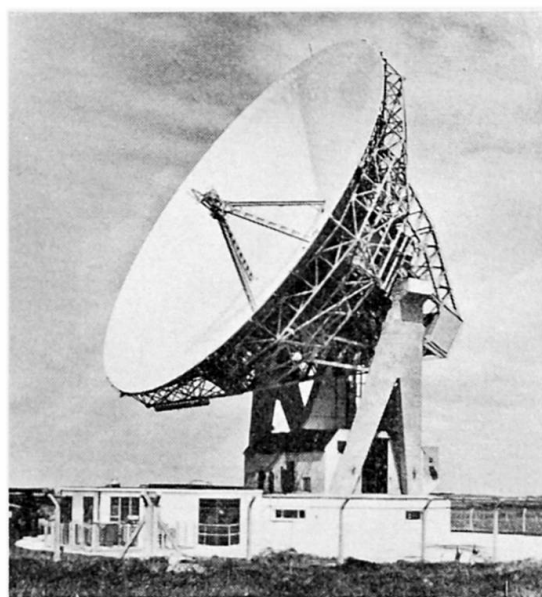Fig. 2. 50-ft. diameter altazimuth instrument at Jodrell Bank.



Fig. 3. 50-ft. polar axis instrument at Jodrell Bank.

Fig. 4. 85-ft. diameter satellite communications aerial at Goonhilly Downs, Cornwall.



theoretical deflections. The final design must also take account of the best possible constructional accuracy which can be obtained — in other words, part of the shape tolerance is always taken up by fabrication errors.

Fig. 1 to 5 illustrate a few examples of recently completed large steerable aerials in Britain, all designed by Dr. H. C. HUSBAND[2]), from which it will be seen that these structures can be highly complex, with many degrees of redundancy.

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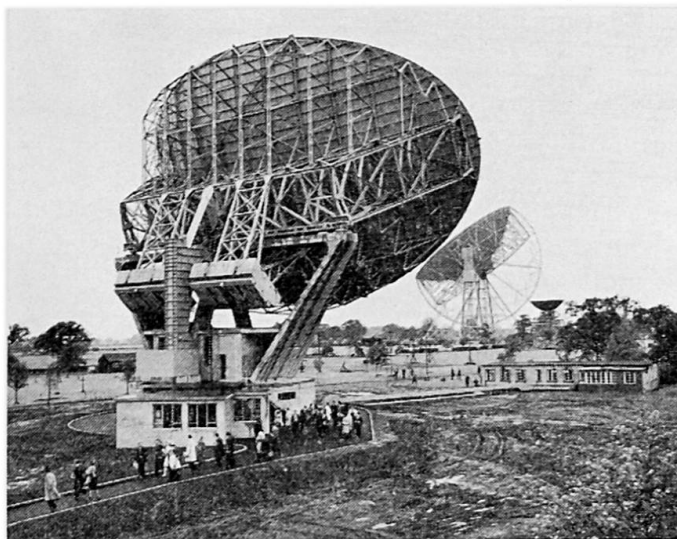[2]) Senior Partner, Husband & Co., Consulting Engineers.

Fig. 5. 125-ft. radio telescope at
Jodrell Bank.

These redundancies are of great advantage in limiting deflections, but complicate the deflection calculations. In addition to calculating the stiffness of a system of space frames supporting the reflector and connecting to a counterweighting system, consideration has also to be given to the stiffening effect of the continuous steel membrane.

A further factor which might increase the calculation problem is that sometimes these aerials have to be designed and constructed in an abnormally short time, perhaps to meet the date for a satellite launching. This applied in the case of the British ground station at Goonhilly Downs, which had to be built very quickly in order to be ready to receive the first transmission from Telstar.

These are obviously cases where an electronic computer becomes a useful if not essential tool at the design stage, but there are no available programmes capable of dealing with the comprehensive solution of interconnecting space frames combined with solid membranes of this type.

However, by adopting standard frame programmes which are readily available, the relative stiffnesses of the various components of the structure can be rapidly assessed, and alternative arrangements compared.

This method has the advantage of requiring the designer to exercise proper engineering judgment at all stages of the design process, and this is very useful because there is often a tendency for younger designers to become blinded by the rolls of figures which the machine turns out, and sometimes to treat them as if they were the Word of God.

## Summary

This contribution deals with some practical considerations during the structural analysis of particular types of interconnected space frames.

## Résumé

Cette contribution traite de quelques problèmes pratiques qui se posent dans le calcul de certains types particuliers de charpentes spatiales solidaires entre elles.


## Zusammenfassung

Der Beitrag gibt einige praktische Überlegungen zur baulichen Ausbildung spezieller räumlicher Tragwerke.