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Calculation of the Increase in Yield Strength due to the Effects of Cold Work of Forming

Calculation de l'augmentation de la limite d'élasticité due au travail à froid

Berechnung der durch die Kaltverformung erhöhten Streckgrenze

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1 Introduction

In cold-formed members strain hardening of material takes place in parts of the section that deform plastically in the course of the forming operation. Plastic deformation raises the yield strength and the ultimate strength as well. The increase in the mechanical properties of the material depends on a number of factors. Likewise, the distribution of higher mechanical properties throughout the section is affected by the method of forming used. Two essentially different kinds of strain hardening of cold-formed members may be discerned:

In the first, hardening occurs by plastic strain in the corners of the section during press brake forming. This strain hardening is restricted to a rather narrow area of the section corners. In roll-forming work, the region of strain hardening is, too, bound to parts of the section (to corners in particular) in which appreciable plastic strains were set up.

In the second kind, sheet steel of which the member is being formed, is subjected to plastic strain prior to the cold forming operation proper. The process usually employed to gain this end, is cold wave forming which causes the sheet to strain plastically throughout its entire width. As a result the higher mechanical properties are distributed through the whole section, and this brings about substantial savings in material consumption. Not uncommon is the increase in yield strength corresponding to an increase in steel quality from St 37 to St 52.

In what follows we shall deal in detail with the conditions under which strain hardening of material occurs in the whole section of light-gauge cold-formed members.

2 Stress-strain dependence in the plastic range

We shall express the dependence between stress and strain in the plastic range by the following law

/1/
$$\sigma = F(\varepsilon)$$

where according to the Huber--Mises--Hencky theory, σ at multiaxial state of stress is the stress intensity given by the relation

/2/
$$\sigma = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}$$

when expressed in terms of principal stresses, or by

/3/
$$\sigma = \sqrt{\sigma_x^2+\sigma_y^2+\sigma_z^2-\sigma_x\sigma_y-\sigma_y\sigma_z-\sigma_{zx}-3\tau_{xy}^2-3\tau_{yz}^2-3\tau_{zx}^2}$$

when expressed in terms of stress components in three mutually perpendicular planes.

Similarly, strain at multi-axial state of stress is defined by the intensity relation

/4/
$$\varepsilon = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_3-\varepsilon_1)^2}$$

in terms of principal strains, or by

/5/
$$\varepsilon = \frac{2}{3}\sqrt{\varepsilon_x^2+\varepsilon_y^2+\varepsilon_z^2-\varepsilon_x\varepsilon_z-\varepsilon_x\varepsilon_z-\varepsilon_{yz}+\frac{3}{4}\gamma_{xy}^2+\frac{3}{4}\gamma_{yz}^2+\frac{3}{4}\gamma_{zx}^2}$$

in terms of strain components in three mutually perpendicular planes.

In the plastic range, we shall describe the actual unit logarithmic strain as

/6/
$$\varepsilon = \int_{l_0}^{l}\frac{dl}{l} = lg\frac{l}{l_0} - lg\frac{l_0+l-l_0}{l_0} = lg\,(1+\varepsilon')$$

where ε' is the unit strain referred to the original dimension

/7/ $$\varepsilon' = \frac{\Delta l}{l_o} .$$

Eq. /1/ holds good under the following assumptions:

1 The material is isotropic during plastic straining

2 Compared to the plastic ones, elastic strains are slight

3 During deformation, the ratio of principal strains remains the same

4 The principal axes of strain remain constant

5 The stress-strain dependence is identical for tension and compression

6 The Bauschinger effect is absent

For steels in the plastic range in which strain hardening takes place, function F in law /1/ can be expressed as

/8/ $$\sigma = K \varepsilon^n$$

where K is the modulus of plasticity, and n the constant hardening exponent. This way of expressing function σ is possible for those materials, for which the dependence between the actual stress (i.e. referred to an instantaneous dimension) and the actual unit strain in the plastic range is linear in the logarithmic plot. The value of the modulus of plasticity K varies from 5 000 to 8 000 kp/cm^2, that of the strain hardening exponent n , from 0.13 to 0.28 depending on the kind of steel used; either can be expressed as a function of the original tensile yield strength of the virgin material, viz.:

/9/ $$K = 2,80\ \sigma_u - 1,55\ \sigma_y$$

/10/ $$n = 0,225 \frac{\sigma_u}{\sigma_y} - 0,120$$

3 Fundamental relations

Prior to the cold-forming of the member, the sheet is cold-waved in the longitudinal direction in minute waves. For an element taken from a sheet thus waved, with axes marked according to the notation of Fig.1, the condition of zero strain in the longitudinal direction

/11/ $$\varepsilon_y = 0$$

implies - under the observance of constant volume - that

/12/ $$\varepsilon_z = -\varepsilon_x$$

As there is no shear strain in the plane perpendicular to axis (i.e. in the plane of the sheet),

/13/ $$\gamma_{xy} = 0$$

We shall futhermore assume that there is no shear strain in the plane perpendicular to axis x , either, so that

/14/ $$\gamma_{yz} = 0$$

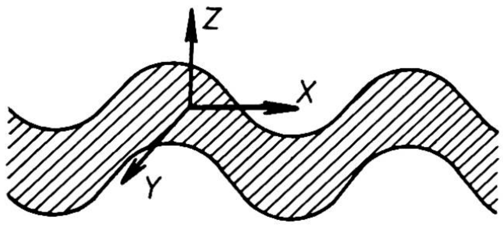

Fig.1

The decisive one is then the shear strain in the plane per - pendicular to axis y where

/15/ $$\gamma_{zx} \neq 0$$

On substituting relations /11/ to /15/, eq. /5/ of unit strain takes on the following form

/16/ $$\varepsilon = \frac{2}{3}\sqrt{3\varepsilon_x^2 + \frac{3}{4}\gamma_{zx}^2}$$

4 Parameters of cold waving

Consider a steel sheet cold waved in accordance with notation of Fig.2. D denotes the length of the wave, V the depth of the wave, R the radius of the circular arc of the wave. The rolls can also be set up so that the waves do not mesh completely; as a result the sheet is deformed in half waves with different lengths and different radii of curvature. Denote by a , b the length of the half-waves, by R , Z their radius of curvature, and by v_1, v_2 their depth. The half-length of the roll wave is denote by L , the thickness of the sheet by t . The characteristic parameter of the amount of waving is the wave depth, or the drop of the sheet. As a result of this drop, the sheet deforms like a built-in beam with instantaneous span l . The drop is caused in part by bending, in part by shear strain depending on the bending and shear rigidities.

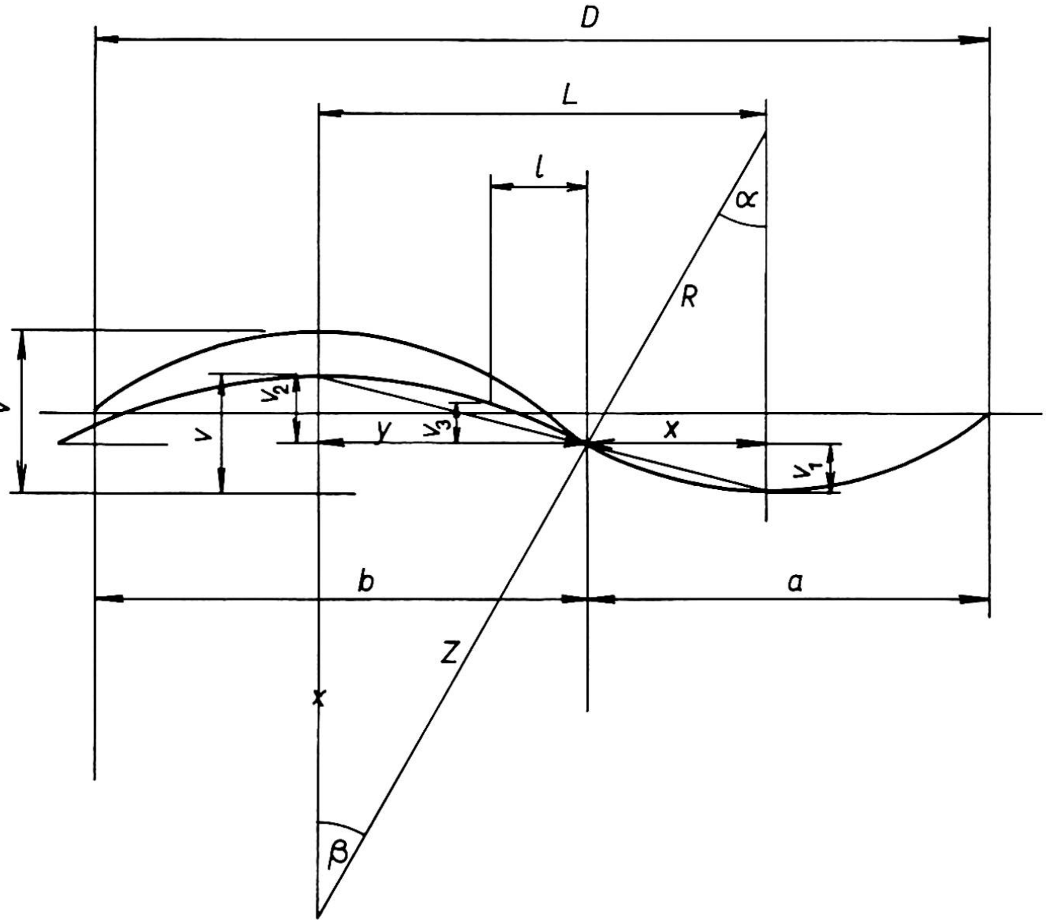

Fig.2

In the calculation that follows we shall substitute for the actual dimensions their relative values referred to the half-length of the wave, $L = \frac{D}{2}$, viz.:

/17/ $$\rho = \frac{R}{L}, \quad \tau = \frac{t}{L}, \quad \nu = \frac{v}{L}, \quad \lambda = \frac{l}{L}, \quad x = \frac{X}{L}, \quad y = \frac{Y}{L}, \quad z = \frac{Z}{L}$$

5 Strains produced by cold waving

Since span l varies proportional to v, we shall introduce the concept of effective span λ_{ef} defined by the following relation

/18/ $$\lambda_{ef} = \frac{1}{\nu}\int \lambda \, d\nu = 1 - 2\frac{\rho}{\nu}\lg\left(1+\nu^2\right) \doteq 1 - 2\rho\nu$$

The mean shear strain will now be γ_z

/19/ $$\gamma_z = \frac{\nu_s}{\lambda_{ef}} = \frac{\nu}{(1-2\rho\nu)\left[1+\frac{0,353}{\tau^2}\left(1-4\rho\nu+4\rho^2\nu^2\right)\right]}$$

At $x=\frac{l}{2}$, $y=\frac{t}{2}$ the unit strain due to bending has its maximum of

/20/ $$\varepsilon_o = \pm \frac{3\nu\tau\varkappa}{1+\varkappa}$$

where $\varkappa$ is the coefficient of rigidity in bending and shear

/21/ $$\varkappa = \frac{0,353}{\tau^2}\left[1-4\rho\nu+4\rho^2\nu^2\right]$$

Since the width of the sheet undergoes no change during the pass through the rolls, stretching in the transverse direction is also experienced in addition to the shear strain and the unit strain due to bending. The pertinent unit strain is constant across the whole sheet width and given by the difference between the wave lengths and the original width of the sheet; its magnitude is

/22/ $$\varepsilon_p = \frac{1}{2\cdot 3}\left(\frac{2\nu}{1+\nu^2}\right)^2 + \frac{1\cdot 3}{2\cdot 4\cdot 5}\left(\frac{2\nu}{1+\nu^2}\right)^4 + \cdots$$

6 Calculation of the increase in yield strength

Given the parameters of waving ρ , ν , τ and knowing the mechanical properties of virgin material σ_u, σ_y , formulae /18/ to /21/ will serve us in the calculation of effective span λ_{ef}, coefficient of rigidity $\varkappa$, average shear strain γ_z , unit strain due to bending ε_o, unit strain due to transverse stretching ε_p, the maximum unit strain in the upper half of the section

/23/ $$\varepsilon_{xh} = \varepsilon_o + \varepsilon_p$$

the comparative strain

/24/ $$\varepsilon_h = \frac{2}{3}\sqrt{3\,\varepsilon_{xh}^2 + \frac{3}{4}\gamma_{zx}^2}$$

and on substituting in the equation

/25/ $$\sigma_h = K\,\varepsilon_h^n$$

the increased yield strength of the upper fibers σ_h .
Similarly for the lowe fibers

/26/ $$\sigma_d = K\,\varepsilon_d^n$$

For the unit strain varying in accordance with Fig.3a, the average increased yield strength of the whole section will be

/27/ $$\sigma = \frac{\sigma_h + \sigma_d}{2}$$

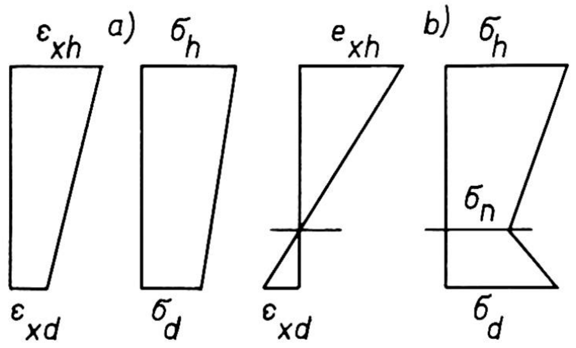

Fig. 3

So long as bending predominates in the unit strain, i.e. ε_{xd} and ε_{xh} are opposite signs /Fig.3b/ we shall also calculate the value of yield strenth in the neutral axis, σ_n . The yield strength of the whole section is the weighted mean obtained from the values of yield strength in the upper fibers, σ_h, σ_d, through the use of the formula

/28/ $$\sigma = \frac{1}{2}\left(\sigma_n + \frac{\sigma_h\,\varepsilon_{xh} + \sigma_d\,\varepsilon_{xd}}{\varepsilon_{xh} + \varepsilon_{xd}}\right)$$

7 Comparison between calculations and results of tests

The tests were made by VÖST-Linz using material strain hardened by variously shaped waves of different depths. In all cases, the steel sheet was 1.5 mm thick; the variables were the length of the wave, D , and the radius of the arc, R ,and the examined depen - dence was that on the depth of depression v . The results of tensile tests, and the curves of yield strength, strength and elongation are in the diagrams. In these diagrams, solid dots denote the values of yield strength, strength and elongation determined by tests. Ase the measured values fail to lie on a continuous curve, the re - gion of variance indicating the probable course of the individual quantities is marked out adjacent to the points. Circles on the yield strength curve denote the values calculated according to the theory discussed in this paper.

Fig. 4 gives the results for a wave with length $D = 6.3$ mm, radius $R = 1.6$ mm. The yield strength increases from 33.5 kp/mm^2, to 65.8 kp/mm^2, the elongation decreases from 28.5 % to 5.7 %. The calculated values of yield strength σ_y lie in the middle of the region determined by the tests. The percent increase in sttength is less than that in yield strength.

Fig. 5 corresponds to a wave $D = 10$ mm, $R = 3{,}15$mm. The yield strength increases from 40 kp/mm^2 to 66.4 kp/mm^2, the elongation drops from 30.7 % to 4.5 %. The agreement between the calculated and measured values is again good.

8 Conclusion

An examination was made of the increase in yield strength due to the effect of cold work of forming. The yield strength of the whole member is already raised by higher yield strength in the corners of the cold-formed section. In hot-rolled, press brake-formed members the increase thus achieved is about 10 %. In hot-rolled, cold roll-formed steel members, it can attain up to 20 %.

The highest increase in ultimate strength of the whole section about 70 % is obtained by increasing the yield strength through cold-waving of the steel sheet prior to the cold-forming operation proper. The paper derives a method which makes it possible to calculate the increased yield strength of a sheet thus cold-waved to a relatively high degree of accuracy. The theoretically obtained values of increased yield strength are in very good agreement with the results of tests.

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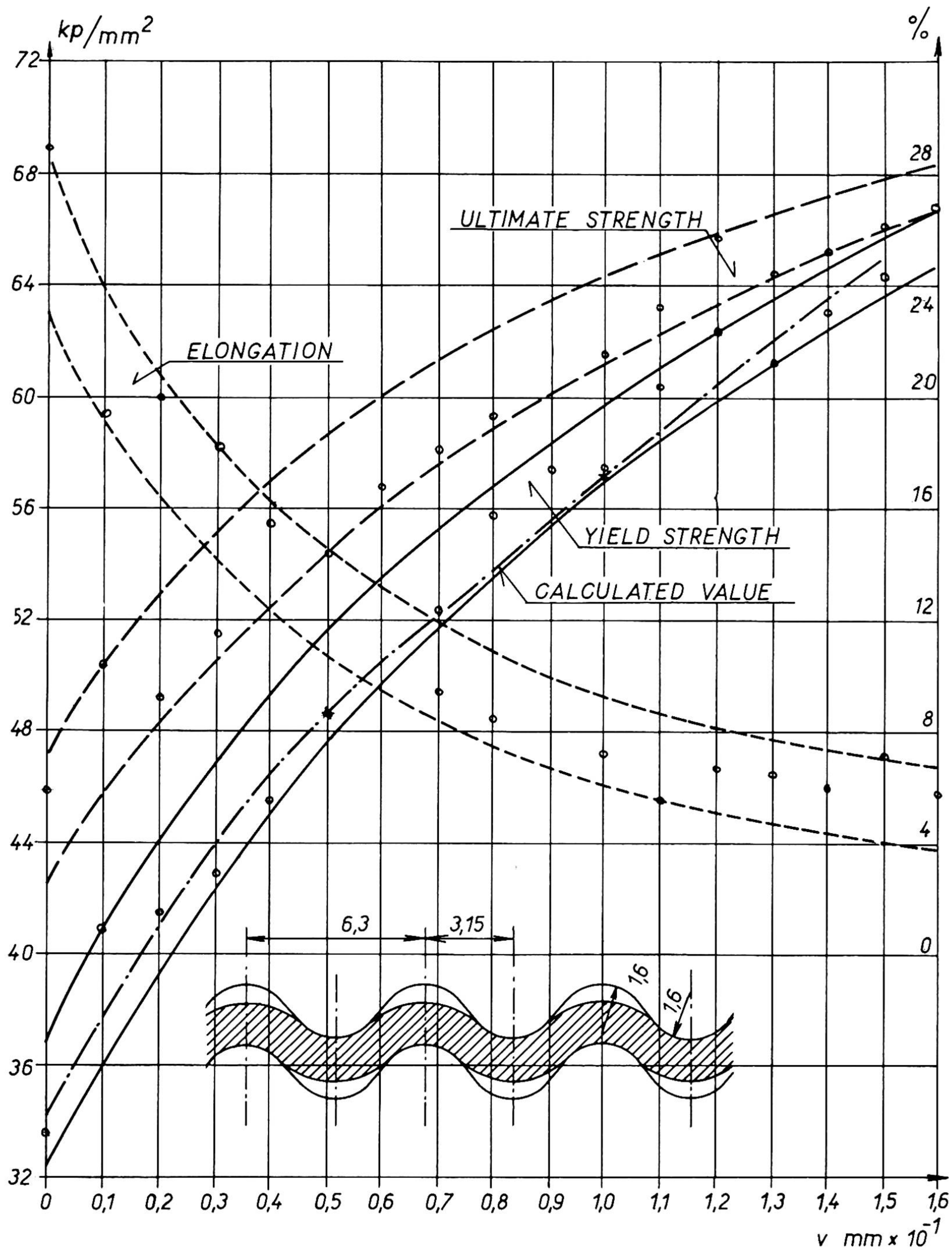
kp/mm²
%
72
68
64
60
56
52
48
44
40
36
32
28
24
20
16
12
8
4
0
ULTIMATE STRENGTH
ELONGATION
YIELD STRENGTH
CALCULATED VALUE
6,3
3,15
1,6
1,6
0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0,9
1,0
1,1
1,2
1,3
1,4
1,5
1,6
v mm x 10⁻¹

Fig. 4

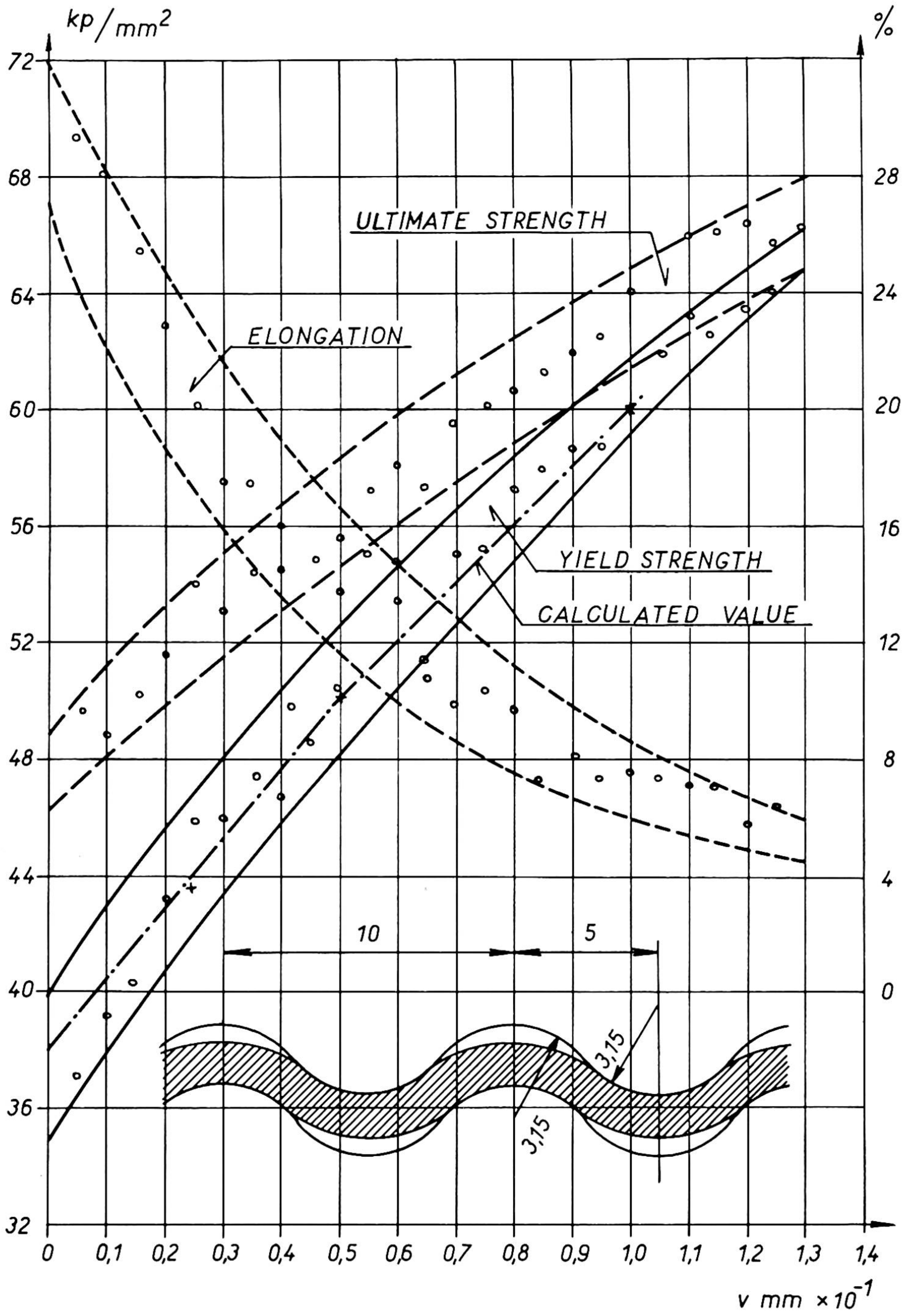
kp/mm²
%
72
68
64
60
56
52
48
44
40
36
32
28
24
20
16
12
8
4
0
ULTIMATE STRENGTH
ELONGATION
YIELD STRENGTH
CALCULATED VALUE
10
5
3,15
3,15
0 0,1 0,2 0,3 0,4 0,5 0,6 0,7 0,8 0,9 1,0 1,1 1,2 1,3 1,4
v mm ×10⁻¹

Fig. 5

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SUMMARY

The effect of strain hardening on the mechanical properties of cold-formed members depends on many factors of the method of forming used. Strain hardening can occur in the corners of the section, or - if the members are made in a special way - in the whole section. Brake-forming raises the yield strength by about 10 %. For cold-rolled members, the increase in yield strength is about 20 %. The highest increase in yield strength of the whole section - about 70 % - is achieved by raising the yield strength of the whole section through cold-waving of the sheet prior to the cold-forming operation proper.

The paper presents a method which makes it possible to calculate the increased yield strength of a sheet thus cold-wave formed to a relatively high degree of accuracy.

The relation between stress and strain in the plastic range is expressed by the parabolic law

$$\sigma = K \varepsilon^n$$

The modulus of plasticity K and the strain hardening exponent n vary with the type of steel used in dependence on the strength-yield strength ratio. Analytic relations for the strain and shear strain and for their composite effect are derived for sheets cold-formed in waves of different shapes. As the calculations imply, shear strains exercise the decisive effect on strain hardening and the increase in yield strength.

For sheets with differently shaped waves the theoretical values were compared with the results of tests made in the whole range of deformation, and found to agree well.

RÉSUMÉ

Le durcissement dû à une déformation à froid dépend de beaucoup de facteurs de la méthode de formation utilisée. La méthode la plus efficace (augmentation de 70 %) tend à durcir toute la section de la tôle en lui donnant une ondulation à froid avant l'opération propre de mise en forme définitive. Le présent travail permet de calculer la valeur exacte du durcissement pour ce dernier cas.

ZUSAMMENFASSUNG

Die Wirkung der Härtung auf die mechanischen Eigenschaften kaltgereckter Stähle hängt von mehreren Faktoren der Verformungsweise ab. Die Härtung kann in den Ecken oder, sofern das Stück in spezieller Weise hergestellt worden ist, im ganzen Querschnitt erreicht werden. Die höchste Heraufsetzung der Streckgrenze im ganzen Querschnitt, ungefähr 70 %, ist dadurch erreicht worden, dass man vor der endgültigen Kaltverformung das Blech kalt verwellte. Die vorliegende Arbeit erlaubt die Berechnung der Härtungswerte für den letzten Fall.