The distribution of stresses around the end anchorages of prestressed concrete beams: comparison of results obtained photoelastically with strain gauge measurements and theoretical solutions

The Distribution of Stresses around the End Anchorages of Prestressed Concrete Beams. Comparison of Results Obtained Photoelastically with Strain Gauge Measurements and Theoretical Solutions

Répartition des contraintes dans les ancrages d'extrémités des poutres précontraintes et comparaison des résultats de la photo-élasticité avec les mesures de tension et les solutions théoriques

Die Spannungsverteilung bei den Endverankerungen vorgespannter Träger und Vergleich der fotoelastischen Resultate mit Spannungsmessungen und theoretischen Lösungen

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Introduction

The heavy loads applied to cables or bars for prestressing concrete beams are anchored at the ends of the beams and set up high stresses in the concrete around the anchorages. To avoid failures under excessive compressive or tensile stresses, the concrete anchorage blocks have to be reinforced with mild steel.

There are two methods which can be used to design the mild steel required, one was given by the late professor G. Magnel [1][1]) and the other by Y. Guyon [2]. In both methods, the end-anchorage problem was simplified by their authors who assumed a two-dimensional stress-distribution and ignored the effect of the inclination of the cables and the cable ducts. Figure 1 shows the type of problem considered by Magnel and Guyon.

In assessing the form of distribution, Magnel considered the end block of a prestressed concrete beam acting as a beam subjected to a bending stress $\widehat{xx}$, a shear stress $\widehat{xy}$, and a compressive stress $\widehat{yy}$. He assumed that along any longitudinal section the stresses $\widehat{xx}$ and $\widehat{xy}$ vary in the direction of the length of the beam in the form of a cubic parabola, and the stress $\widehat{yy}$ remains constant.

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[1]) Numbers refer to items in the list of references.

Guyon obtained a theoretical solution [3] of the semi-infinite solid under a point load. To introduce the effect of the longitudinal boundaries, he made assumptions which are not compatible with the equations of equilibrium or the equation of compatibility of the Mathematical Theory of Elasticity.

These considerations and also failures of end anchorages in practical cases of prestressed concrete beams designed by both methods indicate that the simplified solutions obtained by Magnel and Guyon lead to designs which are

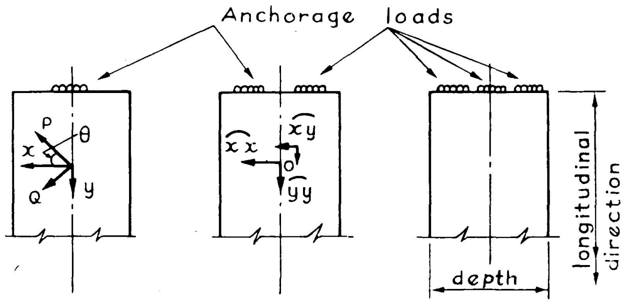


Fig. 1. Problem considered by Magnel and Guyon.

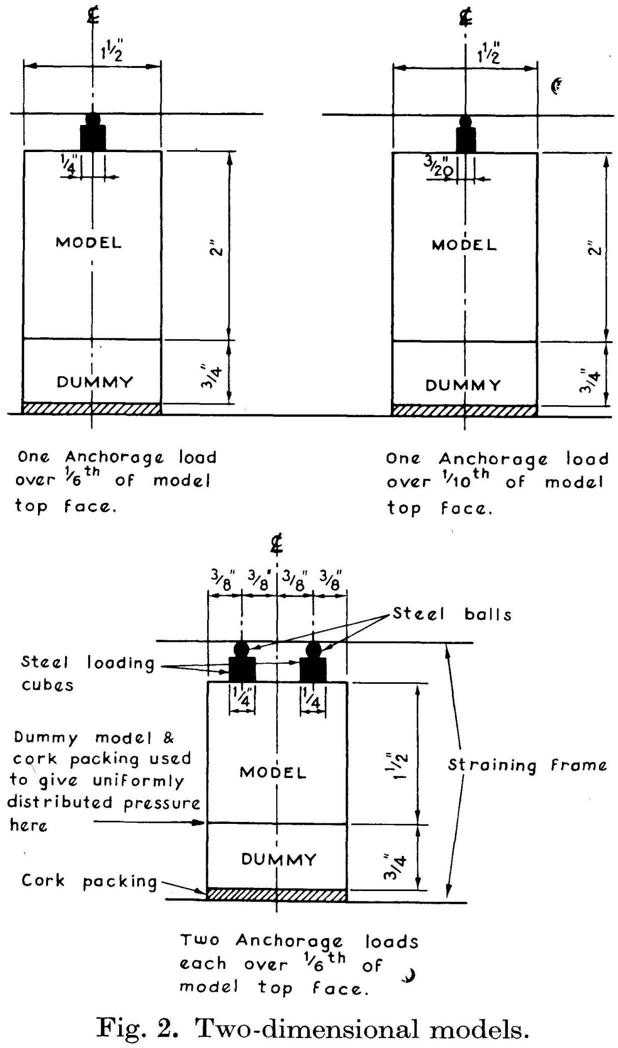


Fig. 2. Two-dimensional models.

not safe. This is also shown from the results of photoelastic [4] tests carried out on a number of models.

In photoelasticity [5], scale models made in transparent materials are loaded at high temperatures and are then allowed to cool under load. The residual stress system which is "frozen" into the model reproduces the system due to the load and can be investigated. The stresses are measurable from a pattern of interference fringes which are seen in the model when viewed in a polariscope. Two-dimensional observations on slices cut from the three-dimensional models give the magnitudes of the shear stresses and the directions of the principal stresses in the planes of the slices, while the separate principal stresses can be determined by calculations based upon the differential equations of equilibrium.

Three-dimensional stress distributions can be affected by differences in Poisson's ratio between prototypes and models, and electrical resistance strain gauge measurements on concrete beams were taken and compared with the photoelastic results. Additional comparisons were made between the photoelastic results and simplified mathematical solutions.

The Two-dimensional Photoelastic Tests

These were preliminary to the three-dimensional investigation, and were designed to give a check to the solutions given by Magnel and Guyon, who claimed some agreement with early photoelastic work carried out in Holland [6].

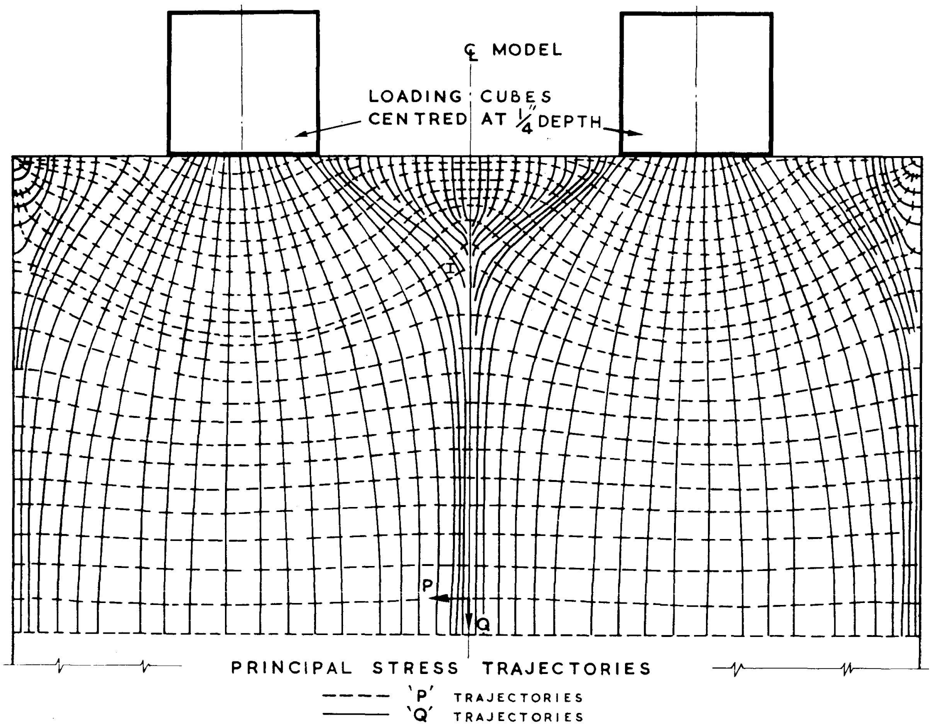


Fig. 3.

Some of the two-dimensional models investigated are shown in figure 2, where the method of loading is also indicated. Figure 3 gives typical principal stress trajectories, i. e. principal stress envelopes, and Figure 4 typical contours of maximum shear.

The Separation of the Principal Stresses in Two-dimensional Models

Referring to figure 1, *ox*, *oy* are the axes of reference, P, Q, the principal stresses, θ the inclination of the P-principal stress to the x axis and $\widehat{xx}$, $\widehat{yy}$, $\widehat{xy}$ the three stresses at the point 0. Using the well known equation:

$$\widehat{xy} = \tfrac{1}{2}(Q - P)\sin 2\,\theta, \tag{1}$$

the shear stress $\widehat{xy}$ was calculated from the quantities $(Q - P)$ and θ obtained photoelastically. From this the shear slope $\dfrac{\partial\,\widehat{xy}}{\partial\,y}$ was obtained and used in the equation of equilibrium in Cartesian co-ordinates:

$$\frac{\partial\,\widehat{xx}}{\partial\,x} + \frac{\partial\,\widehat{xy}}{\partial\,y} = 0, \tag{2}$$

to give values of $\dfrac{\partial\,\widehat{xx}}{\partial\,x}$. Values of the transverse stress $\widehat{xx}$ were then obtained by carrying out a step by step integration and starting from a point where the value of $\widehat{xx}$ was known. Using then the equations:

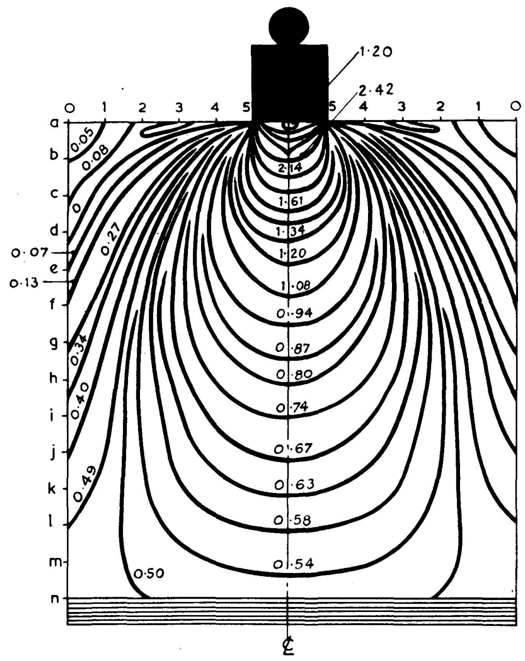


Fig. 4. Note. Numbers shown on curves multiplied by the mean compression give the maximum shear.

$$\widehat{xx} = \frac{P+Q}{2} + \frac{P-Q}{2}\cos 2\theta,$$
$$\widehat{yy} = \frac{P+Q}{2} - \frac{P-Q}{1}\cos 2\theta, \tag{3}$$

$(P+Q)$, $\widehat{yy}$, P, Q were computed.

Alternatively, the Lamé-Maxwell equations:

$$\frac{\partial P}{\partial s_1} + \frac{P-Q}{\rho_2} = 0,$$
$$\frac{\partial Q}{\partial s_2} + \frac{P-Q}{\rho_1} = 0 \tag{4}$$

where (s_1, ρ_1), (s_2, ρ_2) are the arcs and radii of curvature of the principal stress trajectories, were used where it was convenient, or to check the previous method. Figure 5 gives typical graphs of principal stresses.

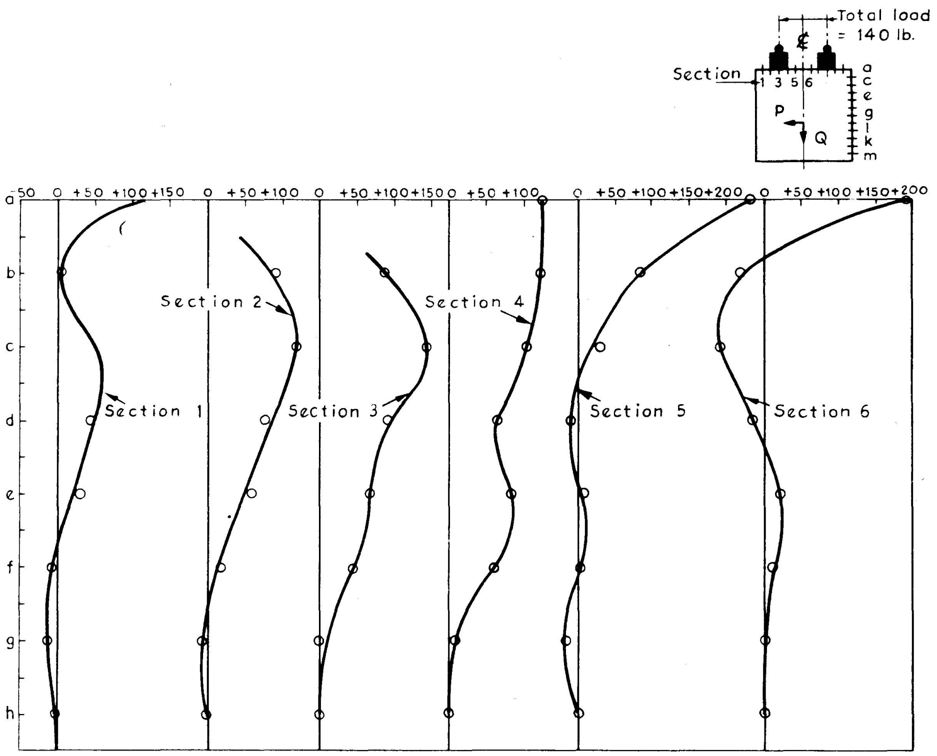


Fig. 5. Principal stress "P" in lb/in². Sign convention: (+ve) signs indicate tensile stresses, (−ve) signs indicate compressive stresses.

The Three-dimensional Photoelastic Tests

The three-dimensional models were made of "Araldite B" hot setting resin [4, 7], true to shape of the concrete prototypes, and approximately $^1/_{25}$ full size.

The prestressing cables were represented by brass wires, and the FREYSSINET or MAGNEL anchorages by round or rectangular washers.

Figure 6 shows the photoelastic pattern of the model of a prestressed concrete beam as viewed in the polariscope. This beam was used in the construction of the railway gantry given in figure 7. Typical photoelastic curves observed on a slice cut along the longitudinal plane of symmetry of the model are shown in figures 8 and 9.

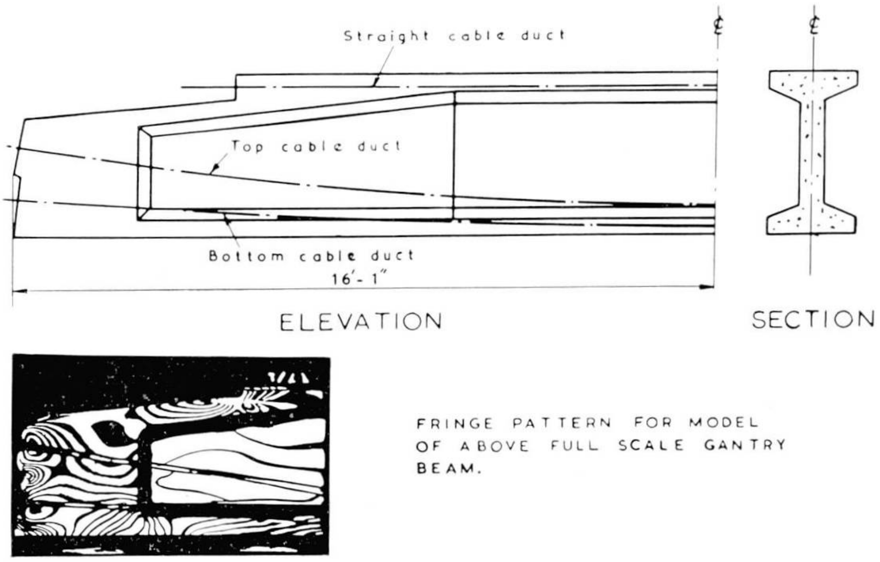


Fig. 6. Gantry Beam.

Fig. 7.

The separation of the principal stresses was carried out by integrational methods similar to those described for the two-dimensional models. Contours of the transverse principal stresses obtained by these methods on a model of uniform rectangular section prestressed with two cables are given in figures 10 and 11. The contours obtained from the corresponding two-dimensional photoelastic models and those obtained by GUYON's solution are given in

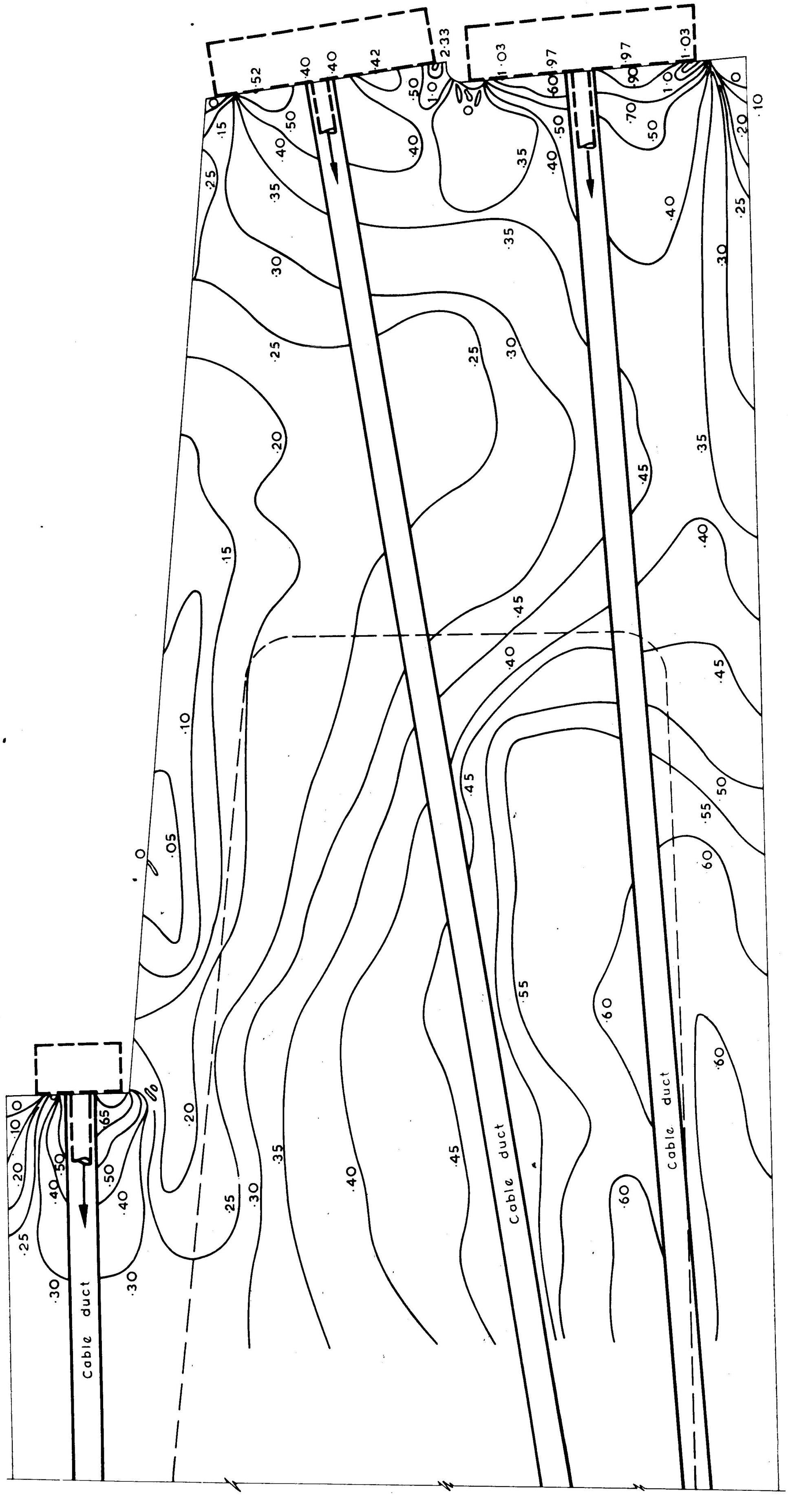


Fig. 8. Gantry beam. Photoelastic model. Central slice. Isochromatics giving R. retardations in fractions of wavelengths.

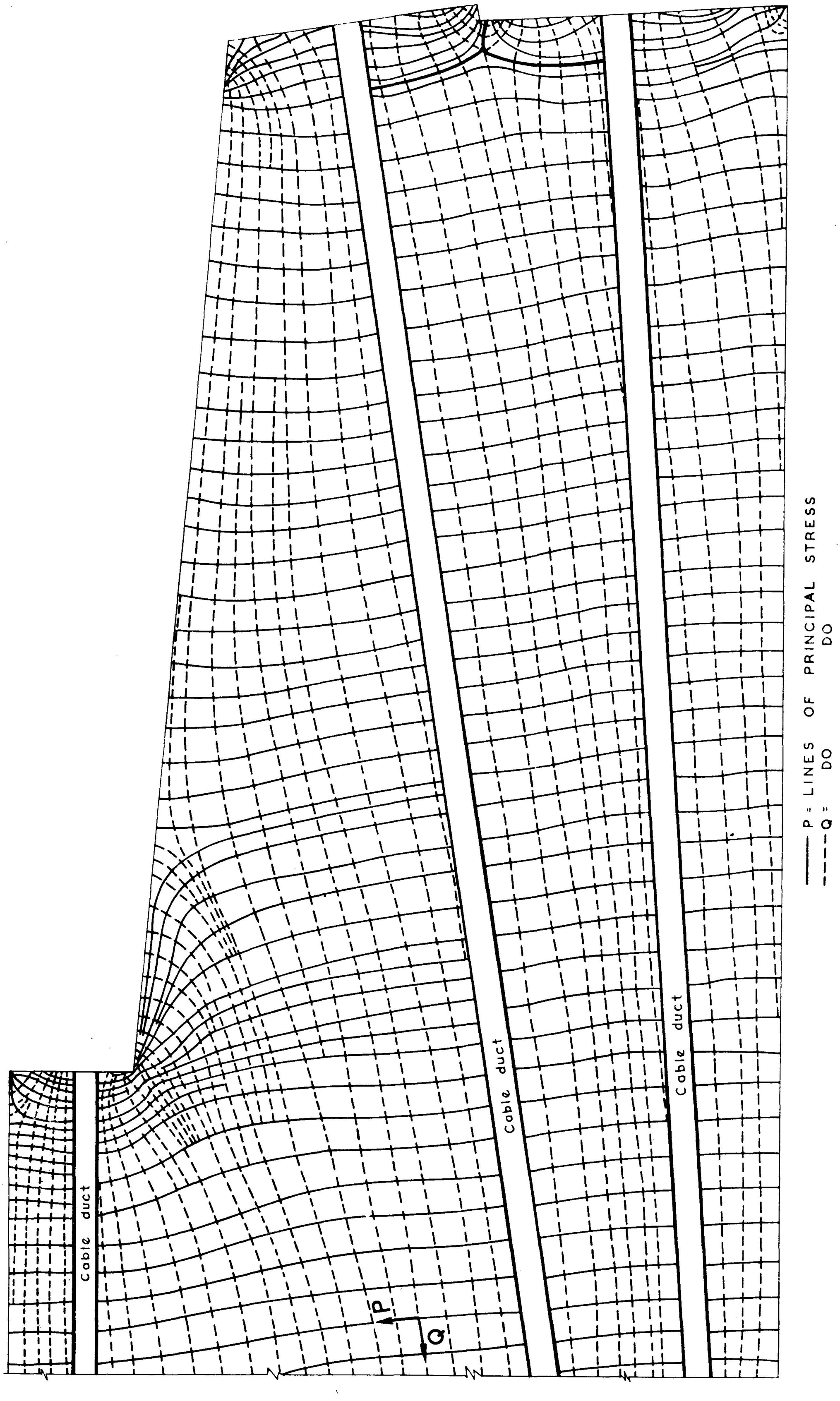


Fig. 9. Gantry beam. Photoelastic model. Principal stress trajectories. Central slice.

figure 12. GUYON's results appear quantitatively similar to the two-dimensional photoelastic results but give stresses which are different in distribution and considerably smaller than those obtained in the three-dimensional models. This is also shown from figure 13, where experimental and theoretical stresses were plotted.

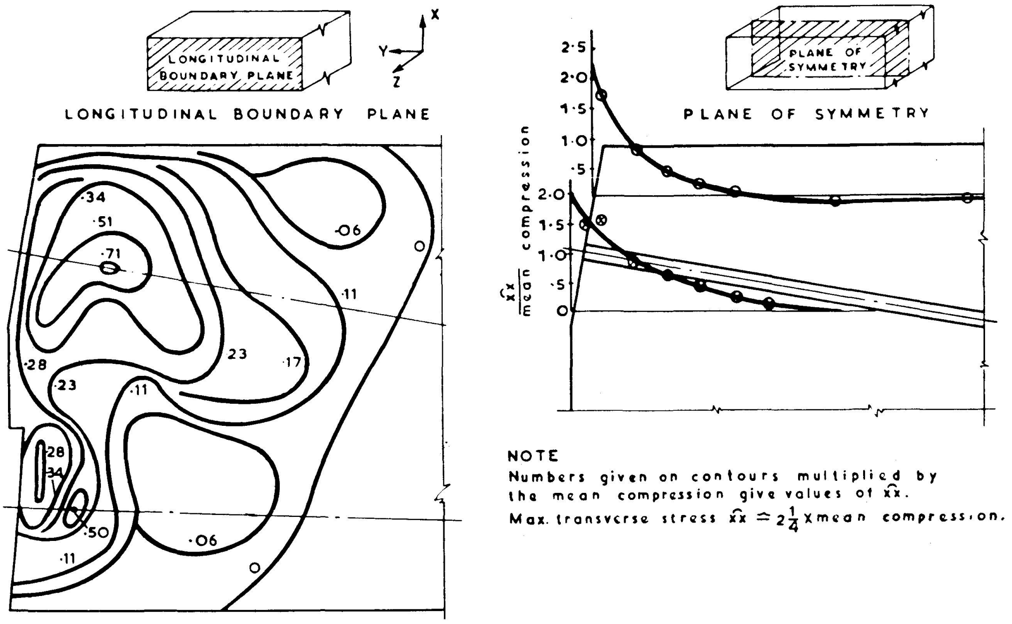


Fig. 10. Prestressed model of uniform rectangular section. Three-dimensional photoelastic results. Transverse stress $\widehat{XX}$.

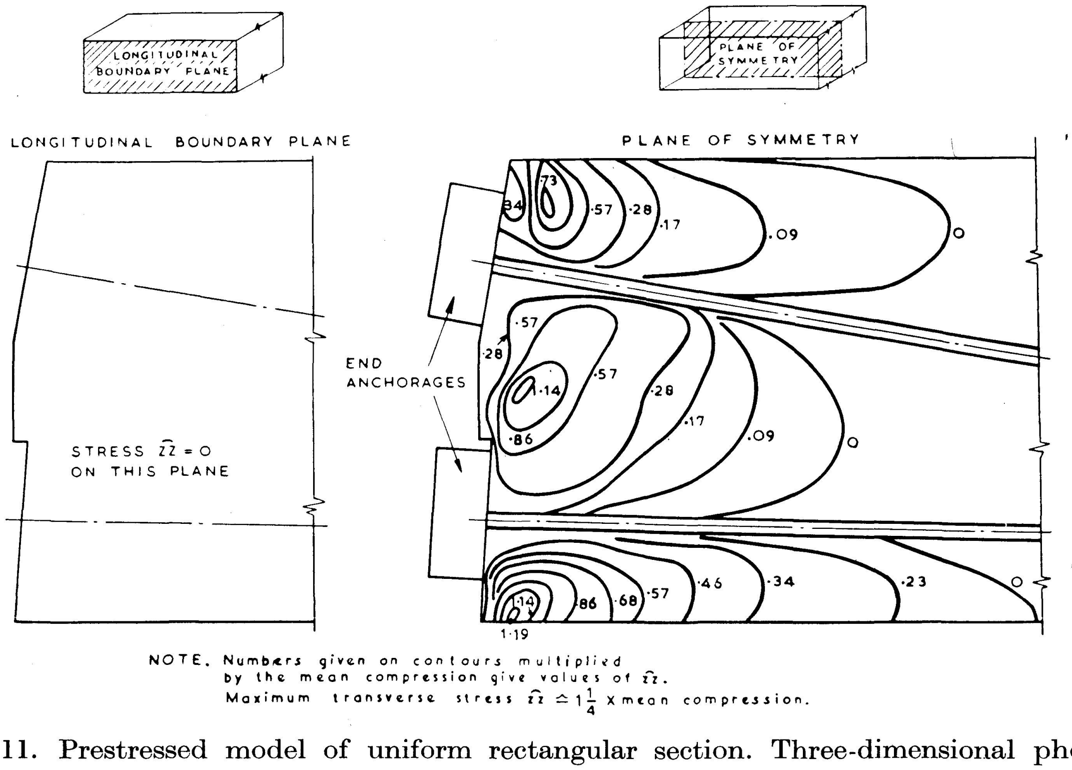
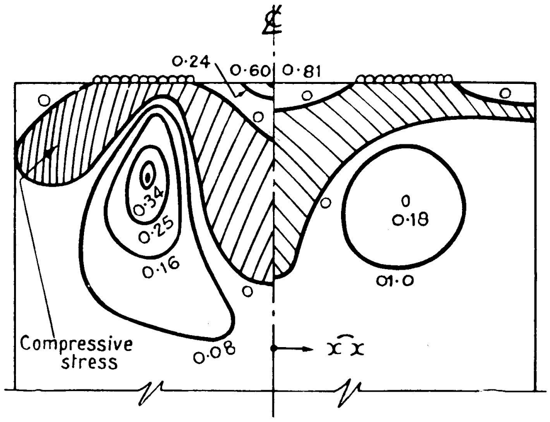


Fig. 11. Prestressed model of uniform rectangular section. Three-dimensional photoelastic results. Transverse stress $\widehat{ZZ}$.

Two-dimensional photoelastic modul



GUYON's solution

Fig. 12. Contours of transverse stress $\widehat{XX}$. Numbers marked on curves multiplied by the mean compression give the stress $\widehat{XX}$.

The Strain Gauge Measurements on Concrete Beams

To justify the use of plastic models representing concrete structures the strains due to prestressing and dead loads were measured on the surface and internally in a full size concrete beam used in the construction of the railway gantry shown in figure 7.

It was necessary to measure strains in two directions on the surface, and strains in four directions on the plane of symmetry, to calculate the stresses in these planes and compare them with those obtained photoelastically.

Electrical resistance strain gauges [8, 4], were fixed to the concrete surface, or to small steel frames to make strain gauge units which were waterproofed and cast into the concrete. Four of these units were wired to the reinforcement at each point in the plane of symmetry where the strains were required to be measured. On the concrete surface the strains were measured by 4 inch long electrical resistance strain gauges fixed to the concrete, or by a mechanical type strain gauge. Figure 14 shows the concrete end block during the tests.

The comparison of stresses calculated from the strains and the corresponding photoelastic stresses, given in figure 15, shows that no serious error is introduced by the difference of Poisson's ratio between the materials used for the models and the prototype, or through using an isotropic homogeneous elastic material for the photoelastic models representing concrete units.

The Mathematical Analysis of the Problem

It was first considered that in the two-dimensional problem, useful information could be obtained from a simplified mathematical investigation of the effect of interference caused by the two anchorages.

The longitudinal boundaries of the anchorage block were ignored, and a solution was obtained [9] by superposing a number of simple solutions derived from the Airy stress function.

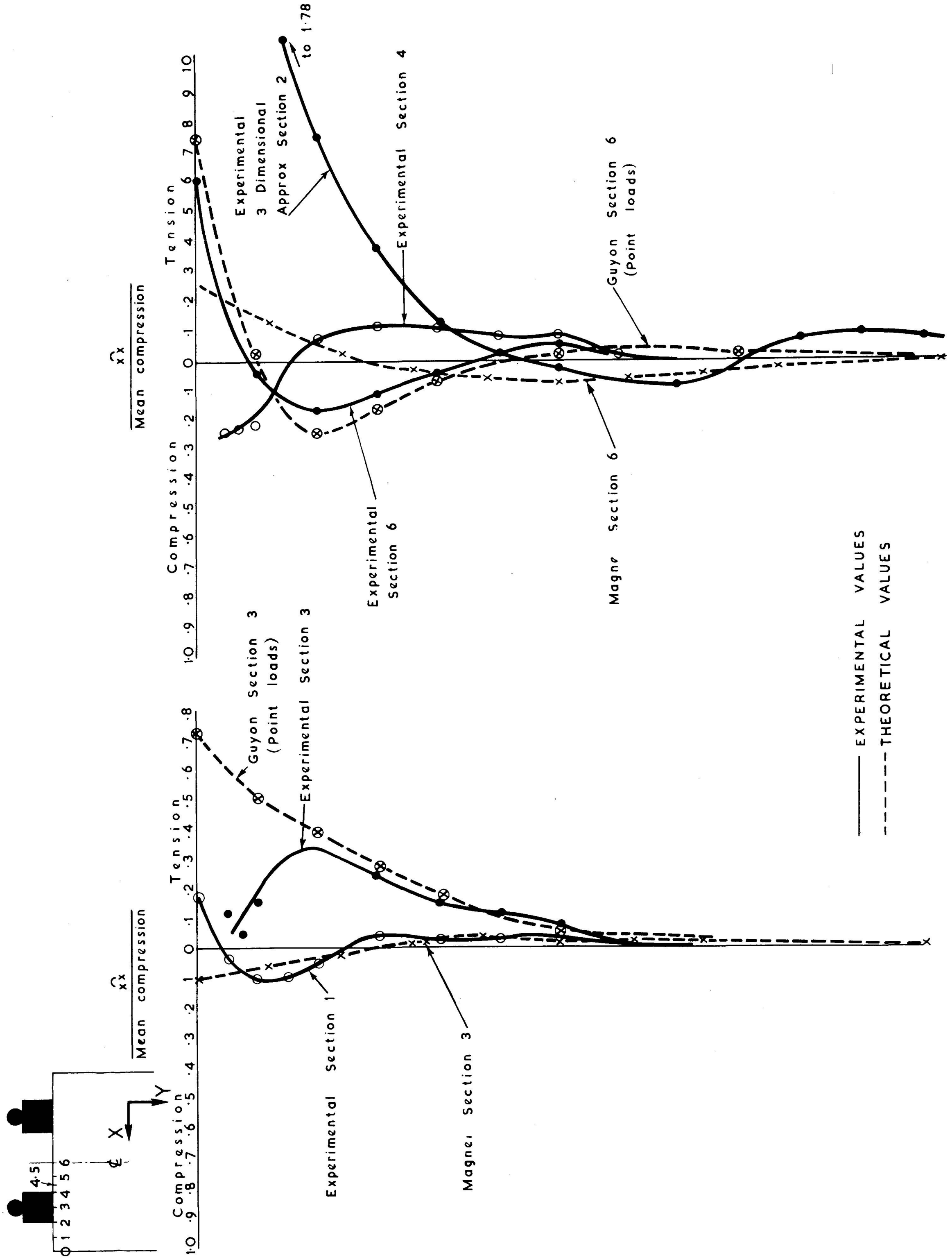


Fig. 13. Two-dimensional model. Transverse stress XX.

$$x = -\frac{P}{2\pi}(x^2+y^2)\tan^{-1}\left(\frac{y}{x}\right).$$

The curves corresponding to the photoelastic patterns were derived by using complex potentials.

Referring to figure 3, the isotropic point on the centre line was obtained mathematically at 0.35 ins. from the nearest edge of the model as compared with 0.20 ins. observed photoelastically. Also, the principal stress trajectories which approximately follow the centre lines of the loading cubes were confirmed in the mathematical solution.

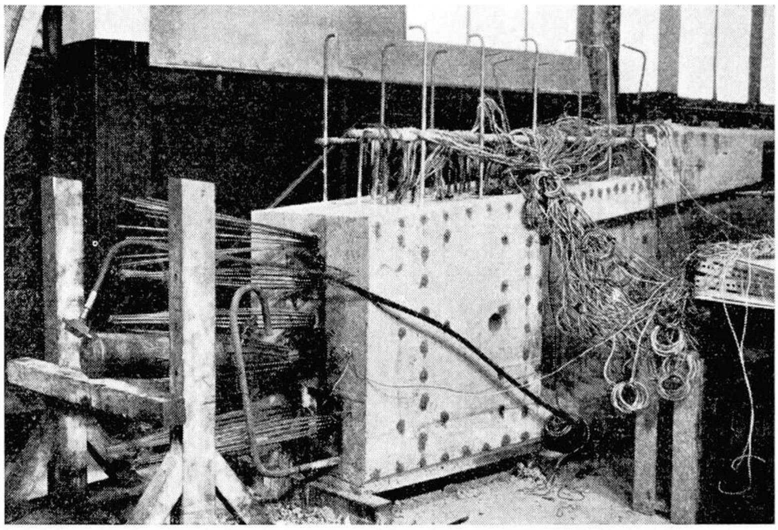


Fig. 14.

Conclusions

The stress distribution obtained by using simplified two-dimensional photoelastic models representing end blocks of prestressed concrete beams did not agree with previous experimental work and the theoretical solutions given by Magnel and Guyon. In the case of two anchorages the photoelastic results were confirmed by a simplified mathematical solution.

The use of photoelastic models to represent prestressed concrete end blocks was justified and the effect of Poisson's ratio was found to be negligible, by comparing the results of the three-dimensional photoelastic analysis carried out by the "frozen stress" technique and the full scale tests on concrete beams where surface and internal strains were measured, and used to calculate the stresses.

The transverse tensile stresses around the end anchorages of the three-dimensional models were found considerably greater than the corresponding stresses obtained by Guyon and Magnel.

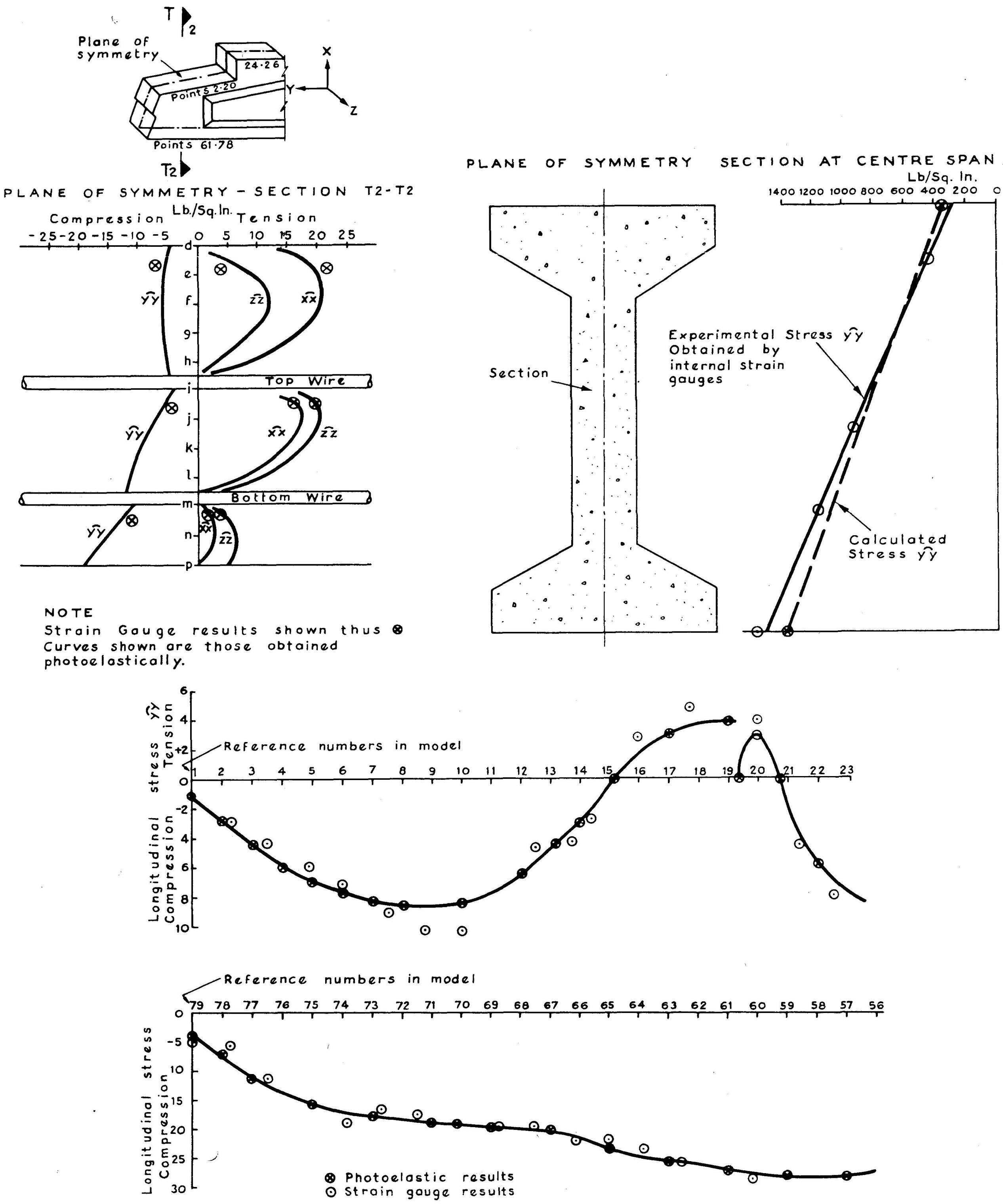


Fig. 15. Gantry beam. Comparison of photoelastic and strain gauge results.

Acknowledgements

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References

1. MAGNEL G.: "Prestressed Concrete" (Concrete Publications Ltd., 1950).
2. GUYON Y.: "Prestressed Concrete" (Contractors Record and Municipal Engineering, 1953).
3. — ("The International Association of Bridge and Structural Engineering", Vol. 11).
4. CHRISTODOULIDES S. P.: "A Photoelastic Investigation into End Anchorages of Prestressed Concrete Beams" (Ph. D. Thesis, University of London, 1955).
5. a) JESSOP H. T., HARRIS F. C.: "Photoelasticity" (Cleaver-Humes Press Ltd., 1949).
   b) FROCHT M. M.: "Photoelasticity" (Chapman and Hall Ltd., Vol. I, 1941, Vol. II, 1948).
6. "Ingenieur", Jaarg 41, 1926.
7. SPOONER H. J., McCONNELL L. D.: "An Ethoxylene resin for photoelastic work" (British Journal of Applied Physics, 1953, Vol. 4, pp. 181—184).
8. TROTT J. J.: "Prestressed Concrete — The Transverse Distribution" (Department of Scientific and Industrial Research, Road Research Laboratory, Note No. RN/1741/JJT, May 1952).
9. CHRISTODOULIDES S. P.: "A Two Dimensional Investigation of the End Anchorages of Post-tensioned Concrete Beams" (Structural Engineer, April 1955).

Summary

A photoelastic investigation of the stresses in the end anchorages of post-tensioned concrete beams was carried out on models of varied sections. The principal stresses in two-dimensional and three-dimensional models were calculated from the values of the shears and the directions of the principal stresses observed photoelastically. This was achieved by using the equations of equilibrium to integrate along a line starting from a point where a stress was known.

A strain gauge unit was designed to cast internally in the concrete and measure the strain in one direction. Four electrical resistance strain gauges were connected to form a Wheatstone bridge, and were suitably fixed on the unit. A number of these units were waterproofed and cast at various points around the end anchorages of a concrete beam, which was one of the precast members used in the erection of a railway overhead travelling gantry.

The strains on the surface of the concrete beam were measured by a mechanical strain gauge, and by electrical resistance strain gauges stuck on the concrete. The three-dimensional stresses computed from the strains agreed reasonably well with those obtained photoelastically.

The stress distribution affected by the interference between two cable anchorages was studied in a simplified mathematical solution, and found to be in agreement with the two-dimensional experimental results.

Résumé

Une étude photo-élastique des tensions dans les ancrages d'extrémité des poutres en béton post-contraintes a été effectuée sur des modèles de différentes sections. La contrainte principale dans les formes bi- et tri-dimensionnelles a été calculée à partir des efforts tranchants et de la direction observée photo-élastiquement de cette contrainte principale. Ce résultat a été obtenu en utilisant les conditions d'équilibre par intégration le long d'une droite, qui part d'un point avec contrainte connue.

Un élément de mesure de contrainte, à insérer dans le béton, a été étudié en vue de la mesure des contraintes suivant une direction. Quatre extensomètres électriques ont été associés sous la forme d'un pont de Wheatstone et fixés d'une manière appropriée à l'élément de mesure. Un certain nombre d'éléments de mesure de ce modèle ont été traités en vue de leur étanchéité à l'eau et insérés en différents points autour des ancrages d'extrémité, dans la poutre de béton. Il s'agissait de l'une des poutres préfabriquées qui étaient destinées à la construction d'un pont au-dessus d'une voie ferrée.

Les contraintes superficielles dans le béton ont été déterminées à l'aide d'extensomètres mécaniques et électriques collés sur le béton. Les contraintes tri-dimensionnelles calculées à partir des mesures concordaient d'une manière satisfaisante avec les résultats photo-élastiques.

La répartition des contraintes résultant du chevauchement de deux ancrages de câbles a été déterminée sous la forme d'une solution mathématique simplifiée; une bonne concordance a été constatée avec les résultats bi-dimensionnels expérimentaux.

Zusammenfassung

Eine fotoelastische Untersuchung der Spannungen bei den Endverankerungen von nachträglich vorgespannten Betonträgern wurde an Modellen verschiedener Querschnitte angestellt. Die Hauptspannung in zwei- und dreidimensionalen Formen wurde berechnet aus den Schubkräften und der fotoelastisch beobachteten Richtung der Hauptspannung. Dies wurde erreicht unter Benützung der Gleichgewichtsbedingungen durch Integration entlang einer Geraden, die von einem Punkt mit bekannter Spannung ausgeht.

Es wurde ein Spannungsmeßelement entworfen, das, im Beton eingegossen, die Spannungsmessung in einer Richtung gestattet. Vier elektrische Spannungsmesser wurden in Form einer Wheatstoneschen Brücke verbunden und in

geeigneter Weise am Meßelement befestigt. Eine Anzahl solcher Meßelemente wurde gegen Wasser gedichtet und an verschiedenen Punkten um die Endverankerungen im Betonträger eingegossen; der Träger war einer der vorfabrizierten Balken, die für die Errichtung einer Eisenbahnüberführung dienten.

Die Oberflächenspannungen des Betons wurden durch mechanische und elektrische Spannungsmesser, die auf den Beton aufgeklebt wurden, bestimmt. Die aus den Messungen errechneten dreidimensionalen Spannungen stimmten befriedigend überein mit den fotoelastischen Ergebnissen.

Die Spannungsverteilung durch Überdeckung zweier Kabelverankerungen wurde in einer vereinfachten mathematischen Lösung untersucht; es wurde Übereinstimmung mit den zweidimensionalen Versuchsresultaten gefunden.