Cyclic loading of portal frames: theory and tests

Cyclic Loading of Portal Frames. Theory and Tests

Portiques soumis à des cycles de charges. Théorie et essais

Zyklische Belastungen von Portalrahmen. Theorie und Versuche

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I. Introduction

The plastic methods for the design of steel frames are now well known, and in Great Britain alone more than 200 frames have been designed on this basis. The basic purpose of these methods is the provision of a *load factor* against collapse; the members of a frame are therefore proportioned so that the working loads, if multiplied by a suitable load factor, would just produce *plastic collapse*. In the absence of instability effects, plastic collapse will occur when a sufficient number of *plastic hinges* have formed to transform the frame into a mechanism. Several analytical methods now exist for determining the collapse load and mechanism for any given frame, and these methods have proved to be much simpler and more rapid in their application than the corresponding elastic methods of analysis. However, it is not possible to make the plastic methods of analysis the basis of a comprehensive design method until a fuller understanding of the phenomena of buckling of partially yielded members has been attained. Meanwhile, applications of the plastic methods are restricted to frames in which there is ample bracing against buckling.

Plastic design methods should only be employed when it is clear that the only way in which failure can occur is by plastic collapse, or at least that any other type of failure is far less likely. Thus for some structures, such as crane-bearing portals, failure may effectively occur when the deflections exceed permissible values, and a plastic design procedure might not be appropriate. If the loading on a structure is variable, and a number of repetitions of the peak loads of the order of several hundred thousand is likely to occur, the most probable type of failure is by fatigue, and so plastic design methods could not be used.

Apart from such cases as these, there remains another possibility to be considered, namely that a failure by plastic action other than plastic collapse may occur when the peak loads on a structure may be applied repeatedly for a comparatively small number of times [1, 2]. If a simple shed type of frame is considered as an example, the live loading will consist of snow and wind loads. It is possible for peak values of these loads to occur separately or in combination several times during the expected lifetime of the frame, although there can be no prior knowledge of the actual sequence of loading which will occur. *Variable repeated loading* of this kind can ultimately cause a failure by *incremental collapse* if the load intensities are sufficiently high. This type of collapse can occur if two or more critical load combinations follow one another in a more or less definite cyclic order; unacceptably large deflections might then be produced by applying a number of cycles of the order of magnitude of ten, with peak loads well below those required for plastic collapse. Failure by incremental collapse is caused by the successive formation of plastic hinges in the course of a loading cycle, at positions in the frame such that if all the hinges rotated simultaneously, the frame would deform as a mechanism. This process will be discussed more explicitly below.

Incremental collapse can generally occur when the peak load intensities are lower than the values required to cause plastic collapse. For structures which are subjected to variable repeated loading the question then arises as to whether failure by incremental collapse is more likely to occur than failure by plastic collapse. The importance of this question is obvious; if frames subjected to variable repeated loading were more likely to fail by incremental collapse than by plastic collapse, then the simple plastic design procedure would not be valid for such frames. However, Horne [4] has shown that if the theoretical predictions concerning incremental collapse are correct, this will not be the case. He argued that several applications of peak loads with values considerably in excess of the working values are required to produce incremental collapse, and that the probability of the occurrence of all these peak load combinations in the right sequence is very much less than the probability of occurrence of the single even greater peak load which would cause plastic collapse. If Horne's conclusions are accepted, it is thus only necessary to show that the theoretical predictions concerning incremental collapse are borne out in practice in order to confirm that the simple plastic design procedure can be applied when variable repeated loading is anticipated.

The object of this paper is to compare the actual behaviour of model steel frames under variable repeated loading with the predictions of theory. To this end, some analytical results are first presented, and then compared with the results of experiments on small rectangular portal frames of mild steel. It is shown that the predictions of theory are in reasonable agreement with the experimental results. The present paper gives full details of the experiments and of the comparison between theoretical and experimental results, supplementing preliminary accounts already published [2, 3]. MASSONNET [8] has described tests on beams, leading to conclusions similar to those derived from the present work.

It should be mentioned that under variable repeated loading another type of failure, termed *alternating plasticity*, is possible [1, 2]. This could occur if a member was bent back and forth repeatedly so that a particular fibre would yield alternately in tension and compression. This would produce what is basically a severe type of fatigue loading which could lead to fracture of the member. However, from tests carried out at the University of Sheffield [1, 2] it appears that mild steel beams can withstand fifty cycles of a loading producing a severe alternating plasticity condition without any signs of failure. Cases of variable repeated loading on a structure in which the number of peak load reversals would be of this order of magnitude are rare, and so the possibility of failure by alternating plasticity is not normally considered. A test has also been reported from Cambridge [2] in which a beam withstood a million applications of a loading which produced the fully plastic moment without showing any signs of fracture.

II. Illustrative Step-By-Step Calculations

Failure by incremental collapse is not so obvious to visualize as is plastic collapse under steady loading. However, it is possible to indicate the essential features of the process by means of step-by-step calculations for a portal frame, which are based on certain simplifying assumptions. A brief summary of some calculations of this kind will now be given.

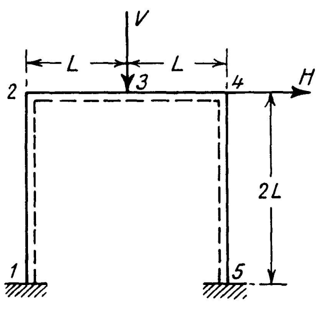
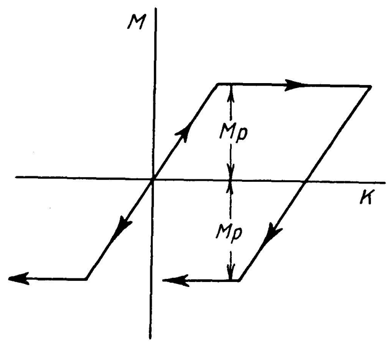


Fig. 1.

Fig. 2.



The calculations refer to the rectangular portal frame of span and height $2L$ which is shown in fig. 1. In this frame all the joints are supposed to be rigid, and the stanchions are assumed to be rigidly built in at their feet. To simplify the calculations it is assumed that each member of the frame has the same fully plastic moment M_p and behaves elastically unless the magnitude of the bending moment becomes equal to M_p, so that the bending moment-curvature relation is of the ideal type shown in fig. 2. In practice, of course, mild steel beams always develop some plastic flow before the fully plastic moment is attained, but the neglect of this effect does not influence the qualitative significance of the results.

When this ideal bending moment-curvature relation is assumed, the behaviour of the frame may be traced very simply. Whenever changes of load are taking place which are causing rotations of plastic hinges, the bending moments at these hinges will remain constant at the fully plastic moment, so that the changes in these bending moments will be zero. The *increments* of load will therefore cause *increments* in the remaining bending moments and in the deflections of the frame, which can be calculated by assuming that there are pin-joints at those positions where rotations of the plastic hinges are occurring and that elsewhere the frame is behaving elastically. Whenever another plastic hinge forms, or one or more of the hinges ceases to rotate, a new step in the calculations must begin in which the positions assumed for the pin-joints are modified appropriately.

Proportional Loading to Collapse

For purposes of comparison with the incremental collapse load tests, two of the frames tested were subjected to proportional loading until failure occurred by plastic collapse. Thus the loads H and V (see fig. 1) were each made equal to W, say, and the value of W was increased steadily until plastic collapse took place. Step-by-step calculations for this type of loading are summarised in fig. 3, in which the value of the non-dimensional load parameter

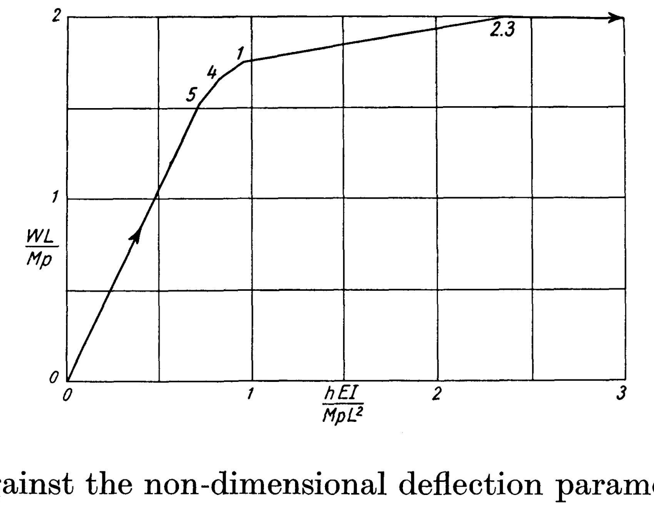


Fig. 3.

$\frac{WL}{M_p}$ is plotted against the non-dimensional deflection parameter $\frac{hEI}{M_p L^2}$, where h is the horizontal deflection occurring at the top of either stanchion and EI is the flexural rigidity of each member. It will be seen that until W reaches the value $1.53 \frac{M_p}{L}$ the frame behaves elastically. At this value of W a plastic hinge forms at cross-section 5 (see fig. 1). For further increases of load it is found that hinges form successively at the cross-sections 4 and 1, and finally at cross-section 2 when $W = 2\frac{M_p}{L}$.

There is then a sufficient number of plastic hinges to form the sidesway mechanism illustrated in fig. 4, and so deflections can continue to grow under constant load due to motion of this collapse mechanism. The plastic collapse load W_c thus has the value $2\frac{M_p}{L}$. With the simplified assumptions underlying these calculations, it is found that a plastic hinge would also form at cross-section 3 when W reached the value W_c, so that the mechanism of fig. 5 could be an alternative collapse mechanism. However, as will be seen later, the effects of the deflections developed prior to collapse and of the finite size of members cause the frame to collapse by sidesway, the bending moment at cross-section 3 at collapse being in practice slightly less than M_p.

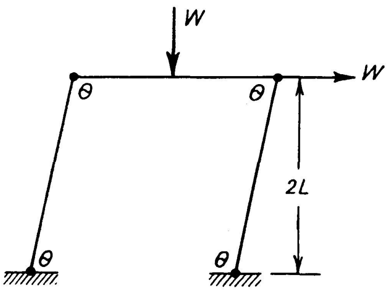
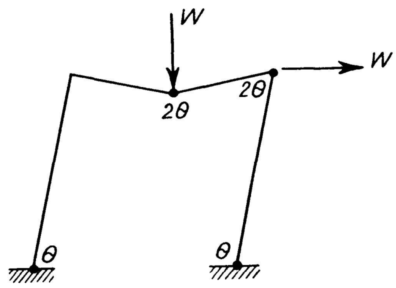


Fig. 4.

Fig. 5.



As is well known, the value of the plastic collapse load and the corresponding mechanism of collapse can be ascertained directly without recourse to step-by-step calculations of this kind. These calculations were only made to provide a comprehensive picture of the behaviour of the frame under proportional loading.

Cyclic Loading

The effect of the repetition of a particular cycle of loading will now be considered. This cycle, which is illustrated in fig. 6, consists of the simultaneous application of horizontal and vertical loads, each of magnitude W, and their removal, followed by the application of a horizontal load of magnitude W and its removal. The effect of the repetition of this cycle depends on the value of W, as indicated in fig. 7. This figure illustrates the growth of horizontal deflection, expressed non-dimensionally as $\frac{h\,E\,I}{M_p L^2}$, with the number of complete cycles of loading to which the frame is subjected, at various values of the load parameter $\frac{W L}{M_p}$ which are appended to the curves. The deflections which are plotted are those occurring at the beginning of each cycle of loading, when horizontal and vertical loads, each of magnitude W, are being applied.

If W is less than $1.675\frac{M_p}{L}$ it is found that there is no cyclic loading effect. For values of W between $1.675\frac{M_p}{L}$ and $1.78\frac{M_p}{L}$ the deflection which can be built up is limited, even though an infinite number of loading cycles is permitted. During each cycle of loading, rotations of plastic hinges occur at different cross-sections at various stages of the cycle. The magnitudes of these rotations become smaller and smaller, and finally cease altogether. Ultimately, therefore, the residual stresses in the frame adjust themselves so that all further

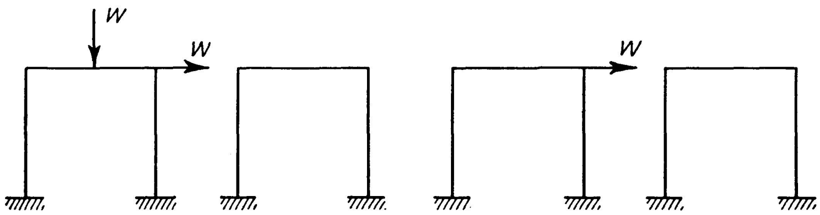


Fig. 6.

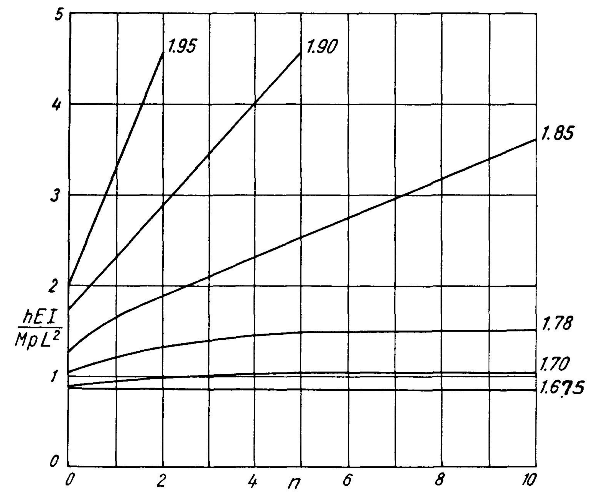


Fig. 7.

changes in the loads are borne by purely elastic changes of bending moment, and the frame is then said to have *shaken down*. However, when W exceeds the value $1.78\frac{M_p}{L}$, the deflections increase steadily with the number of cycles of loading, so that there is no limit to the deflections which can develop. Thus the incremental collapse load, denoted by W_s, has the value $1.78\frac{M_p}{L}$ in this case.

The fundamental change in the nature of the response to cyclic loading when W exceeds $1.78\frac{M_p}{L}$ is associated with the number of plastic hinges which undergo rotation during the loading cycles. Above this value of W plastic hinges form and undergo rotation at the cross-sections 4 and 5 whenever the horizontal and vertical loads are applied together, and at cross-sections 1 and 2 whenever the horizontal load is applied alone. Thus in one complete cycle of loading, increments in plastic hinge rotation occur at the cross-sections 1, 2, 4, and 5. If these hinges all formed simultaneously they would permit a mechanism motion of the sidesway type illustrated in fig. 4. Now the essence of a mechanism motion of this kind is that if the hinges were smooth pins the motion would occur without introducing any bending moments into the frame. It follows that if, as a result of a loading cycle, the *increments* of plastic hinge rotation are in conformity with a mechanism motion, the bending moment distribution at the end of the cycle will be the same as at the beginning. In this particular case this actually happens after only one or two cycles when W exceeds $1.78\frac{M_p}{L}$, the increments of plastic hinge rotation during each cycle being in conformity with the requirements of the sidesway mechanism of fig. 4. Thus at the start of each cycle of loading the bending moment distribution is precisely the same as at the start of the previous cycle, and so the same changes in the plastic hinge rotations occur during every cycle. This implies that the same changes in the deflections also occur during each cycle. However, when W lies between $1.675\frac{M_p}{L}$ and $1.78\frac{M_p}{L}$, the fully plastic moment is never attained at cross-section 2, and changes only occur in the plastic hinge rotations at cross-sections 1, 4 and 5 during each loading cycle. These hinges are not sufficient to constitute a mechanism motion, and so the bending moment distribution at the beginning of each cycle changes progressively. The possibility of achieving a condition in which the changes during each cycle are identical is thus precluded, and in fact the changes in the plastic hinge rotations and the deflections decrease steadily in such a way that the total deflection is limited.

Step-by-step calculations were also carried out for another cycle of loading, which is illustrated in fig. 8. This cycle differs only from that of fig. 6 in that the vertical load is reversed rather than reduced to zero at each alternate application of the horizontal load. The results of these calculations are given in fig. 9. It will be seen by comparison with fig. 7 that there is no difference in principle between the behaviour under the two cycles of loading.

Incremental Collapse Loads

For practical loadings on structures, such as the wind and snow loading case already cited, it often happens that the extreme possible limits of each load can be specified in terms of a single parameter W, although the actual loading programme is not known in advance. It has been shown that under these circumstances there exists a unique value W_s of the load which is termed the incremental collapse load, such that if W is less than W_s incremental collapse cannot occur no matter what sequence of loading takes place, whereas if W exceeds W_s incremental collapse can occur, and will occur if the sequence of loading follows the right pattern. Thus for the frame just considered, if both H and V can vary between limits of zero and W, W_s has the value $1.78\frac{M_p}{L}$, and repetition of the cycle of loading illustrated in fig. 6 is one sequence of loading which would cause incremental collapse if W exceeded W_s. Similarly, if the limits of V are changed to $(W, -W)$, the value of W_s is $1.60\frac{M_p}{L}$, and incremental collapse would be caused by repetition of the cycle shown in fig. 8. Analytical methods are available for the direct determination of incremental collapse loads, without having recourse to step-by-step calculations [1, 2]. Moreover, the limiting deflections which result if a large number of cycles of load are applied when W is equal to the incremental collapse load W_s can also be calculated directly.

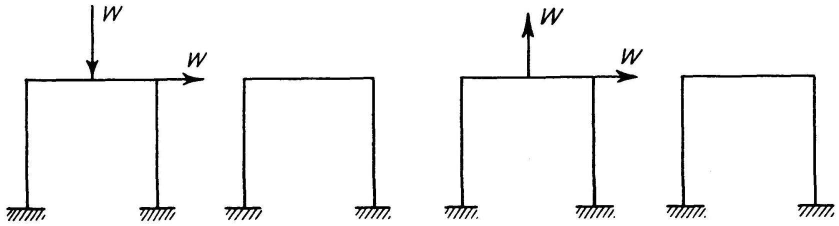


Fig. 8.

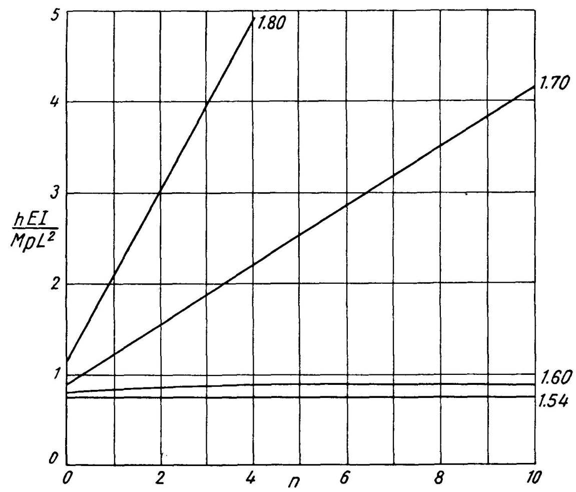


Fig. 9.

III. Secondary Effects

Values of Plastic Collapse and Incremental Collapse Loads

The values of plastic collapse and incremental collapse loads which emerged from the step-by-step calculations were obtained in accordance with the usual assumptions of a structural analysis, namely that the depths of members can be neglected in comparison with the dimensions of the frame, and that the deflections are so small that the equations of equilibrium are identical with those for the undistorted frame. However, these secondary effects can be shown to alter plastic and incremental collapse loads for actual frames by appreciable amounts. Since one of the objects of this paper is to compare observed values of plastic and incremental collapse loads, and since the theoretical values of these loads differ by not more than 20%, it is important to ascertain the small corrections which should be made to allow for these effects. A further small correction to the values of the incremental collapse loads will be made because of the lack of perfect fixity of the feet of the frames in the tests. However, such a lack of full rigidity is known to have no effect on the values of plastic collapse loads.

Consider first the plastic collapse load, whose uncorrected value is $2\frac{M_p}{L}$. The horizontal deflection h_c of the beam at the point of collapse is

$$h_c = \frac{7\,M_p\,L^2}{3\,E\,I}.$$

This value of h_c was derived by assuming the ideal bending moment-curvature relation of fig. 2, but it has been shown that quite accurate estimates can be obtained in this way [5]. As already mentioned, the feet of the frame in the tests were not perfectly fixed, and it might be supposed that the effect of lack of rigidity would be to increase the value of h_c. However, this is not so; this is essentially because plastic hinges form at the feet of both stanchions before the collapse load is attained. The bending moment distribution throughout the frame is of course uniquely determined at collapse, and the geometrical considerations which lead to the derivation of the deflections only involve the discontinuity of slope at the foot of each stanchion; whether this occurs entirely in plastic hinges or partly by rotation of the clamps is immaterial. Consequently, the above value of h_c can be used to estimate the effect of deflections on the value of the collapse load.

Assuming that collapse occurs by the sidesway mechanism, the effect of deflection on the collapse load can be estimated by the method of virtual work. Fig. 10 shows diagrammatically the deflected form of the frame at the

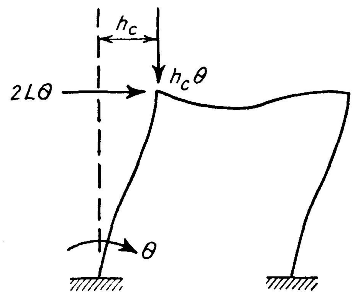


Fig. 10.

point of collapse. If arbitrary small rotations each of magnitude θ then occur at the tops and bases of the stanchions, the entire beam will undergo a vertical deflection $h_c\,\theta$ in addition to the horizontal deflection $2\,L\,\theta$, it being presumed that this latter deflection is small compared with h_c. Thus the vertical load W contributes to the work done during collapse. Equating the work done during collapse to the work absorbed in the plastic hinges,

$$W_c \times 2L\theta + W_c \times h_c\theta = 4M_p\theta,$$

so that
$$W_c = \frac{2M_p}{L}\left(1 - \frac{h_c}{2L}\right), \tag{1}$$

since h_c is small in comparison with L.

To estimate the corrective term $1 - \frac{h_c}{2L}$ in eq. (1) it is sufficient to use the nominal dimensions $L = 3$ in. and $b = d = {}^1/_4$ in., b and d being the breadth and depth of the members respectively. From preliminary tests described later it was found that $M_p = 9300\,\mathrm{bd}^2$ lb.in. and $E = 13{,}200$ tons per sq.in.; with these values $h_c = 0.317$ in. and $\frac{h_c}{2L} = 0.053$, so that the effect of deflection is to reduce W_c by 5.3%.

The form of eq. (1) shows that W_c decreases as the deflection increases. Without strain hardening the frame is unstable, and hence one may question the significance of the computation of W_c for the particular deflection h_c. The frame does in fact continue to carry a constant or slightly increasing load, as shown by the test curves, figs. 19, 20. This is evidently because strain hardening occurs at the plastic hinges. However, strain hardening is comparatively insignificant until the deflection h_c is exceeded, and the collapse load computed at this deflection is therefore considered a good estimate of the maximum load.

In this frame there is no appreciable correction for the effect of normal force on the fully plastic moment in either stanchion.

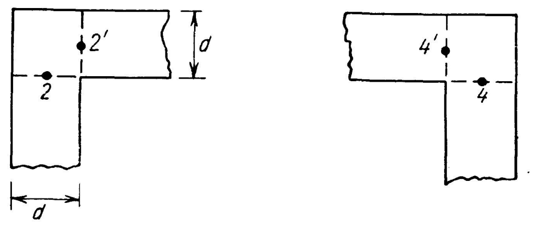


Fig. 11.

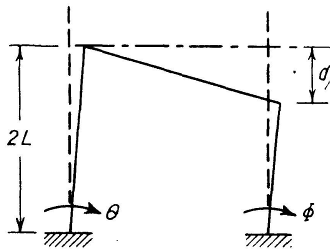


Fig. 12.

The value of the collapse load is also affected by the fact that the members are of finite depth. At the lefthand knee joint, which is depicted in fig. 11, the plastic hinge must form either at section 2 or section 2′, and should be considered to rotate about an axis passing through the centre line of the appropriate member at either of these sections. In fact, it can be shown by considering rotational equilibrium of the joint that the bending moment at section 2 must be slightly less than that at section 2′. The hinge must therefore form at section 2′ in the beam. From similar considerations it follows that the hinge at the right hand knee joint forms at section 4 in the stanchion rather than at section 4′ in the beam.

The effective height of the right hand stanchion is thus reduced to $2L - \frac{d}{2}$, while the effective height of the left hand stanchion remains at $2L$, since d is small compared with L. The collapse mechanism is therefore as shown in fig. 12. Since the horizontal movement at the top of each stanchion is the same, the angle ϕ is given by

$$\phi = \frac{2L\theta}{2L-\frac{d}{2}} = \theta\left(1+\frac{d}{4L}\right),$$

approximately, and the work equations is

$$W_c \times 2L\theta = M_p(2\theta+2\phi),$$

so that $$W_c = 2\frac{M_p}{L}\left(1+\frac{d}{8L}\right). \tag{2}$$

The small corrective term $1+\frac{d}{8L}$ can be estimated by using the nominal dimensions $d=\frac{1}{4}$ in. and $L=3$ in. With these values $1=\frac{d}{8L}=1.010$, so that the effect of the finite depth of the members is to increase W_c by 1.0%. Since the effect of the deflection is to lower the collapse load by 5.3%, the two effects combined give a nett reduction of 4.3%.

Similar corrections can be derived for the values of the incremental collapse loads. The analyses are far more tedious, owing to the fact that incremental collapse loads must be calculated from the elastic solutions for the frame for the application of each load applied separately, and so details will not be given here. For the case in which the load limits on H and V are both (W, O) it is found that there is an increase of 1.5% due to the finite size of members and a decrease due to deflections of the same magnitude. When the load limits on H and V are respectively (W, O) and $(W, -W)$, there is an increase of about 1.3% due to the finite size of members while the effect of deflections is negligible.

Turning now to the effect of the lack of perfect fixity at the feet of the portal frames, it must be stated at the outset that there is insufficient evidence to enable a really accurate assessment of the magnitude of the correction to be made. As will be seen when the experimental results are given, the horizontal deflections of the frames in the elastic range were 23% greater than the deflections computed for frames with rigid joints and rigidly held feet. It is of course a common experience in conducting tests of this kind that ideal conditions of fixity are not realized, but it is always difficult to decide just where the flexibility is occurring. In the particular arrangement adopted in the tests it is probable that the knee joints of the frames were comparatively rigid, and that the flexibility occurred at the feet. Even if this view is accepted, there are three possible sources of flexibility at the feet, for there could be incomplete restraint against both vertical and horizontal movement and also against rotation. However, since it is far more difficult to provide restraint against rotation, it will be assumed that the increases of deflection are due solely to flexibility at the feet in the rotational sense.

If it is assumed that at each foot a restraining moment $\lambda\theta$ is developed when the rotation is θ, then an elementary analysis shows that in the elastic range the horizontal deflection h due to a horizontal load H at beam level is given by

$$h = \frac{HL^3}{EI}\left(\frac{10+18k}{21+9k}\right), \qquad \text{where} \quad k = \frac{EI}{\lambda L}.$$

A central vertical load on the beam does not, of course, produce any horizontal deflection in the elastic range. With the feet fully clamped, k is zero, and $h = \frac{10HL^3}{21EI}$. From the experiments, the actual horizontal deflection is found to be $\frac{12.3HL^3}{21EI}$, and this corresponds to a value of k of 0.18. Thus the value of λ which corresponds to the increased deflections in the elastic range is $\frac{EI}{0.18L}$.

Using this value of λ, it is possible to determine the elastic bending moment distributions due to the separate action of the horizontal and vertical loads on the frame, and hence to calculate the incremental collapse loads. It is found that when the load limits on H and V are both (W, O), the effect is to increase W_s by 1.1%, and when the load limits on H and V are (W, O) and $(W, -W)$, respectively, there is an increase of 1.9%.

The various corrections to the theoretical values of the plastic and incremental collapse loads are summarised in table 1, which also gives the final values of the plastic and incremental collapse loads which will be assumed in analysing the experimental results.

Table 1.

Loading	Corrections				Collapse Load	
	Finite size of members	Deflections	Flexibility	Total	Uncorrected	Corrected
Plastic collapse load $H = V = W_c$	+1.0%	−5.3%	Nil	−4.3%	$2\frac{M_p}{L}$	$1.91\frac{M_p}{L}$
Incremental collapse load $H(W_s, 0)$, $V(W_s, 0)$	+1.5%	−1.5%	+1.1%	+1.1%	$1.78\frac{M_p}{L}$	$1.80\frac{M_p}{L}$
Incremental collapse load $H(W_s, 0)$, $V(W_s, -W_s)$	+1.3%	Nil	+1.9%	+3.2%	$1.6\frac{M_p}{L}$	$1.65\frac{M_p}{L}$

Also of interest for comparison with experimental results is the limiting deflection in the "shake down" range, h_s. This is the maximum horizontal deflection that can occur (under a given load combination) when W does not exceed W_s. Here we take h_s to be the maximum deflection when horizontal and vertical loads are applied together, with $W \leqq W_s$. This magnitude can be found by a method similar to that for the deflection h_c at the collapse load; use is made of the fact that at *one* of the four hinges which participate in the incremental collapse mechanism there is in fact no rotation until $W > W_s$. Just at the critical load W_s there is continuity of slope at this hinge (in the present frame at section 2). Since the complete bending moment distribution is known at the load W_s this continuity condition allows the deflection to be calculated. The value of h_s is found to be effected very little by the imperfect rigidity of the feet, but is not independent of the base rotations as in the case of h_c. Values of h_s are given in table 2.

Table 2.

Load limits	Values of h_s	
	Uncorrected	Corrected
$H(W_s, 0)$; $V(W_s, 0)$	$1.53\dfrac{M_p L^2}{EI}$	$1.53\dfrac{M_p L^2}{EI}$
$H(W_s, 0)$; $V(W_s, -W_s)$	$0.89\dfrac{M_p L^2}{EI}$	$0.87\dfrac{M_p L^2}{EI}$

IV. Experimental Results

Test Programme

The theoretical study of frames under repeated variable loading leads to the prediction, as has been seen, of two remarkable phenomena. One is the possibility of incremental collapse under cyclic loading; the other is the existence of a critical load parameter W_s below which collapse of this or any other kind *cannot* occur whatever number of cycles of any load sequence is applied. It would be of great interest to confirm by experiments that these phenomena occur as quantitatively predicted. Apart from these effects it would be of interest also to investigate experimentally the behaviour of a frame subjected to a relatively small number of cycles of loading of the sort that might easily occur in practice.

It was pointed out above that incremental collapse depends on the formation of plastic hinges, during a sequence of loading, in positions such that if all the hinges occurred simultaneously the frame would be reduced to a mechanism. Each cycle of loading then leads to an increment of deflection of an amount which is proportional to $(W - W_s)$ within a certain range. Thus a sufficient number of repetitions of the cycle leads to incremental collapse by the development of arbitrarily large deflections. However, the number of cycles required to produce a given large deflection will obviously depend on the amount by which W exceeds W_s.

The phenomenon of incremental collapse could be demonstrated for a given frame by performing a series of tests from which a composite load-deflection curve could be plotted, such that if the predictions of the theoretical study are borne out the resulting curve would resemble the load-deflection curve for the usual collapse test under proportional loading. In an ideal programme one would have available a large number of identical frames. To each frame a number of cycles n of a loading sequence would be applied at a certain load level W, and the magnitude W would be increased in suitably small steps from frame to frame. If the number n is large enough, then as soon as the incremental collapse load W_s is exceeded by some small but finite amount a large permanent deflection would be produced; for example, if W exceeds W_s by 2%, then n could be chosen so that a permanent deflection of the order of 10 times the maximum elastic deflection (under proportional loading) would be expected if the actual frame behaves as predicted.

In fact, the actual frame differs from that considered in the theory mainly in undergoing strain-hardening and shape changes as the deformation proceeds. These effects cause the observed load-deflection diagram to differ from the theoretical one, just as in a test under proportional loading. Strain-hardening at the plastic hinges in a mode of incremental collapse causes the effective incremental collapse load to be increased, so that the deflections are limited even though a large number of cycles is applied at a load above the theoretical critical load W_s. However, the influence of strain-hardening must be very closely the same in a test using repeated cyclic loading as in one using proportional loading, provided other conditions are the same. (For example the total time taken for a test should not be such that strain ageing is appreciable in one case and not in the other.) Similarly, the effect of shape changes on the load-deflection diagram should be very nearly the same in the two cases. It would seem, therefore, that such an ideal programme of tests would lead to a load-deflection diagram of shape quite similar to that observed in ordinary collapse tests, but with the ordinates in the large deflection range differing by approximately $W_c - W_s$.

Such an ideal test programme would be costly because of the large number of specimens required, and the results would be difficult to interpret because of the inevitable differences in material properties, notably the value of M_p, from frame to frame. It was considered that a less elaborate test programme, using a small number of frames, would be capable of providing the information sought. Seven nominally identical frames were constructed as described below. Of these, two were subjected to proportional loading so as to obtain the curves for plastic collapse in the usual way. The remaining five were subjected to cyclic loading, three with the cycle shown in fig. 6, and two with that of fig. 8.

Each of the frames tested under cyclic loading was subjected to loads of magnitude extending over the whole range from elastic to somewhat in excess of the theoretical incremental collapse load. At each magnitude a number of load cycles was applied, the number being small (3 or 4) in the low load range $W < W_s$, and somewhat larger (up to 10) in the neighbourhood of W_s or larger. The justification of this procedure is as follows. At load magnitudes W such that $W < W_s$ the effect of previous loading cycles is small, and stabilisation occurs very rapidly (in 2 or 3 cycles). At a load magnitude exceeding W_s the deflection caused by applying a certain number of load cycles may be large compared to the corresponding elastic deflection, and strain hardening may be appreciable. Cyclic loading at one load level would therefore be expected to influence the effect of subsequent cyclic loading at a higher load level. Despite this, it was felt that the differences between cyclic loading below and above W_s would stand out clearly by applying cyclic loading at a series of load levels to the same frame. The load-deflection curve obtained by this procedure will lie closer to the ordinary collapse curve for proportional loading than would the curve obtained in the ideal programme, and is on this account somewhat unconservative. Thus the test programme chosen differs from the ideal programme mainly in the fact too few cycles of loading were applied in the range above the critical load W_s to enable the magnitude of W_s to be determined with real precision from the tests, as would have been possible if a large number of cycles had been applied at each load level. On the other hand, the test programme used has the advantage of enabling information to be obtained on the behaviour of the frame at many load levels, while using a relatively small number of frames.

Fabrication of Test Frames

The incremental collapse and plastic collapse tests were carried out on seven rectangular portal frames of mild steel. Each portal had a nominal span and height of 6 in., measured to centre-lines, and the members were all nominally of $\frac{1}{4}$ in. square cross-section. The frames could be subjected to horizontal and vertical loads, H and V, as in fig. 1.

The frames were all made of material cut from a single $\frac{1}{4}$ in. mild steel plate. A number of specimens cut from the same plate were tested in tension and bending. These tests were necessary for the determination of the value of the fully plastic moment to be used in analysing the results of the frame tests, and they also indicated that the plate was uniform in its properties, so that each frame was effectively made from identical material. Before any tests were performed, the test specimens and frames were all heat treated simultaneously in the same furnace. The heat treatment used was full normalising at 900° C, with a soaking time of 15 min., after which the specimens were cooled in air. This treatment was carried out to produce a complete recrystallisation of the steel, thereby producing a uniform grain size and removing any residual stresses due to machining and welding.

The plate was 20 in. wide and about $3\frac{1}{4}$ in. long. It was first sawn into thirteen strips of width $\frac{1}{4}$ in. and length 20 in., and each of these strips was then cut into three portions, two of length $6\frac{7}{8}$ in. and one of length $6\frac{1}{4}$ in., each length being marked as indicated in fig. 13. The two sides of each length which lay originally in the surface of the plate were not machined, but the other two sides were ground down so that the depth of each specimen was 0.25 in. $\pm$ 0.005 in., the wide tolerance being permitted as uniformity of depth along the length of each specimen was more important than the exact value of this depth. In what follows the breadth b and depth d of each specimen will be taken to refer to dimensions between the unground and ground faces, respectively.

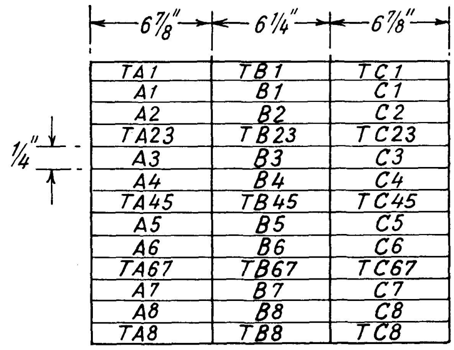


Fig. 13.

Tests of Material Properties

The prefix T in fig. 13 indicates that the length in question did not subsequently form part of a frame, but was tested either in bending or tension.

After testing, portions of the specimens TB 23, TB 45 and TB 67 were analysed. There was no appreciable variation in the results, and the average percentages by weight of the various elements were as given in table 3.

Table 3. Analysis of Steel

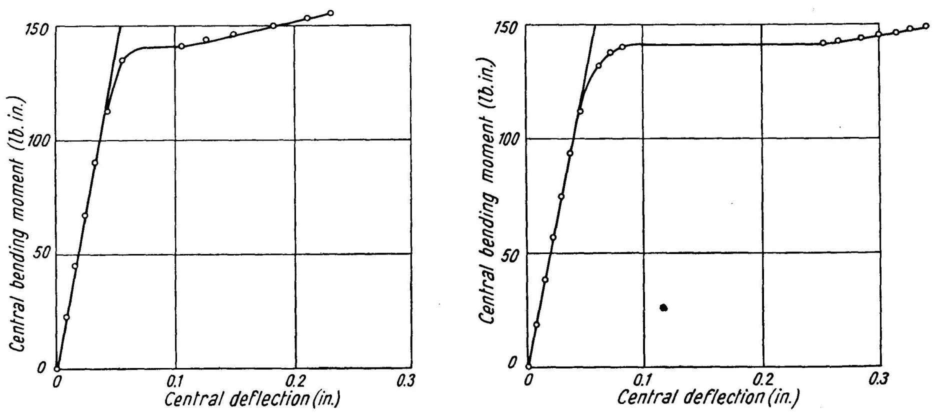
C	Si	Mn	P	S	Cu	Ni	As	Sn	N
0.18	0.22	0.44	0.04	0.03	0.04	0.04	0.04	0.01	0.01

Preliminary Bending and Tensile Tests

The five "TA" specimens (see fig. 13) were tested as simply supported beams over a 6 in. span, subjected to a central concentrated load. The five "TC" specimens were also tested as simply supported beams over a 6 in. span, but the loading consisted of two equal loads applied $\frac{1}{4}$ in. on either side of the centre of the span. The five "TB" specimens were tested in tension.

The results of typical tests on the beams TA 67 and TC 67 are shown in fig. 14, in which values of the central deflection are plotted against the central bending moment, which is proportional to the load. It will be seen that in each case there was a well-defined fully plastic moment at which the deflection increased while the load remains constant.

An analysis of the test results must allow for the variations in the dimensions of the cross-section from specimen to specimen. The fully plastic moment M_p for a beam of rectangular cross-section has the value $\frac{1}{4} b d^2 f_L$, where f_L is the lower yield stress. Thus from each bending test an equivalent value of f_L, independent of the dimensions of the specimens, can be calculated as $f_L = \frac{4 M_p}{b d^2}$. These values of f_L are given in table 4.



TA 67. $M_p = 141$ Lb. in. TC 67. $M_p = 141$ Lb. in.

Fig. 14.

Table 4. Analysis of Bending Tests

Central concentrated load						
Specimen	TA 1	TA 23	TA 45	TA 67	TA 8	
f_L (tons per sq. in.)	17.5	17.1	17.5	16.6	17.7	Average = 17.3
Two-point loading						
Specimen	TC 1	TC 23	TC 45	TC 67	TC 8	
f_L (tons per sq. in.)	16.5	16.9	16.8	16.4	16.5	Average = 16.6

The average value of f_L was 17.3 tons per sq. in. for the "TA" tests and 16.6 tons per sq. in. for the "TC" tests. A small difference of this order of magnitude is to be expected. In the "TA" tests the fully plastic moment was only developed directly beneath the load, so that there were compressive stresses sustaining the load as well as the longitudinal bending stresses which are compressive just beneath the load. These additional compressive stresses reduce the maximum shear stress in the neighbourhood of the load, and so delay the attainment of the fully plastic condition, as pointed out by HEYMAN [6].

In analysing the results of the frame tests the value of M_p for each frame was calculated as $\frac{1}{4} b d^2 f_L$, and the value of f_L was taken as 16.6 tons per sq. in., the average from the "TC" tests. The reason for this choice is that a plastic hinge did not form under the concentrated beam load in any of the tests, so that a value of M_p unaffected by direct compressive stresses was appropriate. The value of M_p for each frame test was therefore taken to be

$$M_p = \tfrac{1}{4} b d^2 \times 16.6 \times 2240 = 9300\, b d^2 \text{ lb. in.}$$

The "TB" specimens were tested in a 2 ton lever type testing machine. Strains were measured over a 2 in. gauge length by a Gerard extensometer. The results of a typical test are shown in fig. 15. For each test Young's modulus was about 13,200 tons per sq. in. The values of the lower yield stress obtained in these tests are given in table 5.

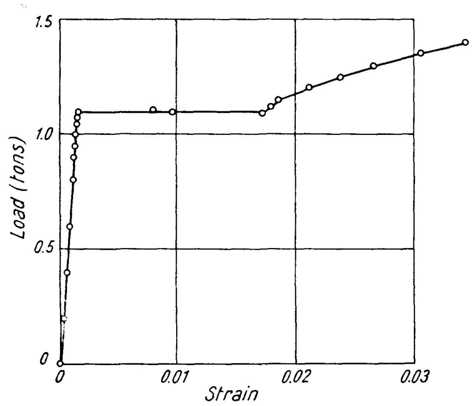


Fig. 15.
$Y = 1.10$ tons.

Table 5. Analysis of Tensile Tests

Specimen	TB 1	TB 23	TB 45	TB 67	TB 8	
f_L (tons per sq. in.)	17.4	17.1	17.3	17.1	17.1	Average = 17.3

The larger value of f_L indicated by the tensile tests as compared with the two-point bending tests is attributed to the higher rates of straining in the former case. It was not felt necessary to modify the result for f_L found on the two-point bending tests, since the strain rates in these tests are probably reasonably close to those of the frame tests.

Plastic Collapse Tests

The apparatus used for the plastic collapse tests was as shown in fig. 16. The vertical load V at the centre of the beam was applied through the same load fitting that was used in the concentrated load beam tests. The horizontal load H was applied to the frame by a hardened steel pin, of $^3/_{32}$ in. diameter, passing through a hole drilled in the knee of the portal. This pin was carried in a pair of ball races to ensure rotational freedom, so that the load was applied along the centre line of the beam. The loads themselves comprised buckets containing weighed quantities of lead shot; the horizontal load chain passed over a pulley not shown in fig. 16.

The nominal dimensions of the frames were as shown in fig. 17, in which the letters on the members correspond to the identification letters shown in

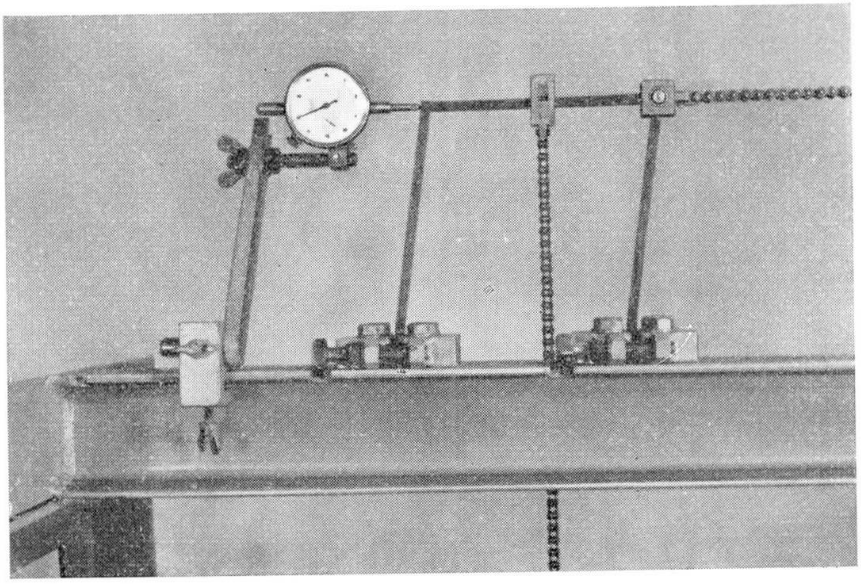


Fig. 16.

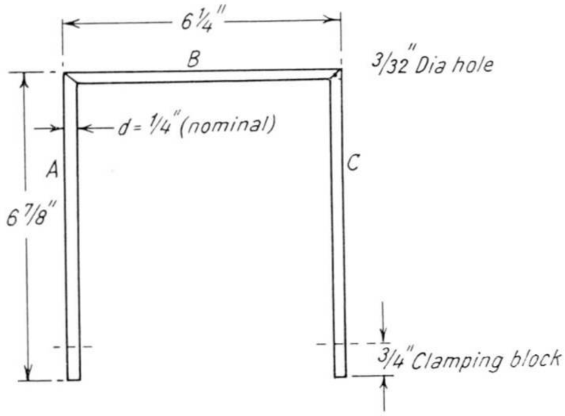


Fig. 17.

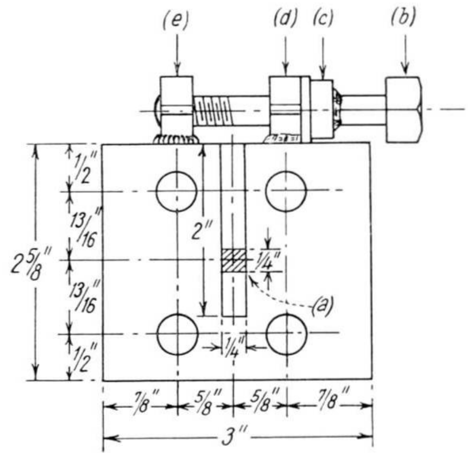


Fig. 18.

fig. 13. Thus frame no. 5 was made up from the lengths A 5, B 5 and C 5. The members were arranged so that their ground faces were perpendicular to the plane of the frame. The welded mitred joints which were used were similar to those used by Baker and Heyman [7].

The feet of the portals were held in clamping blocks, which in turn were bolted to the loading frame shown in fig. 16. Details of a clamping block are given in fig. 18. Each block was $\frac{3}{4}$ in. thick, and a slot of depth 2 in. and width $\frac{1}{4}$ in. was cut. The foot of a stanchion could be inserted at (a). The slot could then be tightened by means of the $^5/_{16}$ in. bolt (b). This bolt was provided with a collar (c) bearing on the $^5/_{16}$ in. nut (d), which was drilled for clearance of the shank of the bolt and welded to the block. Another $^5/_{16}$ in. nut (e) was welded to the block on the other side of the slot.

The horizontal deflection of the frame at beam level was measured by a dial gauge. The vertical deflection at the centre of the beam was not measured.

Two plastic collapse tests were carried out, on frames 1 and 5. In each of these tests proportional loading was adopted, with $H = V = W$. The results of these tests are shown in figs. 19 and 20 in which W is plotted against h, the horizontal deflection.

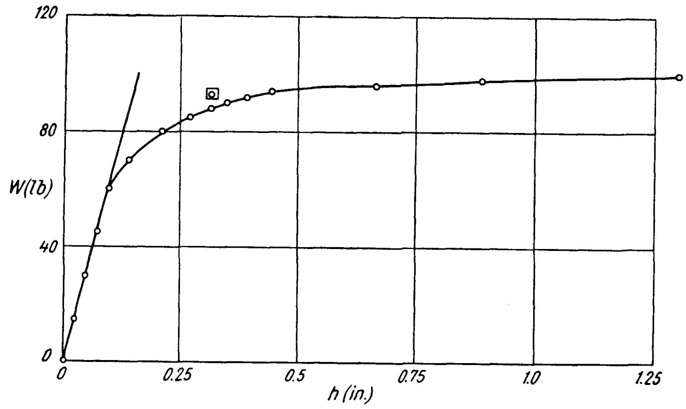


Fig. 19. Collapse Test. Frame No. 1. $H = W$, $V = W$.

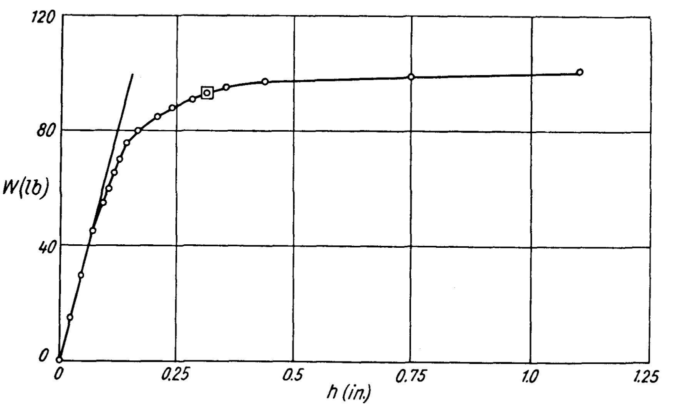


Fig. 20. Collapse Test. Frame No. 5. $H = W$, $V = W$.

In order to see what measure of agreement exists between experiment and theory in these tests, a point is plotted on each load-deflection diagram whose ordinate is the calculated value of the collapse load W_c and whose abscissa is the calculated value of h_c, the horizontal deflection at the point of collapse. This point should then lie on the experimental curve, and any increase of load above the value W_c should be accompanied by a large increase in the horizontal deflection.

It has already been shown that the theoretical values of W_c and h_c are

$$W_c = 1.91\frac{M_p}{L}, \qquad h_c = \frac{7\,M_p\,L}{3\,E\,I}.$$

Taking $M_p = 9300\,b\,d^2$ lb. in. and $E = 13{,}200$ tons per sq. in., as deduced from the bending and tensile tests, the calculations for W_c and h_c are summarised in table 6.

Table 6. Calculated Plastic Collapse Loads and Deflections

Frame No.	b (in.)	d (in.)	L (in.)	M_p (lb. in.)	h_c (in.)	W_c (lb.)
1	0.250	0.249	2.98	144.2	0.314	92.5
5	0.250	0.250	3.00	145.3	0.317	92.4

The points whose ordinates and abscissae are the calculated values of h_c and W_c are shown in figs. 19 and 20, enclosed by a small square. These points lie close to the experimental curves, and the rate of increase of h with W tends to become large if W exceeds W_c. It can therefore be concluded that a good measure of agreement exists between theory and experiment. This confirms that the experimental technique is satisfactory, and that the predicted values of collapse loads and of deflections are in reasonable conformity with the results of model tests.

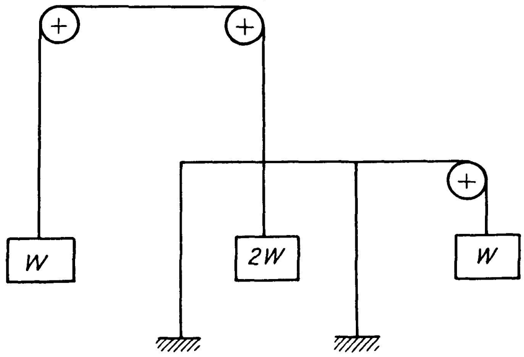


Fig. 21.

Incremental Collapse Tests

The apparatus used for the incremental collapse tests was similar to that used in the plastic collapse tests, as shown in fig. 16. In all, five incremental collapse tests were carried out, on frames 2, 4, 6, 7 and 8, frame 3 having been used for a preliminary test. The load limits employed in these tests were as follows:

$$\text{Frames 2, 4 and 6} \quad H(W, O);\ V(W, O)$$

$$\text{Frames 7 and 8} \quad H(W, O);\ V(W, -W),$$

the cycles of loading being as shown in figs. 6 and 8.

Removal of loads during the cycles was effected by jacking up the lead shot containers. The reversal of the load V from W to $-W$ required when testing frames 7 and 8 was achieved by means of the loading arrangement shown diagrammatically in fig. 21, a jack being placed under the vertical load $2W$.

The procedure adopted in each test was to apply successive equal increments of H and V until these loads were each about 75 lb. At this value of W a few cycles of load, usually about three or four, were carried out. The value of W was then increased by a small amount, of the order of 3—5 lb., and a further few cycles of loading were performed. Further small increases in W were then made, a small number of cycles of loading being carried out at each load level.

The actual details of the loading programme for each frame are given in figs. 22—26. Each of these figures comprises a series of curves, and each curve shows the growth of horizontal deflection with number of cycles of loading for the particular value of W which is appended to the curve. The ordinates

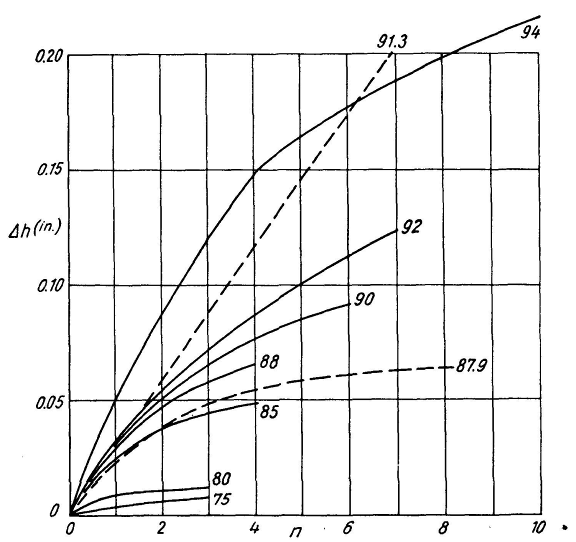


Fig. 22. Frame No. 2. $W_s = 87.9$ Lb.

represent the increment h of the horizontal deflection at each level of loading, taking the deflection which occurs at the first application of $H = W$ and $V = W$ as datum, and the abscissæ represent the number of cycles of loading.

Presentation of the experimental results is completed in figs. 27—31. In these figures the ordinates represent values of W and the abscissæ represent the *total* horizontal deflection h which was developed at this value of W at the end of the cyclic loading, before a further increase in W was made and more cycles of loading were carried out. Each cycle of loading was terminated by the application of the loads $H = W$ and $V = W$ simultaneously, and it is the value of h with both these loads applied which is plotted in figs. 27—31. The dashed-line curves are averaged curves for proportional loading.

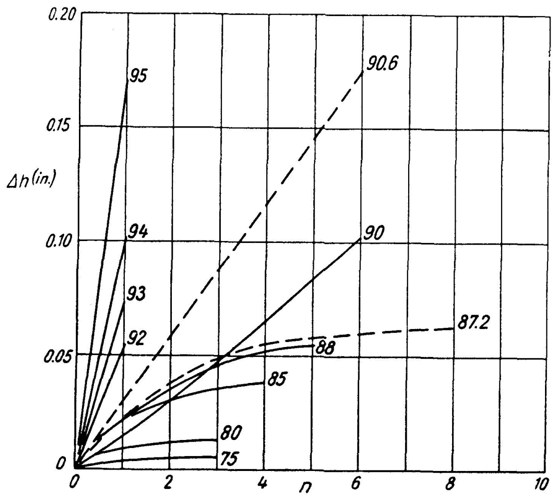


Fig. 23. Frame No. 4. $W_s = 87.2$ Lb.

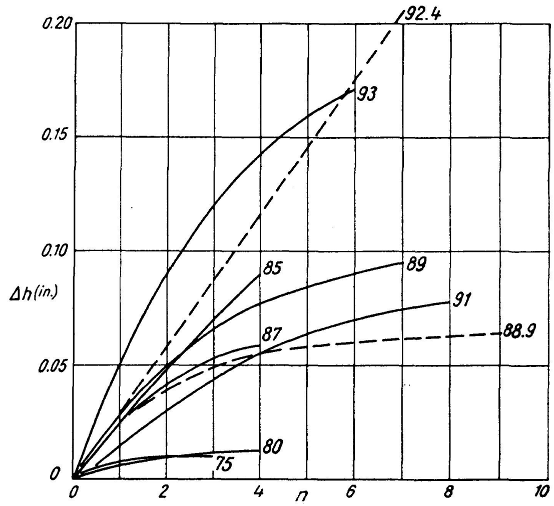


Fig. 24. Frame No. 6. $W_s = 88.9$ Lb.

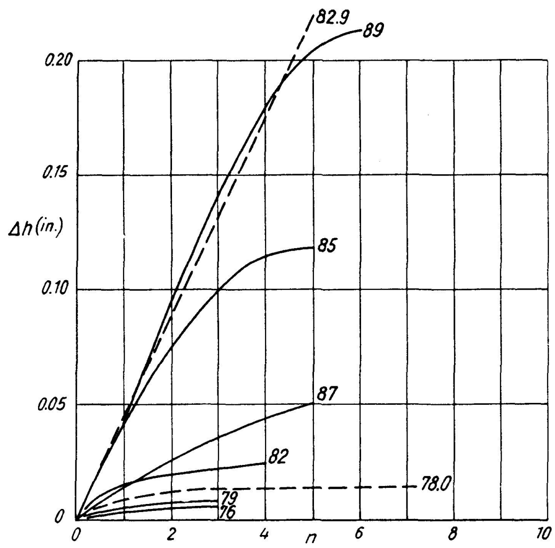


Fig. 25. Frame No. 7. $W_s = 78.0$ Lb.

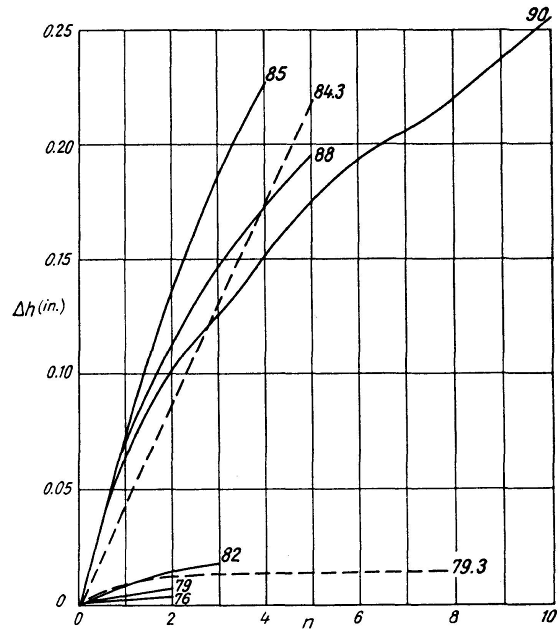


Fig. 26. Frame No. 8. $W_s = 79.3$ Lb.

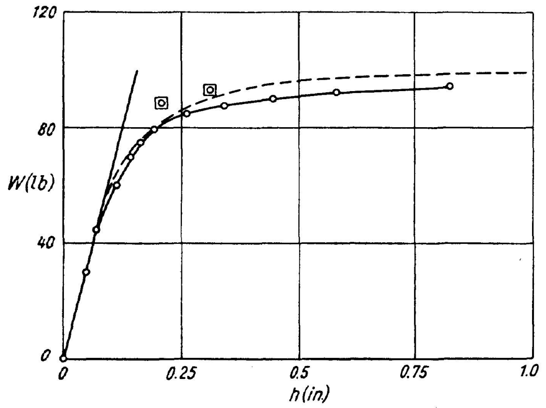


Fig. 27. Shake-Down Test.
Frame No. 2. $H(W, O)$, $V(W, O)$.

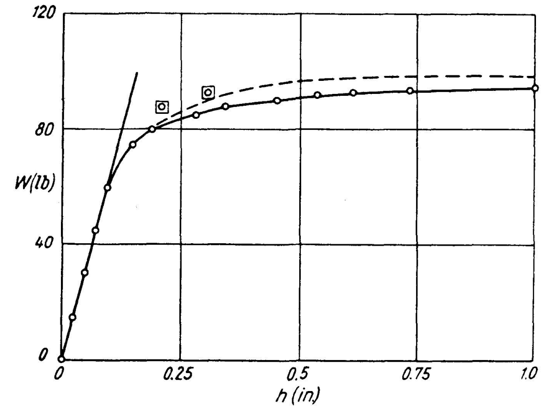


Fig. 28. Shake-Down Test.
Frame No. 4. $H(W, O)$, $V(W, O)$.

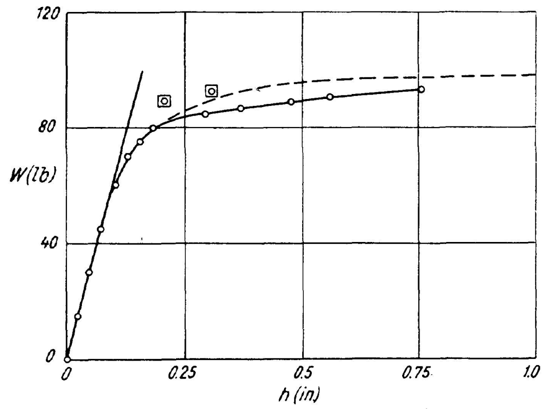


Fig. 29. Shake-Down Test.
Frame No. 6. $H(W, O)$, $V(W, O)$.

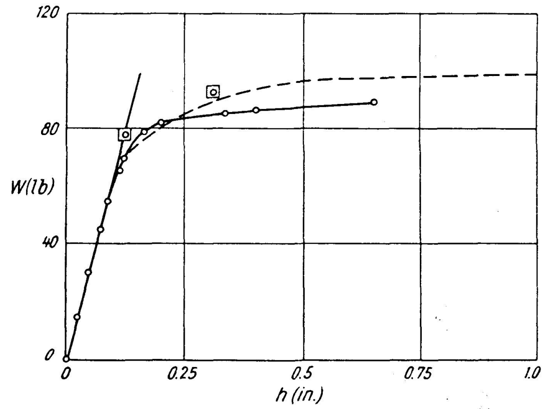


Fig. 30. Shake-Down Test.
Frame No. 7. $H(W, O)$, $V(W, -W)$.

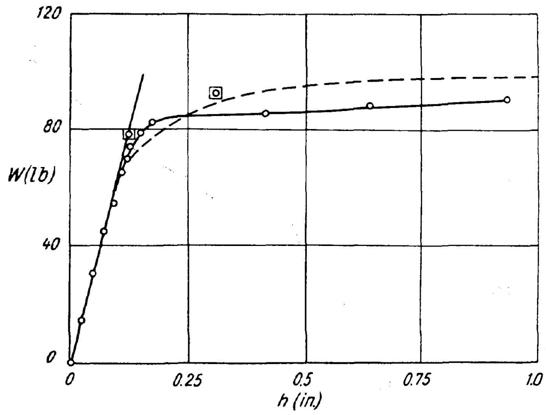


Fig. 31. Shake-Down Test.
Frame No. 8. $H(W, O)$, $V(W, -W)$.

V. Comparisons Between Theory and Experiment

In comparing the experimental results with theoretical predictions, it is seen from table 1 that for the frames 2, 4 and 6, in which the load limits were $H(W, O)$ and $V(W, O)$, the incremental collapse load W_s should be $1.80 \frac{M_p}{L}$.

Moreover, the maximum horizontal deflection h_s which could be developed at this load is seen from table 2 to be $1.53\frac{M_p L^2}{EI}$. For frames 7 and 8, for which the load limits were $H(W, O)$ and $V(W, -W)$, the corresponding results were $W_s = 1.65\frac{M_p}{L}$ and $h_s = 0.87\frac{M_p L^2}{EI}$. Taking $M_p = 9300\,bd^2$ lb. in. and $E = 13{,}200$ tons per sq. in., as before, the calculations for the values of W_s and h_s for each frame are as summarised in table 6.

Table 7. Calculated Incremental Collapse Loads and Deflections

Frame No.	b (in.)	d (in.)	L (in.)	M_p (lb. in.)	h_s (in.)	W_s (lb.)
2	0.252	0.250	3.00	146.5	0.208	87.9
4	0.250	0.250	3.00	145.3	0.208	87.2
6	0.253	0.251	3.00	148.2	0.207	88.9
7	0.250	0.247	3.00	141.8	0.120	78.0
8	0.250	0.249	3.00	144.2	0.119	79.3

The points whose ordinates and abscissæ are the calculated values of W_s and h_s are shown in figs. 27—31, enclosed by a small square. To confirm the predictions of theory, these points should lie on the experimental curves, and should mark the stage at which extremely large deflections first occur due to the repetition of the loading cycles. It will be seen that there exists a good measure of agreement between experiment and theory for frames 2, 4 and 6, but that for frames 7 and 8 the predicted values of W_s are somewhat low.

A further comparison between the experimental results and theory can be made from figs. 22—26, which show the observed values of the increment h in the horizontal deflection plotted against the number of cycles of loading n for each value of W used in the tests. A comparison of these results with the theoretical results given in figs. 7 and 9 will be made. However, this comparison can only be regarded as semi-quantitative for two reasons:

1. The theoretical values of h were calculated by assuming complete rigidity at the feet of the frames, whereas in the tests there was an appreciable degree of flexibility.

2. In deriving the theoretical values of h for any particular value of W it was assumed that in each case the frame was free from residual stress when the loading cycles commenced. However, in the tests the frame would be in a state of residual stress at the commencement of cycles of loading at a new value of W, due to the plastic flow which had taken place in previous cycles.

Nevertheless, it is felt that the theoretical calculations must indicate the general form of the $h-n$ curves which would be expected to occur.

In the tests on frames 2, 4 and 6 the load limits were $H(W, O)$ and $V(W, O)$, and the corresponding theoretical deflection curves are shown in fig. 7. Although curves are given for several values of W, only two values, namely $1.78\frac{M_p}{L}$ and $1.85\frac{M_p}{L}$ will be considered for comparison purposes. The first of these values is the incremental collapse load for the idealized frame, and the second value is the next higher value of W considered in the theoretical calculations. Still higher values were not considered because in the experiments the corresponding deflections were then so high that the material had clearly entered the strain-hardening range. The ratio of these two values of W is 1.039, and so the corresponding theoretical deflection curves will be assumed to correspond to values of W_s and 1.039 W_s, where W_s is the corrected value of the theoretical incremental collapse load for each frame.

For the tests on frames 7 and 8 the load limits were $H(W, O)$ and $V(W, -W)$, and the corresponding theoretical deflection curves are shown in fig. 9. The two values of W taken for comparison purposes were $1.60\frac{M_p}{L}$ and $1.70\frac{M_p}{L}$, the former value being the value of W_s for the idealised frame. The ratio of these two values of W is 1.063, so that the corresponding theoretical deflection curves will be assumed to correspond to values of W_s and $1.063\,W_s$, where W_s is the corrected value of the theoretical incremental collapse load for each frame.

In the theoretical calculations the values of h were multiplied by the factor $\frac{EI}{M_pL^2}$. The numerical value of this factor is

$$\frac{EI}{M_p L^2} = \frac{Ed}{3L^2 f_L} = 7.37\,\text{in.}^{-1}$$

using the nominal values $d = 0.25$ in. and $L = 3$ in., together with $E = 13{,}200$ tons per sq. in. and $f_L = 16.6$ tons per sq. in. Using this factor, values of h can be derived from the curves of figs. 7 and 9.

The theoretical deflection curves derived in this way are shown dotted in figs. 22—26, the corresponding values of W being appended to the curves. Considering first fig. 22, it is seen that the theoretical curve for $W = W_s = 87.9$ lb. lies between the experimental curves for 85 lb. and 88 lb. being nearer the former. The theoretical curve for $W = 1.039\,W_s = 91.3$ lb. is seen to be linear, whereas the experimental curves for 88, 90, 92 and 94 lb. all show a pronounced decrease of slope with increase of n, presumably due to strain-hardening. However, the initial slopes of the experimental curves for 88, 90 and 92 lb. are all close to the slope of the theoretical line for 91.3 lb., and the experimental curve for 94 lb. has a much greater initial slope. Thus there is a reasonable measure of agreement between experiment and theory in this case.

In fig. 23 it will be noted that the experimental results are somewhat erratic, presumably due to strain-hardening effects. In particular, the curve for a load of 90 lb. lies initially below the curves for 85 lb. and 88 lb. Bearing this in mind, there is again a reasonable measure of agreement between experiment and theory.

In fig. 24 it will again be seen that the experimental results are erratic. However, the theoretical curve for $W = W_s = 88.9$ lb. compares well with the experimental curves for 87 lb. and 89 lb.

For the remaining two experiments the load limits on V were changed from (W, O) to $(W, -W)$. As will be seen from figs. 25 and 26, when $W = W_s$ there is only a very small increase of h due to cyclic loading, whereas when W exceeds W_s the effect is very pronounced. This sharp distinction in the form of the curves occurred experimentally, but at higher values of W than the theoretical value of W_s. Thus in fig. 25 the curves change form in this way when W exceeds 82 lb., as compared with the theoretical value of W_s of 78.0 lb., and in fig. 26 the change also occurs when W exceeds 82 lb., whereas the theoretical value of W_s is 79.3 lb. From these data, it therefore appears that the experiments are consistent with a value of W_s of about 82 lb., as compared with an average theoretical value of W_s of about 79 lb. for the two experiments.

Conclusions

From the results obtained the following conclusions can be drawn.

1. In the two plastic collapse tests the agreement between experiment and theory was good, thus showing that the experimental technique was satisfactory.

2. Under cyclic loading the frames behaved qualitatively in the manner predicted theoretically. Repetition of the appropriate loading cycle caused little or no increase of deflection below a certain value of W. For greater values of W it was found that the deflections could be made to increase by repeating the loading cycle. The total increase of deflection was found to be limited if W was less than a certain value, whereas if W exceeded this value the total possible increase of deflection became considerably greater and was evidently limited only by the strain-hardening of the members.

3. The calculated values of the incremental collapse loads were compared with the experimental results. For frames 2, 4 and 6, with load limits $H(W, O)$ and $V(W, O)$, the agreement was quite satisfactory. However, for frames 7 and 8, with load limits $H(W, O)$ and $V(W, -W)$, the experimental load-deflection curves showed smaller differences from the curves for proportional loading than is predicted by theory. This effect is probably associated with the differences between the "ideal" and the actual test programmes.

4. For values of W exceeding W_s the theoretical $h - n$ relation is linear, whereas the experimental curves showed a pronounced decrease of slope with increase of n. This effect is apparently due to strain-hardening.

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Summary

The paper is concerned with the growth of deflections of portal frames under repeated cyclic loading due to the formation and rotation of plastic hinges at various stages of each loading cycle. Step-by-step calculations are used to show that if the load intensity exceeds a critical value it is possible for large deflections to be built up by repeated cyclic loading, even though the load intensity is less than that required to cause plastic collapse. The deflections would only be limited in practice by such effects as strain-hardening. This type of cumulative failure is termed incremental collapse, and the critical load intensity above which incremental collapse can occur is termed the incremental collapse load.

Five tests on model rectangular portal frames of mild steel were carried out to confirm the theoretical predictions, and two plastic collapse tests on similar frames were also made for comparison purposes. The results of the tests are analysed in detail, and it is concluded that the measure of agreement between the experimental results and theoretical predictions is satisfactory.

Résumé

Les auteurs étudient l'accroissement des déformations des portiques soumis à des cycles répétés de charges, déformations provoquées par la formation et la rotation de rotules plastiques au cours des diverses phases de chaque cycle de charges. A l'aide de calculs par cheminement, on montre que, si l'intensité des charges dépasse une valeur critique, il peut se produire de grandes déformations sous l'effet de cycles répétés de charges, même si l'intensité des charges est inférieure à celle qui provoque l'effondrement par adaptation plastique. En pratique, les déformations plastiques seraient limitées uniquement par des effets tels que l'écrouissage. Ce mode de mise hors de service par accumulation des déformations est appelé «ruine progressive» et l'intensité critique des charges qu'on ne peut dépasser sans danger de «ruine progressive» s'appelle «charge de stabilisation plastique».

On a essayé cinq modèles de portiques en acier doux pour confirmer les résultats théoriques; à titre de comparaison, deux essais de ruine par adaptation plastique ont été réalisés sur des cadres semblables. Les résultats des essais sont analysés en détail et on en conclut que les expériences sont en accord satisfaisant avec la théorie.

Zusammenfassung

Die Abhandlung befaßt sich mit dem Anwachsen der Verformungen eines zyklisch belasteten Portalrahmens durch Bildung und Drehung plastischer Gelenke während der verschiedenen Stadien jedes Belastungszyklus. Durch schrittweise Berechnungen wird nachgewiesen, daß von einem kritischen Betrage der Belastung an große Verformungen entstehen können durch zyklisches Wiederholen dieser Belastung, deren einmalige Anwendung noch keineswegs einen plastischen Zusammenbruch zur Folge hätte. Diese Verformungen würden in Wirklichkeit nur durch Spannungs-Verfestigung beschränkt werden. Diese Art des kumulativen Versagens wird «schrittweiser Zusammenbruch» genannt, die kritische Belastungsgrenze, oberhalb der es zu einem kumulativen Versagen kommen kann, «Grenzlast für schrittweisen Zusammenbruch».

Zur Bestätigung der Theorie wurden fünf Modellversuche mit rechtwinkligen Portalrahmen aus Flußstahl ausgeführt. Zum Vergleiche der Wirkungen wurden zwei gleiche Rahmen durch einmalige Belastung zum plastischen Zusammenbruch gebracht. Die Versuchsergebnisse wurden detailliert ausgewertet und ergaben eine zufriedenstellende Übereinstimmung mit der Theorie.

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