# Matrix analysis

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## Matrix Analysis

*Analyse matricielle*

*Matrizenrechnung*

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The present discussion is concerned with two subjects.

I. The advantages of the matrix formulation of structural analysis problems as compared with other formulations. This follows the suggestion of the general reporter of subject I — General Questions.

The matrix methods, from the strictly structural point of view, were introduced in the last decade by the pioneering work of FALKENHEINER [1], LANGEFORS [2], and ARGYRIS [3, 4]. Formulations from the topological point of view were proposed by KRON [5], LANGEFORS [6], SAMUELSON [7], and FENVES [8].

The introduction of these formulations and the commencement of the generalised use of digital computers combined to open up new perspectives for the analysis and design of structures.

Some points which appear to constitute definite advantages of the matrix formulation are mentioned below.

1. A unified formulation of the processes of structural analysis (improperly termed "methods") of the first order. To examplify briefly, an equation of the type

$$a^t r a X = R$$

is used indifferently in the force or in the displacement process. It is only necessary to interpret, according to the process used, the matrices that enter into the equation.

2. A rapid and compact presentation of the complete theory of hyperstatic structures.

3. The problem of the modification of structural elements is treated simply as an operation performed with matrices previously obtained, and matrices that define the modification.

4. Application to any structure idealised as an assemblage of a finite number of structural elements. Trusses, rigid frames, aeronautical structures, composed of stringers and shear panels, are all capable of being analysed by the matrix formulation. In [9] examples are given of a rigid frame and an aeronautical structure analysed by the matrix formulation in the IBM-1620 computer of the Instituto Tecnológico de Aeronáutica.

More recently the matrix formulation has been applied to the study of continuous systems such as plane stress systems, plates, shells, and three-dimensional stress systems [10, 11, 12]. In this approach these systems are divided by a mesh into a certain number of structural elements; the assemblage of the resulting structural elements constitutes the idealisation of the continuous system.

5. It is specially suitable for the use in the routines of matrix algebra which exist for the various types of computers [13, 14].

The advantage of other formulations over the matrix formulation is that the former, when applied to particular structural systems, can be more efficient with regard to the preparation of input data, the use of the core memory, and the time of computation.

II. Paper by Michalos and Grossfield, I d 1, Analysis of Interconnected Space Frames.

The approach used is simply the displacement method applied to spatially rigid frames. Since the computational procedure depends markedly on the inversion (Eq. (6)) of the stiffness matrix $K$ (Eq. (4)) a study of the conditioning of $K$ would be desirable.

For an arbitrary space frame there is interaction only between the degrees of freedom corresponding to each joint and the degrees of freedom of the joint adjacent to that joint.

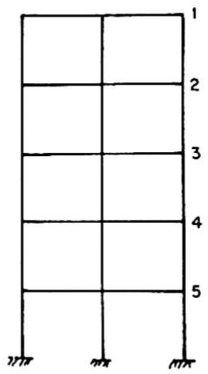


Fig. 1a.

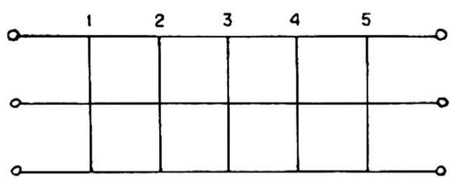


Fig. 1b.

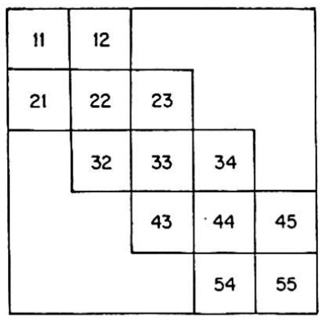


Fig. 2.

For structures such as plane or spatially rigid frames, Fig. 1a, and plane grids, Fig. 1b, there is interaction only between the degrees of freedom of one storey and of the two adjacent storeys, Fig. 1a, and between the degrees of freedom of one cross-beam and of the two adjacent cross-beams, Fig. 1b. This leads to a stiffness matrix $K$ which is a tri-diagonal band matrix. Such a matrix is represented schematically in Fig. 2.

The inversion of a tri-diagonal band matrix is efficiently performed through a process of recurrence and back-substitution [15]. In each step of this process the order of the matrix to be inverted is equal to the number of unknowns of the storey considered, and the data that must be available in the core memory of the computer are only those of that storey and of the two adjacent storeys. By means of this approach CLOUGH [15], and TEZCAN [16] have analysed systems with thousands of degrees of freedom.

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## Summary


A general survey of the main features of the matrix formulation of structural analysis problems is presented.

The conditioning of the stiffness matrix of spatially rigid frames is dis-

cussed in connection with the paper by MICHALOS and GROSSFIELD, I d 1, Analysis of Interconnected Space Frames.

**Résumé**

L'auteur présente les caractéristiques principales de l'application des matrices aux calculs statiques. En rapport à l'article «Calcul des systèmes hyperstatiques tridimensionnels», I d 1, de MM. MICHALOS et GROSSFIELD, il discute la forme générale de la matrice de rigidité des cadres spatiaux.

**Zusammenfassung**

Es wird eine allgemeine Übersicht über die Darstellung der Rechenabläufe der Tragwerksstatik in Matrizenform gegeben. Die Beschaffenheit der Steifigkeitsmatrizen räumlicher Rahmensysteme wird diskutiert im Zusammenhang mit der Arbeit von MICHALOS und GROSSFIELD, I d 1, «Berechnung von räumlichen Netzwerken».