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Decision Theory Approach to Structural Optimization

Optimisation des constructions à l'aide de la "théorie des décisions"

Stellungnahme der Entscheidungstheorie zur Bauwerksoptimierung

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Some of the optimization problems mentioned by M. Cambon are similar in principle to those treated in the theory of decision making under uncertainty that has been developed in recent years. Such theories are of considerable potential value in design since they provide a logical framework in which engineering judgement can be applied realistically and consistently.

1. FUNDAMENTALS OF DECISION THEORY

The theory of decision under uncertainty deals with the problem of choosing "optimum" courses of action when the results of action involve uncertain future events and a variety of possible consequences. To satisfy the basic requirements of the theory, a designer must establish four sets of basic data before decision:

(i) a set of feasible actions a_i from which he must choose one,

(ii) a set of possible outcomes o_{ij} resulting from the choice of any action a_i,

(iii) a probability measure specifying the probability p_{ij} that o_{ij} will occur if he chooses any action a_i,

(iv) a set of value losses L_{ij} specifying the relative undesirability of all outcomes o_{ij}.

Extensions to these basic requirements have been made when experiments are performed prior to final action [1]. Provided that a designer's values follow certain rules of consistency [2], the relative desireability of any action is given by its total expected loss

$$T_i = \sum_j p_{ij} L_{ij} \tag{1.1}$$

and the action with least total expected loss is the optimum action.

In structural design alternative actions may involve different structural forms, materials, construction methods, and member sizes. Outcomes may involve initial costs, construction delays, appearance, serviceability and safety.

Prior to design outcomes cannot be predicted because of uncertainties in costs, material behaviour, foundation conditions and future loadings. In general there are insufficient data to establish statistical distributions and a designer must use judgement to establish subjective probability measures. It has been shown [3] that provided several consistency requirements are satisfied, subjective probability measures obey the conventional rules of probability. However subjective probabilities change as new information becomes available. The formal mechanism for modifying subject distributions is Bayes' Theorem [4]. Applications of subjective probabilities have been made to the strength of concrete structures [5] and the prediction of earthquake probabilities.

In order to formulate an optimization criteria, the relative value of all outcomes must be expressed on a single scale. Some outcomes such as construction costs and time delay involve monetary losses which can be measured directly. The value of other losses must be measured on the same scale as monetary losses from a set of elementary decisions. Thus, for example, structural collapse may only involve economic losses of the order of initial costs but must be assigned much higher relative values for moral, social, and professional reasons.

To illustrate the essentials of a decision theory approach, consider a simple example involving the preliminary design of a structure in which two basic schemes are under consideration. It is assumed that both schemes would be designed to provide the same safety and serviceability. The only variables under consideration are appearance, construction costs and construction delay.

To establish relative values of appearance the following basic decision is made. If both schemes were completed at the same time, the first scheme would be preferable if its initial costs were not more than \$25,000 greater than those of the second scheme. Denoting the losses associated with aesthetics of the schemes as A_1 and A_2 the basic decision implies that $A_2 = A_1 + \$25,000$. After further investigation, a designer might estimate the range of possible initial costs and time delays and their respective probabilities for the two schemes as shown in Table 1.

TABLE 1

Scheme	Outcome Losses and Probabilities					
	Initial Costs ($000)	Prob.	Time Delay ($000)	Prob.	Appearance ($000)	Prob.
I	90 100 110 120 130	0.1 0.3 0.3 0.2 0.1	0 5 10 15	0.2 0.3 0.4 0.1	A_1	1
II	80 90 100 110	0.1 0.4 0.4 0.1	0 5 10	0.4 0.4 0.2	A_1 + 25	1

The numbers in this example were chosen to illustrate the common problem of conflicting objectives. The first scheme is more aesthetically pleasing than the second but probably would cost more and is less likely to be completed on time. The total expected loss for the first scheme is A_1 + $116,000 while the second scheme has an expected loss of A_1 + $124,000. Hence the first scheme is preferable to the second.

2. APPLICATIONS TO SAFETY ANALYSIS

Although decision theory can be applied to any situation in which a designer can formulate the necessary sets of basic data, existing studies have concentrated on design strength decisions. In many cases, alternative actions can be grouped into schemes with alternatives in a scheme essentially identical in appearance, functional efficiency and ease of construction. Optimization within such a scheme involves the choice of member sizes and proportions on the basis of initial costs and safety alone. Solution of such restricted optimization problems is the purpose of standard design procedures.

Each alternative in a scheme is associated with a variety of possible resistances and future loads. Since future conditions are uncertain, probability measures must be applied to the families of possible loads and resistances. Such probabilities must be at least partially subjective to allow for uncertainties in workmanship, methods of analysis, and load idealization. Using well known methods of analysis, load and resistance distributions can be combined to obtain failure probabilities [6].

If attention is restricted to one class of failure, the decision table for optimization within a scheme takes the form shown in Table II.

TABLE II

Alternatives	Outcome Losses and Probabilities			
	Success	Probability	Failure	Probability
a_1	S_1	$(1 - P_1)$	F_1	P_1
a_2	S_2	$(1 - P_2)$	F_2	P_2
"	"	"	"	"
"	"	"	"	"
a_n	S_n	$(1 - P_n)$	F_n	P_n

Where S_i and F_i are the success and failure losses if alternative a_i is chosen.

In view of the definition of a basic scheme, it is reasonable to assume the following loss relationships.

$$S_i = I_i + S \tag{2.1}$$

$$F_i = I_i + S + F \tag{2.2}$$

where I_i is the initial cost of alternative a_i. The constants S and F are success and failure losses in excess of initial costs for the scheme. With these losses, the optimum alternative has a total loss given by:

$$T_{opt} = \min (I_i + p_i F) \tag{2.3}$$

except for the constant S that is irrelevant to decision. A similar optimization rule has been suggested by other authors [7]. However in this development, the failure loss F is not simply the capitalized economic losses associated with failure but may be, and generally is, greater than such losses.

The relationship between total losses and a safety factor n can be deduced qualitatively for many common structures. Since initial costs generally increase relatively slowly with increasing n, and failure probabilities decreases relatively quickly, total losses decrease quickly, reach a minimum and then increase slowly eventually merging with initial costs. This behaviour is shown symbolically in Figure 1. In practice, costs and failure probabilities would be discontinuous functions of n but the trends indicated can be expected in many situations.

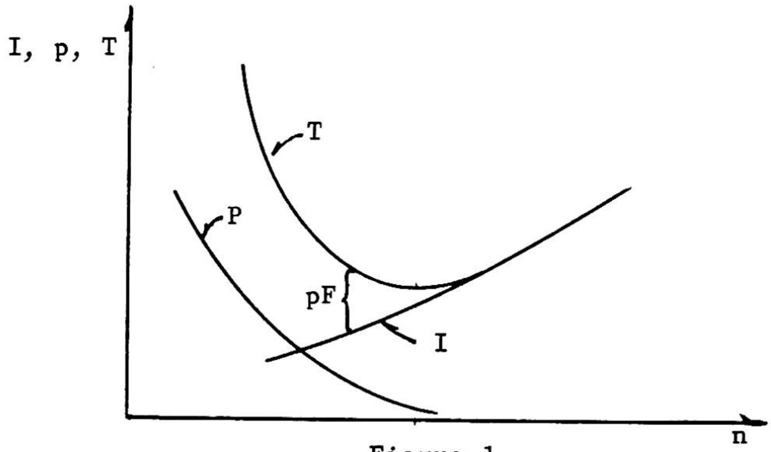

Figure 1

The behaviour of total losses for large n and low failure probability is of particular interest. If, for example, an alternative has an initial cost of \$1 x 10^6, failure losses equal to 100 times initial costs and a failure probability of 1 x 10^{-6}, its total loss is equal to \$1,000,000 + \$100. Obviously the loss of \$100 associated with failure is negligible in comparison to the initial cost.

A formal basis for neglecting failure losses may be established by stating that small variations in initial costs are irrelevant. Such a statement implies that initial costs must be rounded off in forming expected losses. To ensure consistency, failure losses must be rounded to the same level of precision. In the preceeding example, if initial costs are rounded to the nearest \$1,000, any failure probability less than 5 x 10^{-6} results in zero failure loss. In other words, any failure probability less than 5 x 10^{-6} is effectively zero.

With a restriction on sensitivity to initial costs, the general behaviour of initial costs and total losses shown in Figure 1 becomes that shown in Figure 2.

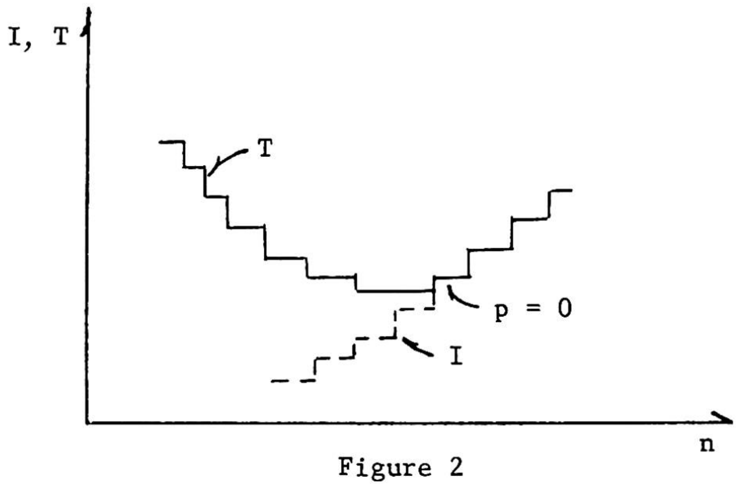

Figure 2

A number of alternatives may have equal initial costs and failure losses, and a number of alternatives may have effectively zero failure probability. The least costly alternative with zero failure loss is always either equally or more safe and costly than the theoretically optimum alternative. Computations under a variety of conditions indicate that an order of magnitude estimate of failure losses may be sufficient for practical applications [8].

An alternative formulation based on the probabilities and values associated with strength alone has been applied by Burnes to the choice of capacity reduction factors in reinforced concrete design [9]. In this study, subjective probability measures and the values associated with various real strengths were obtained from several individuals. After modification of initial probability measures by the use of Bayes' Theorem and experimental data, optimum code values of reduction factors were obtained.

Both of the preceeding formulations admit design based on the traditional concept of absolute safety. Effective absolute safety can result from insensitivity to initial costs and small failure losses. True absolute safety can be achieved if a designer believes that certain conditions cannot occur and assigns zero probability to such conditions. However, there can be no objective procedure for establishing true absolute safety.

3. USES AND LIMITATIONS OF DECISION THEORY

The unfortunate fact that experimental evidence is rarely a sufficient basis for design is well known in structural engineering practice. Inevitably objective fact must be combined with intuition and experience. However, in conventional design decisions rational analysis is confined to the study of physical phenomena and judgement is applied in a more or less arbitrary manner. In a decision theory approach judgement is explicitly recognized as an essential component of design and an attempt is made to analyze its operation. In effect, the art of engineering is assumed to be an intuitive science.

Any formulation recognizing engineering judgement obviously can not yield optimum designs in an absolute sense. All optimization is relative to the alternatives considered, the existing state of technology and powers of individual intuition. However, one can hope to discover the rules of good judgement and to establish a logical framework in which judgement can be used to best advantage. In particular, statements of judgement can be reduced to their simplest and most meaningful form and logical consistency can be achieved.

In the simple theory developed to date, the decision process has been abstracted into two basic components - prediction and value systems. In addition, two fundamentals postulates have been adopted. The first is that a rational individual operates as an intuitive probability analyzer when making predictions. The second is that in an uncertain situation a rational individual operates as an intuitive expected loss minimizer with constant values. The first of these postulates is almost certainly correct in principle. However, subjective probabilities are seldom numerically precise. The second postulate is more questionable and must be verified experimentally.

Possibly the greatest potential use of decision theory is a vehicle for stating opinions in group decisions. At the present time there is often considerable dispute over the numerical factors to be used in standard design procedures. Such differences of opinion probably arise because individuals have different estimates of probability distributions and different value systems. By decomposing the choice of such factors into a large number of separate small statements of opinion, it may be possible to isolate the differences between individuals and establish reasonable agreement.

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