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Objekttyp: **Article**

Zeitschrift: **IABSE congress report = Rapport du congrès AIPC = IVBH Kongressbericht**

Band (Jahr): **10 (1976)**

PDF erstellt am: **24.05.2024**

Persistenter Link: https://doi.org/10.5169/seals-10508

Preponderance of Idealization in Structural Optimization

Prépondérance de l'idéalisation dans les problèmes d'optimisation structurale

Die überragende Bedeutung der Idealisierungen bei der Optimierung von Tragwerken

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The optimal design of a structure may be divided in two steps. In the first one - the *idealization* - the structural problem is put in following mathematical formulation :

"Find $\vec{X}$ such that :

$$f_k(\vec{X}) < 0 \quad \text{for} \quad k = 1,2,\ldots m \; ; \qquad (1)$$
$$h_j(\vec{X}) = 0 \quad \text{for} \quad j = 1,2,\ldots l \; ;$$

and :

$$F(\vec{X}) = \text{minimum (maximum)}"$$

where $\vec{X}$ is a vector which contains the design variables,

f and h are the constraints of the problem,

and F is the objective function to optimize.

The second step - the *solution process* implies (a) the choice of the solving procedure and (b) the search of the solution of the problem formulated as in (1).

In the opinion of the authors, a good idealization is the basic condition for obtaining a good value of the solution, while a more or less refined mathematical treatment of it plays a rather secondary role |1| .

In many papers of the literature, emphasis is too often brought on the choice of the solution procedure rather than on that of a heuristic which does not modify in anyway the sense of the actual problem.

So long as the structural problem is small - about ten variables and constraints - many methods are available in the literature. However, various numerical experiments have shown that the choice of a method depends on the problem to be solved, for most of the algorithms cannot be used economically in all cases |2|. As a consequence, conclusions concerning the use range and the efficiency of an algorithm for a structural problem can rarely be extended to another one.

If emphasis is almost brought on the idealization, the designer may be sure of obtaining a realistic solution of the problem and, in addition, important simplifications in the mathematical treatment of the second step become possible. Indeed, on one way, a judicious choice of variables or an ingenious variable transformation often enable to present the complex problem in a more simple form, and, on another way, by means of a previous evaluation of the several variables, the designer can establish a hierarchy of the variables and divide the complex problem into smaller ones, which are then more easier to solve quickly.

For example, in |3|, MYLANDER demonstrates that a rather simple variable transformation changes a mathematical non-linear and non-convex problem into a linear programming system. It is worthwhile to recall the following basic non-linear problem which is considered as a very difficult one. The objective function is :

$$f(x) = b_o + a_{o1}\, x_1 + \left(\sum_{j=2}^{5} a_{oj}\, x_j\right) x_1 \rightarrow \min$$

subject to constraints :

$$0 \leqslant a_{i1}\, x_1 + \left(\sum_{j=2}^{5} a_{ij}\, x_j\right) x_1 \leqslant b_i \quad i = 1,2,3 \qquad (2)$$

$$x_1 \geqslant 0 \;;\; 1.2 \leqslant x_2 \leqslant 2.4 \;;\; 20.0 \leqslant x_3 \leqslant 60$$

$$9.0 \leqslant x_4 \leqslant 9.3 \;;\; 6.5 \leqslant x_5 \leqslant 7.0.$$

where the values of the constants are :

a_{01}	= - 8,720,288.795	a_{21}	= - 155,011.1055
a_{02}	= - 150,512.524	a_{22}	= 4,360.5334
a_{03}	= - 156.695	a_{23}	= 12.9492
a_{04}	= - 476,470.319	a_{24}	= 10,236.8839
a_{05}	= - 729,482.825	a_{25}	= 13,176.7859
a_{11}	= - 145,421.4004	a_{31}	= - 326,669.5059
a_{12}	= 2,931.1506	a_{32}	= 7,390.6840
a_{13}	= - 40.4279	a_{33}	= - 27.8987
a_{14}	= 5,106.1920	a_{34}	= 16,643.0759
a_{15}	= 15,711.3600	a_{35}	= 30,988.1459
b_o	= - 24,345.0	b_2	= 294,000.0
b_1	= 294,000.0	b_3	= 277,200.0

By putting, according to MYLANDER

$$y_i = x_1 \cdot x_i \qquad i = 2,\ 3,\ 4,\ 5$$

and $\qquad$ (4)

$$y_1 = x_1$$

above non-linear problem takes following linear formulation :

$$g(y) = b_o + \sum_{j=1}^{5} a_{oj}\, y_j \rightarrow \min$$

$$0 \leqslant \sum_{j=1}^{5} a_{ij}\, y_j \leqslant b_i \quad i = 1,\ 2,\ 3 \qquad (5)$$

$$y_i \geqslant 0 \qquad i = 1,\ 2, \ldots,\ 5$$

$$y_2 - 1.2\, y_1 \geqslant 0 \;;\quad 2.4\, y_1 - y_2 \geqslant 0$$

$$y_3 - 20.0\, y_1 \geqslant 0 \;;\quad 60.0\, y_1 - y_3 \geqslant 0$$

$$y_4 - 9.0\, y_1 \geqslant 0 \;;\quad 9.3\, y_1 - y_4 \geqslant 0$$

$$y_5 - 6.5\, y_1 \geqslant 0 \;;\quad 7.0\, y_1 - y_5 \geqslant 0.$$

which may directly solved by means of the classical simplex routine.

The optimal solution, obtained after six iterations, is given by :

$$g = -\,5{,}280{,}344.9$$

$$y_1 = 4.53743 \;;\quad y_2 = 10.88983 \;;\quad y_3 = 272.24584 \qquad (6)$$

$$y_4 = 42.19811 \;;\quad y_5 = 31.76202$$

which in terms of the original variables gives $f = -\,5{,}280{,}344.9$

$$x_1 = 4.53743 \;;\quad x_2 = 2.40000 \;;\quad x_3 = 60.00000 \qquad (7)$$

$$x_4 = 9.30000 \;;\quad x_5 = 7.00000.$$

The solution of the original problem by means of non-linear programming methods |4, 5| lead, after a lot of iterations, to values of f which are 2 or 3 % below the true optimum but, in some cases, with value of the variable x_3 which is about 50 % erroneous.

In |6|, the authors show how a suitable choice of the behaviour model for a complex structural design - indeterminate prestressed bridges - leads to a benefit similar to that obtained by MYLANDER.

The idealization of the problem is based on an approach with sensitivity coefficients, as that proposed by GURUJEE |7|, and on a variable transformation; it is then allowed to solve this complex design problem by means of linear programming, without the actual problem be denaturated and taking account of all the technological requirements (cover thickness, anchorage dimensions, redundant effects of prestressing, friction losses, anchorage slippage,...). After the variable transformation, the problem remains partially non-linear but the authors have shown in |8| that the non-linear term, being of the order of 1 % with respect to its corresponding linear component, may be neglected in practice.

The authors would like to conclude by saying that for optimum design, as for all the other engineering activities, mathematics are a good servant but a bad master.

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SUMMARY

In structural optimization problems, it is nearly always observed that, in the search for a realistic solution, the suitability of idealization is more important than the choice of the solving algorithm.

RESUME

Dans les problèmes de dimensionnement optimal, il est généralement constaté que la recherche d'une solution réaliste dépend davantage de l'idéalisation du problème que du choix de l'algorithme de résolution.

ZUSAMMENFASSUNG

Bei der Optimierung von Tragwerken wird allgemein festgestellt, dass die Suche nach einer realistischen Lösung mehr von der Idealisierung des Problems als von der Auswahl des Lösungsalgorithmus abhängt.