On the analyses of a skew girder bridge by the theory of orthotropic parallelogram plates

On the Analyses of a Skew Girder Bridge by the Theory of Orthotropic Parallelogram Plates

Etude des ponts à poutres obliques par la théorie du parallélogramme orthotrope

Über die Untersuchung von schiefen Balkenbrücken durch die Theorie der orthotropen Parallelogrammplatte

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1. Introduction

It is well known that Y. GUYON and CH. MASSONNET's table and diagram of the distribution coefficients based on the theory of orthotropic rectangular plates are very effective in the experimental analysis and design calculation of right girder bridges. On the contrary, there are very few researches on the skew girder bridge in every country. It would not be too much to say that the research made by T. Y. CHEN, C. P. SIESS and N. M. NEWMARK [1] is the only excellent one. Their research is based on the theory of continuous isotropic parallelogram plates supported by flexible girders. It can be easily assumed that the theory of orthotropic parallelogram plates will also be effective to the same degree in the analysis of skew girder bridges as the orthotropic rectangular plate in the analysis of right girder bridges. From this point of view, assuming a simply supported skew girder bridge structure with cross girders or sway bracings distributed in the direction perpendicular to the main girder as an orthotropic parallelogram plate which is simply supported on the opposite two skew sides, the authors intended to derive skew network finite difference equations for the orthotropic parallelogram plate and to give the influence coefficients of the deflection and bending moment for the several cases of characteristic values of the plates.

2. Skew Network Finite Difference Equation for the Orthotropic Parallelogram Plate

The analysis of the orthotropic parallelogram plate is to obtain the solution of the following fundamental differential equation expressed in Cartesian coordinates:

$$B_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + B_y \frac{\partial^4 w}{\partial y^4} = p, \tag{1}$$
$$2H = \nu_x B_y + \nu_y B_x + 4C.$$

Since the analytical solution of the above differential equation is very difficult, it is better to solve the equation numerically by using the finite difference method than to solve analytically. In the case where the plate is simply supported at the opposite two skew sides and supported by flexible girders at the other two sides (see fig. 1), by using the skew network as shown in fig. 2, the above differential equation can be changed into a finite difference equation for each

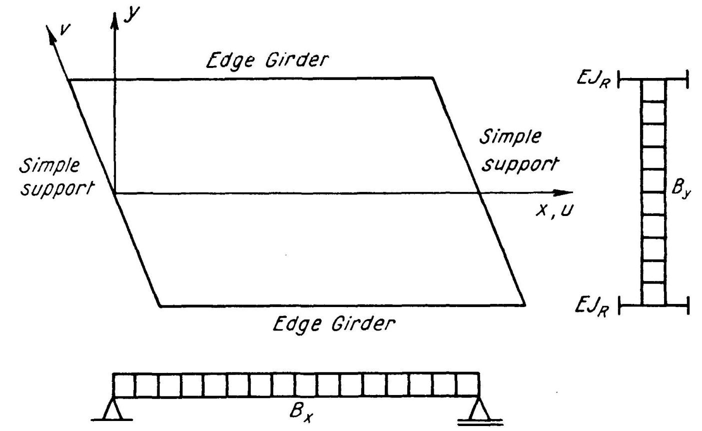


Fig. 1. Assumed Orthotropic Parallelogram Plate, Simply Supported at the Opposite Two Skew Sides and Supported by Flexible Girders at the Other Two Sides.

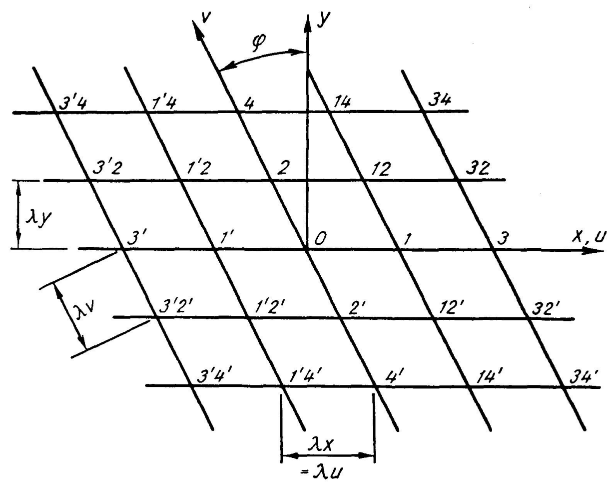


Fig. 2. Skew Network.

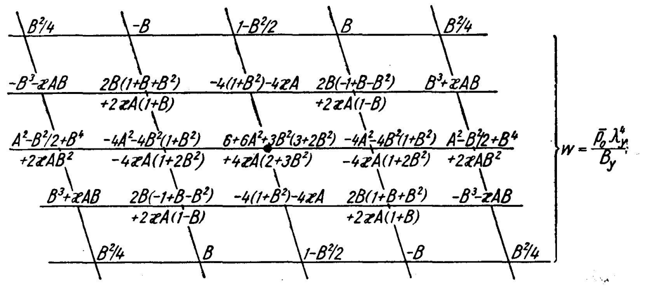
of the following nine types of network points, that is, a) general interior points, b) interior points near the left simple support, c) interior points near the right simple support, d) interior points near the edge girder, e) interior points near the sharp corner, f) interior points near the blunt corner, g) general edge points, h) edge point on the sharp corner, i) edge point on the blunt corner. For these derivations, Poisson's ratio ν_x and ν_y are assumed to be zero and the following notations are used in these equations.

$$A = \frac{K^2}{\alpha}, \quad B = K \tan\varphi, \quad K = \frac{\lambda_y}{\lambda_x}, \quad \alpha^2 = \frac{B_y}{B_x}, \quad J = K^4 \frac{E\,J_R}{\lambda_y B_x}, \quad \kappa = \frac{H}{\sqrt{B_x B_y}}.$$

The values of $\sqrt{A}$ and κ correspond to the values 2θ and α defined by Y. Guyon and Ch. Massonnet.

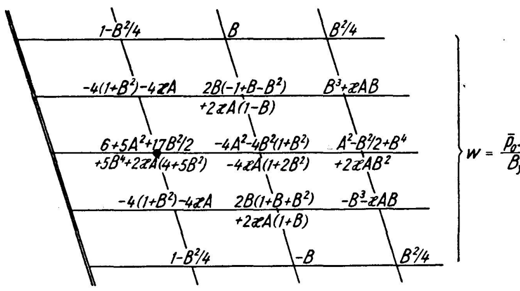
The above nine finite difference equations are shown in eqs. (2) ~ (10).

a) General interior points.



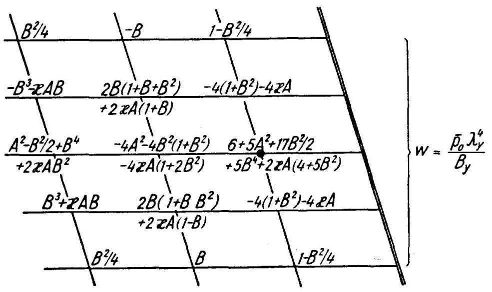
(2)

b) Interior points near the left simple support.



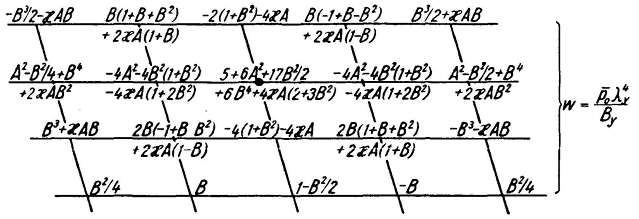
(3)

c) Interior points near the right simple support.



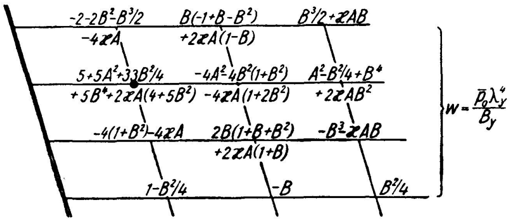
(4)

d) Interior points near the edge girder.



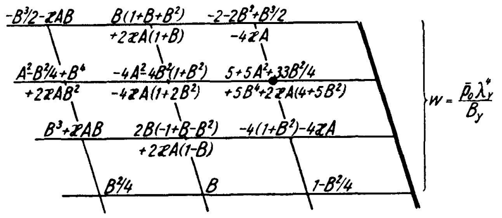
(5)

e) Interior points near the sharp corner.



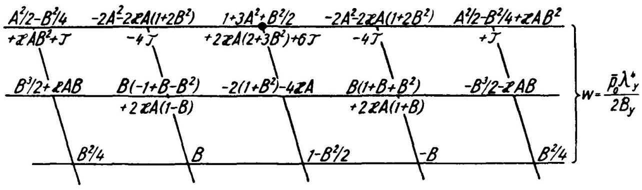
(6)

f) Interior points near the blunt corner.



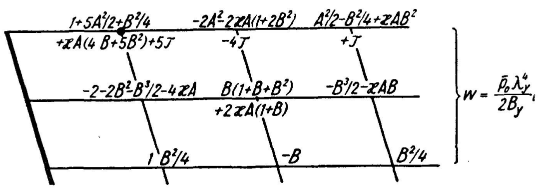
(7)

g) General edge points.



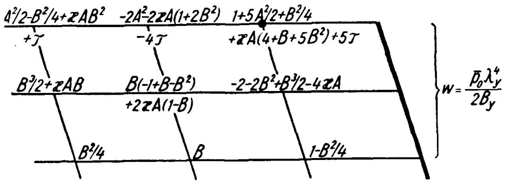
(8)

h) Edge point on the sharp corner.



(9)

i) Edge point on the blunt corner.



(10)

In these equations, the quantity $\overline{p}_0$ is the equivalent combined effects in terms of load per unit of area for all the loads which act upon at point 0. Thus, if at point 0 there act a uniformly distributed load of p_0 per unit of area, a line load of q_0 per unit of length in the x direction, and a concentrated load of P_0, $\overline{p}_0$ is given by

$$\overline{p}_0 = p_0 + \frac{q_0}{\lambda_y} + \frac{P_0}{\lambda_x \lambda_y} = p_0 + \frac{q_0}{\lambda_y} + \frac{K P_0}{\lambda_y^2}.$$

If point 0 lies on an exterior edge of the plate, $\overline{p}_0$ is given by

$$\overline{p}_0 = p_0 + \frac{q_0}{0.5\,\lambda_y} + \frac{P_0}{0.5\,\lambda_x \lambda_y} = p_0 + \frac{2\,q_0}{\lambda_y} + \frac{2\,K P_0}{\lambda_y^2}.$$

Covering the orthotropic parallelogram plate with the skew network and applying the above nine finite difference equations for each network point, we can obtain simultaneous equations. Solving these equations, the values of deflection can be calculated and then the bending moment can also be obtained from the following equations.

a) General interior point (see fig. 2)

$$\begin{aligned} M_x &= -\frac{K^2}{\lambda_y^2} B_x (w_{1'} - 2\,w_0 + w_1), \\ M_y &= -\frac{1}{\lambda_y^2} B_y \{B^2 (w_{1'} - 2\,w_0 + w_1) + 0.5\,B\,(-w_{1'2} + w_{12} + w_{1'2'} - w_{12'}) + (w_{2'} - 2\,w_0 + w_2)\}, \\ M_{xy} &= -\frac{K}{\lambda_y^2} \kappa\,\alpha\,B_x \{B\,(w_{1'} - 2\,w_0 + w_1) + 0.25\,(-w_{1'2} + w_{12} + w_{1'2'} - w_{12'})\}. \end{aligned} \tag{11}$$

b) Point on the left simple support (see fig. 3a)

$$\begin{aligned} M_x &= -\frac{K^2}{\lambda_y^2} B_x z\,(w_{12'} - w_{12}), \qquad M_y = -\frac{1}{\lambda_y^2} B_y (B^2 z - B)\,(w_{12'} - w_{12}), \\ M_{xy} &= -\frac{K}{\lambda_y^2} \kappa\,\alpha\,B_x (B\,z - 0.5)\,(w_{12'} - w_{12}), \\ z &= \left(\kappa \frac{B}{A} + \frac{B^3}{A^3}\right) \Big/ \left(1 + 2\,\kappa \frac{B^2}{A} + \frac{B^4}{A^2}\right). \end{aligned} \tag{12}$$

c) Point on the right simple support (see fig. 3b)

$$\begin{aligned} M_x &= -\frac{K^2}{\lambda_y^2} B_x z\,(w_{1'2} - w_{1'2'}), \qquad M_y = -\frac{1}{\lambda_y^2} B_y (B^2 z - B)\,(w_{1'2} - w_{1'2'}), \\ M_{xy} &= -\frac{K}{\lambda_y^2} \kappa\,\alpha\,B_x (B\,z - 0.5)\,(w_{12'} - w_{1'2'}). \end{aligned} \tag{13}$$

d) Point on the exterior edge (see fig. 3c)

$$\begin{aligned} M_x &= -\frac{K^2}{\lambda_y^2} B_x (w_1 - 2\,w_0 + w_{1'}), \qquad M_y = 0, \\ M_{xy} &= -\frac{K}{\lambda_y^2} \kappa\,\alpha\,B_x \{B\,(w_{1'} - 2\,w_0 + w_1) + 0.5\,(-w_{1'} + w_1 + w_{12} - w_{12})\}. \end{aligned} \tag{14}$$

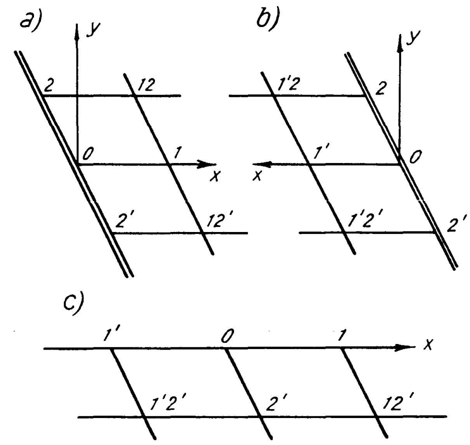


Fig. 3. Points on the Simple Supports and on the Exterior Edge.

The assumption of $\nu_x = \nu_y = 0$ is same as in the case of the derivation of the finite difference equations.

The detailed description of the above eqs. (2) $\sim$ (14) requires so much space that it is omitted and only the results of the derivation are written here (see references [2]).

3. Tabulation of the Influence Coefficients of Deflection and Bending Moment

If the tables of the influence coefficients of the deflection and bending moment were completed, these tables would become of great help for the designers of the skew girder bridges. From this point of view, it is necessary

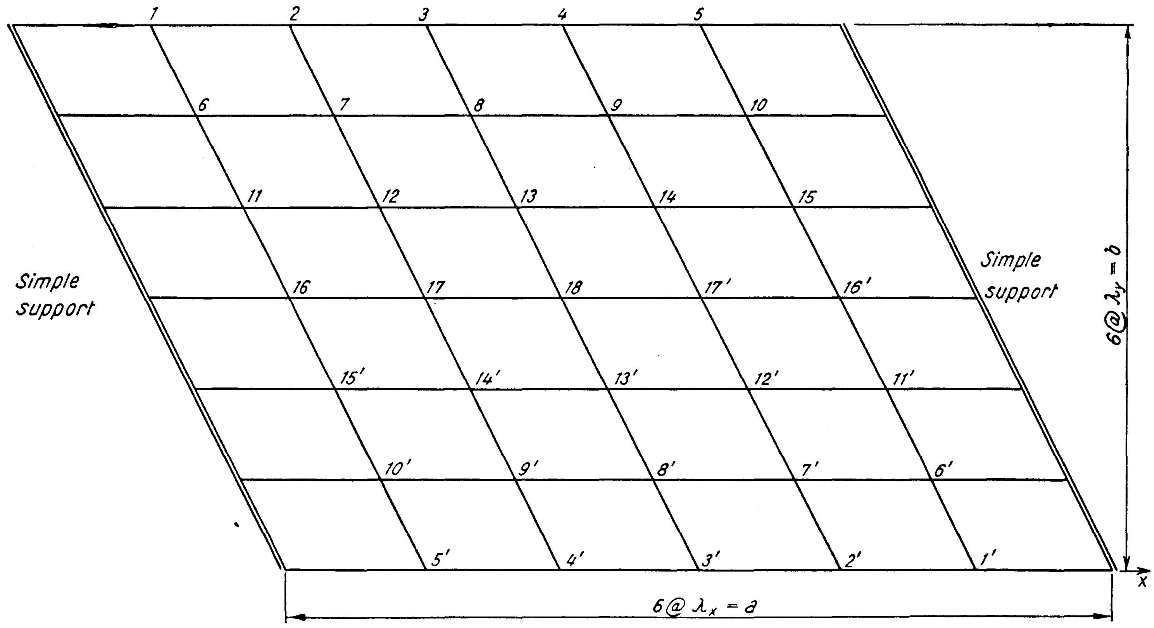


Fig. 4. Orthotropic Parallelogram Plate, Simply Supported at the Opposite Two Skew Sides and Free at the Other Two Sides, and its Skew Network.

to calculate the influence coefficients of deflection and bending moment for the various combinations of the above characteristic values A, B and κ.

The authors are now calculating the table of influence coefficients of deflection and bending moment of points $1 \sim 18$, covering the orthotropic parallelogram plate with the skew network as shown in fig. 4, for the various combinations of the characteristic values of the plate, $A = 1, 4, 9, 16, 25, 36$;

Table 1. Influence Coefficients of Deflection w for the Case of $A=1$, $B=0.5$
(unit: $10^{-6} P a^2/B_x K$)

point	$\kappa=0$				$\kappa=1$			
	w_3	w_8	w_{13}	w_{18}	w_3	w_8	w_{13}	w_{18}
1	37 708	23 973	14 239	7 901	18 419	11 720	7 208	4 443
2	71 613	44 670	25 687	13 551	33 418	21 818	13 721	8 622
3	91 814	56 639	31 196	15 354	40 841	27 448	17 668	11 317
4	77 459	51 896	28 698	13 144	32 171	23 979	16 461	10 970
5	44 791	31 492	17 657	7 530	16 672	13 372	9 843	6 903
6	30 052	21 146	13 553	8 183	15 177	11 912	7 854	5 231
7	50 850	37 500	24 437	14 778	25 313	20 594	14 741	10 129
8	56 639	44 882	29 789	17 953	27 448	24 653	18 495	13 106
9	45 121	36 287	25 944	16 287	20 479	19 467	16 197	12 264
10	23 652	19 378	14 691	9 799	9 950	9 995	9 068	7 391
11	18 884	16 298	12 975	9 354	11 136	10 057	8 547	6 389
12	30 349	27 216	23 512	17 499	17 384	16 913	15 694	12 257
13	31 196	29 789	28 798	22 084	17 668	18 495	19 221	15 648
14	24 537	23 232	22 762	19 490	12 884	14 093	15 295	13 873
15	12 794	12 125	12 095	11 193	6 173	7 049	8 016	7 913
16	9 349	10 642	11 513	10 865	7 610	7 907	8 157	7 663
17	14 602	17 034	19 770	20 291	11 427	12 591	14 032	14 386
18	15 354	17 953	22 084	25 522	11 317	13 106	15 648	17 918
17′	12 278	14 049	17 212		8 247	9 870	12 120	
16′	6 479	7 356	9 029		3 954	4 895	6 200	
15′	3 074	5 739	8 753		5 112	5 894	6 755	
14′	4 996	9 069	14 230		7 573	9 115	11 338	
13′	5 451	9 540	15 193		7 427	9 254	11 989	
12′	5 049	7 555	12 002		5 412	6 920	9 162	
11′	2 292	3 944	6 336		2 608	3 424	4 642	
10′	−1 354	1 655	5 378		3 539	4 417	5 709	
9′	−2 030	2 688	8 673		5 208	6 719	9 006	
8′	−1 752	3 172	9 541		5 064	6 716	9 254	
7′	−1 055	2 862	7 990		3 668	4 981	6 996	
6′	− 517	1 630	4 450		1 767	2 460	3 530	
5′	−6 842	−3 279	1 133		2 537	3 434	4 808	
4′	−9 012	−3 423	3 465		3 688	5 131	7 380	
3′	−7 596	−1 752	5 451		3 558	5 064	7 427	
2′	−4 620	12	5 742		2 576	3 742	5 794	
1′	−1 879	588	3 710		1 259	1 867	2 825	

Table 2. Influence Coefficients of Deflection w for the Case of $A=4$, $B=0.5$
(unit: $10^{-6}\,Pa^2/B_x K$)

point	$\kappa=0$				$\kappa=1$			
	w_3	w_8	w_{13}	w_{18}	w_3	w_8	w_{13}	w_{18}
1	63 877	15 144	−2 361	−4 092	39 195	18 977	8 363	3 527
2	116 084	26 474	−5 051	−7 778	70 381	34 694	15 542	6 642
3	140 931	30 559	−7 461	−10 104	85 230	42 564	19 349	8 376
4	115 485	25 802	−8 044	−9 862	68 545	37 692	17 865	7 897
5	63 276	14 291	−5 536	−6 324	36 694	21 806	10 858	4 930
6	20 452	16 782	7 297	2 395	23 489	21 302	11 794	5 621
7	31 921	29 640	12 708	4 134	38 977	38 393	21 750	10 533
8	30 559	33 357	13 764	4 388	42 564	46 394	26 770	13 164
9	18 209	19 644	9 287	2 985	33 458	37 564	23 870	12 208
10	3 476	3 683	2 178	735	17 441	20 268	13 962	7 474
11	−3 251	8 255	19 062	12 339	11 575	14 523	16 880	9 877
12	−5 952	13 362	35 254	22 272	18 410	24 308	30 922	18 450
13	−7 461	13 764	43 615	26 879	19 349	26 770	38 001	22 973
14	−6 985	10 085	34 444	23 984	15 053	21 343	30 662	20 637
15	−4 292	4 943	18 269	14 134	7 788	11 311	16 565	12 169
16	−5 960	2 195	14 193	22 503	5 183	7 724	12 305	15 931
17	−9 568	3 783	24 039	41 413	8 087	12 399	20 747	29 409
18	−10 104	4 388	26 879	51 145	8 376	13 164	22 973	36 398
17′	−7 995	3 859	22 251		6 480	10 367	18 347	
16′	−4 295	2 241	12 338		3 339	5 431	9 756	
15′	−2 919	243	5 536		2 238	3 696	6 848	
14′	−4 759	555	9 328		3 471	5 830	11 067	
13′	−5 125	803	10 477		3 582	6 109	11 821	
12′	−4 150	813	8 856		2 770	4 783	9 364	
11′	−2 274	507	4 993		1 428	2 495	4 940	
10′	−128	−52	109		1 012	1 793	3 617	
9′	−536	−180	509		1 576	2 823	5 773	
8′	−789	−230	804		1 632	2 955	6 109	
7′	−737	−200	792		1 267	2 315	4 831	
6′	−424	−111	466		657	1 211	2 549	
5′	1 250	−414	3 049		536	1 008	2 169	
4′	1 939	−704	4 868		827	1 573	3 421	
3′	1 976	−789	5 125		850	1 632	3 582	
2′	1 517	−658	4 053		655	1 271	2 812	
1′	797	−366	2 177		338	662	1 477	

Table 3. Influence Coefficients of Deflection w for the Case of $A=9$, $B=1$
(unit: $10^{-6}Pa^2/B_xK$)

point	$\kappa=0$				$\kappa=1$			
	w_3	w_8	w_{13}	w_{18}	w_3	w_8	w_{13}	w_{18}
1	98 918	17 948	− 8	−1 118	54 119	15 956	4 118	1 004
2	177 070	32 012	− 436	−2 152	95 599	30 133	8 118	2 037
3	212 187	38 314	−1 234	−2 850	113 625	38 078	10 699	2 762
4	178 134	34 266	−1 767	−2 835	91 917	35 062	10 474	2 796
5	100 065	20 275	−1 401	−1 847	49 275	21 005	6 734	1 871
6	21 544	37 469	11 362	1 670	23 071	26 068	9 117	2 594
7	35 353	67 557	20 843	2 964	36 640	46 961	17 584	5 181
8	38 314	81 498	25 619	3 442	38 078	56 480	22 511	6 877
9	30 924	67 571	23 434	2 984	28 686	46 181	21 003	6 774
10	16 681	37 475	14 133	1 728	14 244	25 115	12 807	4 396
11	−1 358	14 023	35 185	11 073	7 234	13 194	22 561	8 107
12	−1 738	23 348	63 504	20 235	10 842	21 305	41 128	15 839
13	−1 234	25 619	76 737	24 794	10 699	22 511	50 055	20 506
14	− 467	20 929	63 424	22 621	7 820	17 316	40 992	19 304
15	− 65	11 455	35 099	13 620	3 790	8 814	22 402	11 879
16	−1 840	1 679	13 645	34 820	1 967	4 518	11 931	22 164
17	−2 829	2 940	22 644	62 904	2 863	6 866	19 347	40 505
18	−2 850	3 442	24 794	76 089	2 762	6 877	20 506	49 412
17′	−2 161	3 006	20 214		1 987	5 106	15 800	
16′	−1 128	1 714	11 050		951	2 513	8 062	
15′	− 537	−390	1 726		498	1 303	4 162	
14′	− 874	−540	3 004		715	1 920	6 345	
13′	− 939	−469	3 497		682	1 875	6 374	
12′	− 757	−295	3 039		487	1 366	4 749	
11′	− 422	−133	1 729		231	661	2 346	
10′	− 73	−257	−378		126	360	1 273	
9′	− 129	−404	−530		182	526	1 898	
8′	− 150	−418	−469		174	511	1 875	
7′	− 129	−325	−305		125	372	1 382	
6′	− 72	−172	−142		60	180	678	
5′	64	− 75	−554		42	128	484	
4′	96	−131	−888		59	183	706	
3′	96	−150	−939		56	174	682	
2′	71	−128	−746		39	124	493	
1′	37	− 73	−404		18	59	238	

Table 4. Influence Coefficients of Deflection w for the Case of $A=16$, $B=1$
(unit: $10^{-6} Pa^2/B_x K$)

point	$\kappa=0$				$\kappa=1$			
	w_3	w_8	w_{13}	w_{18}	w_3	w_8	w_{13}	w_{18}
1	112 928	9 784	−1 712	− 586	69 017	15 439	2 942	523
2	201 245	17 399	−3 237	−1 076	121 914	29 044	5 740	1 044
3	239 734	20 733	−4 185	−1 345	144 671	36 475	7 473	1 390
4	201 482	18 478	−4 047	−1 262	118 240	33 613	7 246	1 384
5	113 182	10 906	−2 562	− 782	64 249	20 286	4 638	914
6	11 326	48 600	7 799	− 102	21 608	33 474	8 663	1 778
7	18 835	86 976	14 073	− 250	34 630	60 137	16 539	3 495
8	20 733	104 046	17 004	− 427	36 475	72 197	20 971	4 568
9	17 044	87 029	15 344	− 503	28 118	59 664	19 523	4 448
10	9 370	48 656	9 159	− 361	14 335	32 898	11 942	2 864
11	−2 597	9 155	47 006	7 591	4 843	12 127	30 710	8 060
12	−4 079	15 339	84 150	13 692	7 398	19 669	55 515	15 503
13	−4 185	17 004	100 726	16 541	7 493	20 971	67 072	19 788
14	−3 209	14 079	84 127	14 926	5 617	16 407	55 466	18 522
15	−1 688	7 806	46 982	8 910	2 805	8 513	30 652	11 390
16	− 778	−368	8 913	46 886	938	2 907	11 404	30 541
17	−1 258	−509	14 928	83 953	1 401	4 401	15 533	55 246
18	−1 345	−427	16 541	100 512	1 390	4 568	19 788	66 789
17′	−1 081	−244	13 690		1 030	3 467	15 492	
16′	− 589	− 97	7 589		508	1 747	8 047	
15′	− 51	−359	−349		169	587	2 758	
14′	− 90	−523	−480		250	882	4 257	
13′	− 105	−603	−399		245	880	4 345	
12′	− 91	−476	−225		180	657	3 301	
11′	− 52	−256	− 87		89	326	1 665	
10′	16	− 52	−358		30	112	579	
9′	24	− 85	−572		45	167	876	
8′	24	− 94	−603		43	166	880	
7′	18	− 78	−478		32	124	662	
6′	10	− 44	−258		16	61	331	
5′	8	15	− 54		8	30	167	
4′	12	24	− 93		12	45	248	
3′	13	24	−105		11	43	245	
2′	11	19	− 89		7	32	181	
1′	6	10	− 50		4	16	90	

Table 5. Influence Coefficients of Bending Moment M_x for the Case of $A=1.0$ and $B=0.5$
(unit: $10^{-4}\,P/K$)

point	$\kappa=0$				$\kappa=1$			
	$M_{x,3}$	$M_{x,8}$	$M_{x,13}$	$M_{x,18}$	$M_{x,3}$	$M_{x,8}$	$M_{x,13}$	$M_{x,18}$
1	2 133	1 771	1 239	770	1 088	777	511	342
2	5 528	3 771	2 375	1 336	2 670	1 735	1 088	692
3	12 439	6 230	2 702	1 378	5 793	3 277	1 824	1 066
4	5 883	5 314	2 786	1 132	2 610	2 520	1 749	1 112
5	2 556	2 849	1 764	685	1 015	1 140	966	696
6	2 492	1 543	1 025	687	1 241	1 099	548	391
7	4 935	3 200	2 052	1 306	2 592	1 777	1 216	837
8	6 016	5 751	3 287	1 736	3 275	3 328	2 154	1 350
9	3 722	3 130	2 598	1 611	1 658	1 707	1 694	1 297
10	1 712	1 427	1 248	979	674	687	787	730
11	1 787	1 410	735	637	1 062	865	559	448
12	3 225	2 734	1 847	1 408	1 837	1 760	1 311	1 007
13	2 882	3 311	4 076	2 585	1 855	2 179	2 683	1 852
14	2 297	1 964	1 772	1 985	1 062	1 199	1 290	1 450
15	1 140	905	728	914	461	513	529	677
16	866	1 036	948	621	765	777	695	491
17	1 304	1 705	2 014	1 547	1 169	1 337	1 463	1 191
18	1 445	1 743	2 584	3 766	1 095	1 375	1 860	2 543
17′	1 297	1 225	1 377		694	846	1 008	
16′	694	601	603		312	385	443	
15′	278	554	857		508	599	640	
14′	480	842	1 445		739	935	1 205	
13′	610	870	1 495		605	902	1 252	
12′	1 006	694	1 043		462	600	776	
11′	314	352	503		216	283	360	
10′	− 92	136	521		341	442	581	
9′	−247	220	811		487	658	919	
8′	− 33	286	884		452	623	890	
7′	43	283	753		312	431	593	
6′	37	164	414		147	206	283	
5′	−549	−329	54		235	333	478	
4′	−701	−345	268		332	480	716	
3′	−562	−151	308		307	451	673	
2′	−306	− 94	616		215	317	622	
1′	−100	81	397		103	154	225	

Table 6. Influence Coefficients of Bending Moment M_x for the Case of $A=4.0$, $B=0.5$ (unit: $10^{-4}P/K$)

point	$\kappa=0$				$\kappa=1$			
	$M_{x,3}$	$M_{x,8}$	$M_{x,13}$	$M_{x,18}$	$M_{x,3}$	$M_{x,8}$	$M_{x,13}$	$M_{x,18}$
1	4 232	1 716	–155	–370	2 477	1 477	714	314
2	9 332	3 167	–398	–715	5 470	3 002	1 387	602
3	18 105	3 956	–714	–952	11 353	4 569	1 885	787
4	8 500	3 398	–790	–945	5 649	3 792	1 774	760
5	4 167	1 873	–521	–609	2 356	1 952	1 053	477
6	1 973	1 093	598	247	2 082	1 432	948	491
7	3 223	2 888	1 161	326	3 899	3 219	2 173	954
8	3 183	6 275	1 469	409	4 587	6 059	2 840	1 282
9	1 797	2 232	990	272	2 930	3 157	2 391	1 210
10	318	236	205	71	1 377	1 355	1 262	726
11	–276	1 022	957	1 068	1 125	1 314	1 041	776
12	–543	1 809	2 734	2 050	1 838	2 437	2 500	1 608
13	–658	1 992	6 311	2 688	1 905	2 851	5 191	2 467
14	–531	1 388	2 660	2 360	1 363	1 894	2 478	2 071
15	–310	660	884	1 323	676	915	1 014	1 087
16	–559	314	1 335	1 276	505	753	1 100	951
17	–891	519	2 376	3 304	785	1 234	2 083	2 353
18	–924	597	2 701	10 789	797	1 291	2 469	5 032
17′	–713	546	2 059		595	947	1 601	
16′	–376	319	1 074		301	478	767	
15′	–274	35	536		214	360	666	
14′	–445	75	886		330	565	1 100	
13′	–478	110	997		336	581	1 156	
12′	–387	115	871		256	442	847	
11′	–211	73	497		131	227	430	
10′	– 12	– 7	10		95	171	350	
9′	– 53	– 25	50		148	268	554	
8′	– 78	– 33	86		152	277	577	
7′	– 73	– 28	92		117	215	443	
6′	– 42	– 15	56		60	112	230	
5′	114	– 58	288		50	95	206	
4′	176	– 98	459		77	147	322	
3′	179	–136	483		78	152	333	
2′	137	– 92	381		60	117	257	
1′	72	– 51	204		31	61	134	

Table 7. Influence Coefficients of Bending Moment M_x for the Case of $A=9$, $B=1$ (unit: $10^{-4}P/K$)

point	$\kappa=0$				$\kappa=1$			
	$M_{x,3}$	$M_{x,8}$	$M_{x,13}$	$M_{x,18}$	$M_{x,3}$	$M_{x,8}$	$M_{x,13}$	$M_{x,18}$
1	7 474	1 654	33	− 97	3 744	1 265	340	84
2	15 582	3 006	11	−190	8 135	2 575	692	172
3	24 901	3 726	− 95	−256	14 304	3 895	985	244
4	15 681	3 356	−175	−260	7 911	3 604	1 014	254
5	7 580	1 965	−139	−171	3 452	2 008	657	174
6	2 069	2 644	1 034	173	2 197	1 840	744	221
7	3 446	5 816	1 944	303	3 770	4 056	1 532	453
8	3 726	10 032	2 506	338	3 940	7 134	2 304	642
9	2 914	5 817	2 305	282	6 449	3 984	2 153	658
10	1 549	2 644	1 365	163	1 156	1 764	1 229	431
11	−135	1 342	2 423	1 005	715	1 269	1 513	652
12	−173	2 297	5 419	1 882	1 068	2 190	3 483	1 370
13	− 95	2 506	9 557	2 423	1 010	2 317	6 476	2 111
14	7	1 952	5 410	2 224	687	1 518	3 471	1 987
15	27	1 043	2 417	1 314	324	724	1 499	1 137
16	−171	159	1 317	2 388	186	445	1 143	1 476
17	−260	277	2 226	5 358	266	672	1 992	3 423
18	−250	337	2 424	9 493	249	648	2 113	6 414
17′	−192	307	1 881		173	451	1 368	
16′	− 99	176	1 003		82	217	649	
15′	− 50	−35	163		45	123	409	
14′	− 82	−46	283		63	178	619	
13′	− 88	−37	342		59	169	595	
12′	− 71	−20	310		42	120	415	
11′	− 46	− 9	178		19	58	200	
10′	− 7	−24	−33		11	32	119	
9′	− 13	−37	−45		16	56	174	
8′	− 15	−39	−37		15	45	167	
7′	− 13	−30	−22		10	32	120	
6′	− 7	−15	− 9		5	15	58	
5′	6	− 8	−52		4	12	43	
4′	8	−13	−84		5	16	62	
3′	9	−15	−89		5	15	58	
2′	6	−13	−71		3	10	41	
1′	3	− 8	−39		1	5	19	

Table 8. Influence Coefficients of Bending Moment M_x for the Case of $A=16$, $B=1$
(unit: $10^{-4}\,P/K$)

point	$\kappa=0$				$\kappa=1$			
	$M_{x,3}$	$M_{x,8}$	$M_{x,13}$	$M_{x,18}$	$M_{x,3}$	$M_{x,8}$	$M_{x,13}$	$M_{x,18}$
1	8 829	919	−150	− 54	4 967	1 286	256	46
2	17 950	1 652	−288	− 99	10 524	2 557	508	110
3	27 627	2 011	−390	−126	17 708	3 673	695	126
4	17 972	1 800	−388	−119	10 282	3 400	697	127
5	8 853	1 056	−247	− 73	4 653	1 942	449	86
6	1 092	3 677	730	− 3	2 070	2 431	736	158
7	1 796	7 675	1 334	− 14	3 513	7 032	1 477	314
8	2 012	12 271	1 652	− 36	3 706	8 854	2 112	457
9	1 622	7 679	1 497	− 48	2 509	5 192	1 971	429
10	884	3 682	886	− 35	1 214	2 384	1 147	278
11	−252	885	3 523	710	473	1 184	2 169	2 841
12	−392	1 497	7 397	1 297	720	1 989	4 779	1 379
13	−391	1 653	11 943	1 607	706	2 117	8 339	1 998
14	−287	1 335	7 394	1 326	507	1 469	4 775	1 875
15	−148	731	3 520	861	248	725	2 164	1 093
16	− 73	−36	862	3 510	88	283	1 095	2 143
17	−118	−49	1 456	7 378	130	376	1 876	4 752
18	−126	−36	1 671	11 923	127	430	1 998	8 311
17′	−101	−13	1 297		92	313	1 378	
16′	− 55	− 3	710		45	155	679	
15′	− 5	−34	−34		16	55	268	
14′	− 9	−18	−46		23	82	412	
13′	− 10	−56	−34		22	80	408	
12′	− 9	−45	−12		16	59	297	
11′	− 5	−23	− 2		8	29	147	
10′	1	− 5	−34		3	10	54	
9′	2	− 8	−54		4	15	81	
8′	2	− 9	−57		3	15	80	
7′	1	− 7	−45		3	12	59	
6′	1	− 5	−24		1	5	30	
5′	1	1	− 5		1	3	15	
4′	1	3	− 9		1	4	22	
3′	1	2	−10		1	3	22	
2′	1	2	− 9		1	3	16	
1′	0	1	− 6		0	1	8	

$B=0.5, 1,0, 1.5, 2.0$ and $\kappa=0, 1.0$. The values of $\sqrt{A}/2$ corresponds to the values of θ (defined by Y. Guyon and Ch. Massonnet) $=$ 0.5, 1.0, 1.5, 2.0, 2.5 and 3.0.

Instead of calculating the matrix inversion of the given stiffness matrix of $(6-1)\times(6+1)=35$ elements, we are calculating the inverse matrix of the stiffness matrices of 18 (17) elements for the loading symmetrical (anti-symmetrical) about the diagonal, to obtain the necessary values by adding or subtracting the solution of the above both cases. The digital computers, UNIVAC-120 and Bendix G-15D, are used for these matrix inversions.

It requires much space to write the influence coefficients of deflection and bending moment of all points $1 \sim 18$ in detail. Therefore, as an example, only the influence coefficients of deflection w and bending moment in x direction, M_x, of the points 3, 8, 13 and 18 are shown in tables $1 \sim 8$ for the case of $J=0$ (that is, the plate is free at the two sides in y direction).

4. Experimental Verification of the Theory

In order to check the effectiveness of the author's method of analysis for the skew girder bridges, experimental researches were made on three models, that is, a) a skew grillage girder bridge consisting of composite main and cross girder sections, b) a skew grillage girder consisting of aluminium round bars for main and cross girders, c) a skew grillage girder bridge consisting of main and cross girder sections of plastic material of polymethachrilmethyl, Acrylite. Now that the detail of the result of experimental research for the model a) was already published [3], it is omitted to describe here and the results for the model b) and c) will be described.

I. *Model.* The plan and cross section of these models b) and c) are shown in figs. $5 \sim 8$, and the characteristic values of the models b) and c) are as follows:

Model	$\tan\varphi$	a	b	K	$\alpha=\sqrt{B_y/B_x}$	A	B	Fig.
$b)_1$	8/17	405	860	17/8	$(17/24)^2$	9	1	Fig. 5
$b)_2$	32/63	429	845	63/32	$(63/128)^2$	16	1	Fig. 6
$c)_1$	29/42	580	420	42/58	$(1/1.38)^2$	1	0.5	Fig. 7
$c)_2$	11/20	660	600	10/11	$(1/2.2)^2$	4	0.5	Fig. 8

II. *Experiment.* For the model b), a concentrated load $P=5$ kg was applied to the mid-span point of each main girder and the deflection was measured at these points by dial gauges. For the model c), a concentrated load $P=5$ kg was applied to the points corresponding to the network points of the skew network of the orthotropic plate, and the deflection was measured at these points by dial gauges, and the strain of the girder was measured by electric wire resistance strain gauges cemented into the web plate of the main girder.

III. *Experimental result.* The results of experimental researches for the model b) and c) were shown in figs. $9 \sim 14$. For the sake of convenience, these figures show only the influence values of deflection, bending moment and distribution coefficient for points 3, 8, 13 and 18, along the mid-span line and the

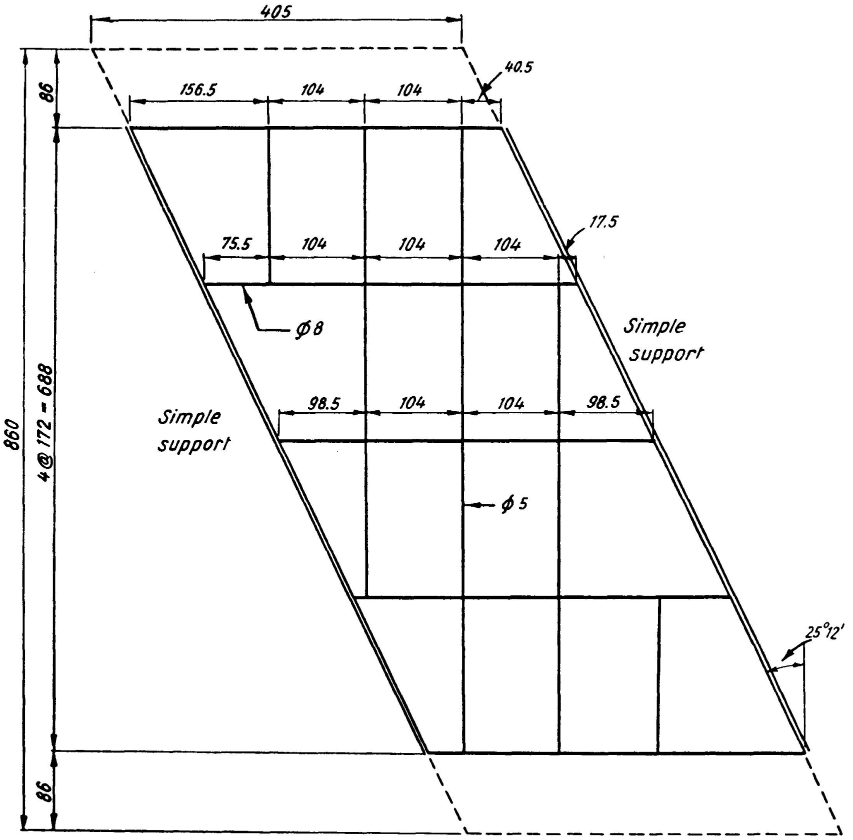


Fig. 5. The Plan of the Skew Grillage Girder ($A = 9$, $B = 1$).

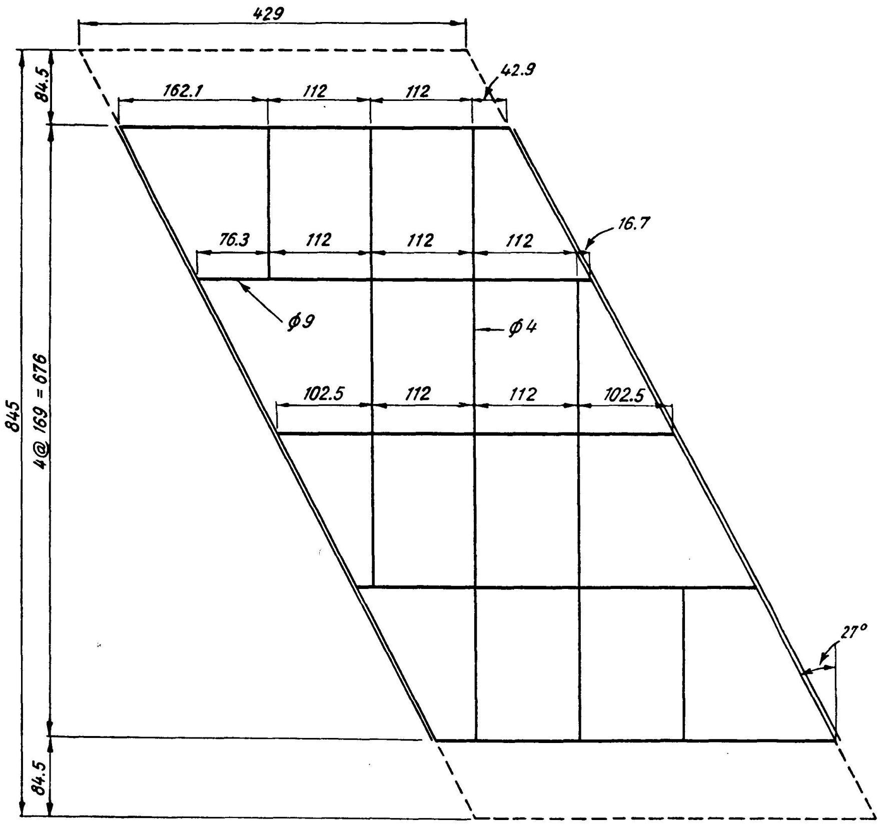


Fig. 6. The Plan of the Skew Grillage Girder ($A = 16$, $B = 1$).

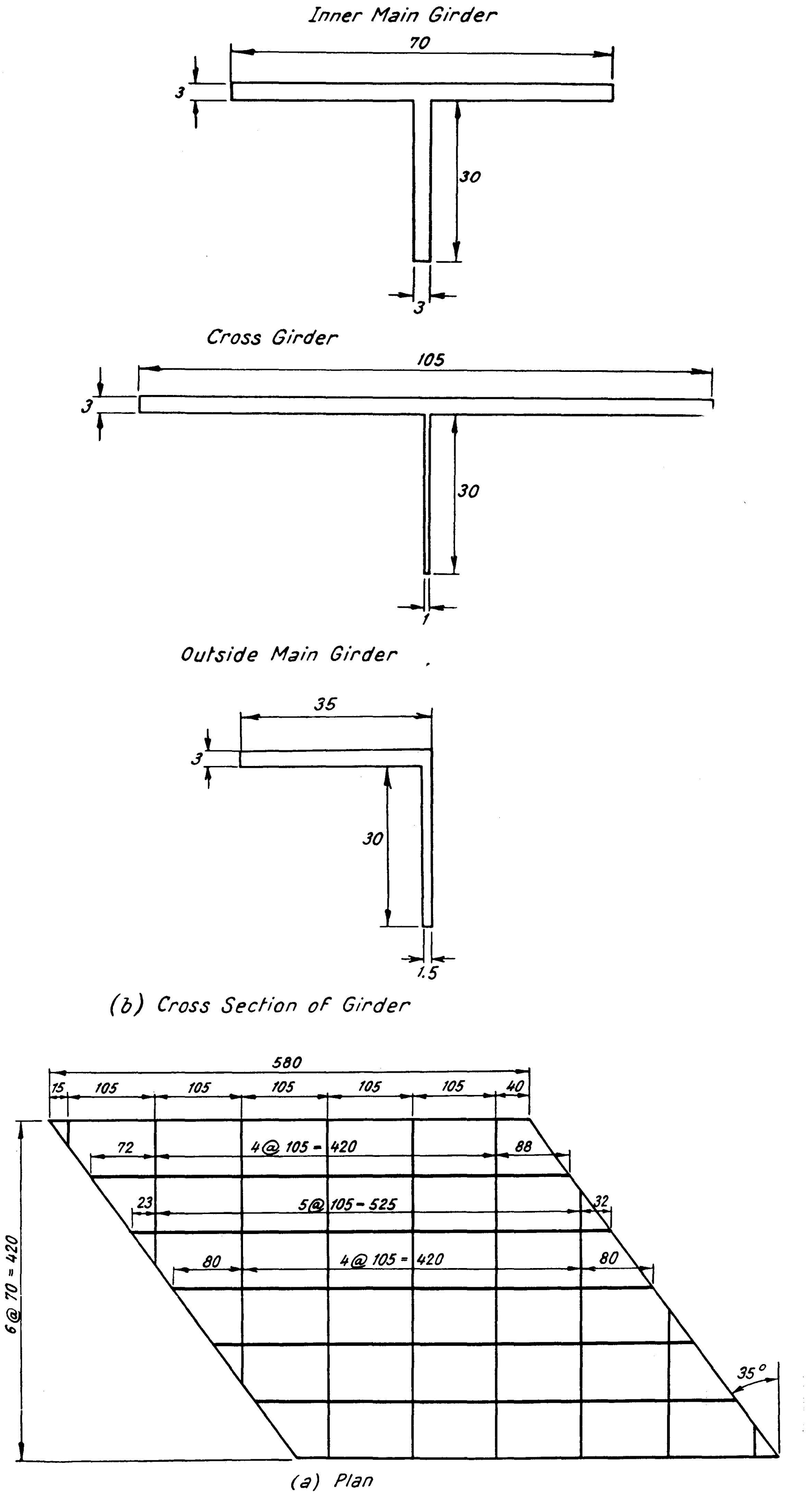


Fig. 7. The Plan of the Skew Grillage Girder Bridge ($A = 1$, $B = 0.5$) and the Cross Section of the Main and Cross Girders.

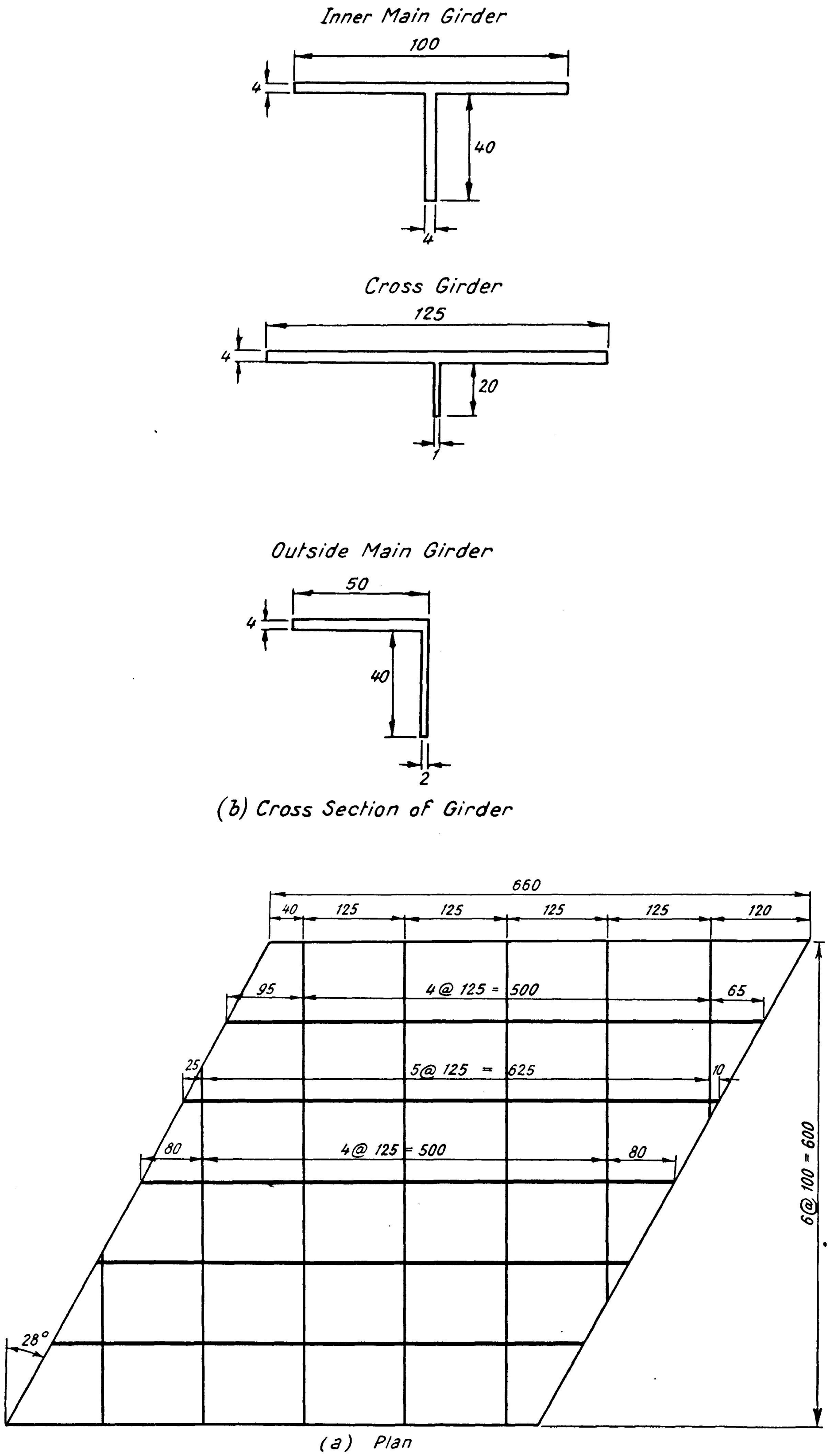
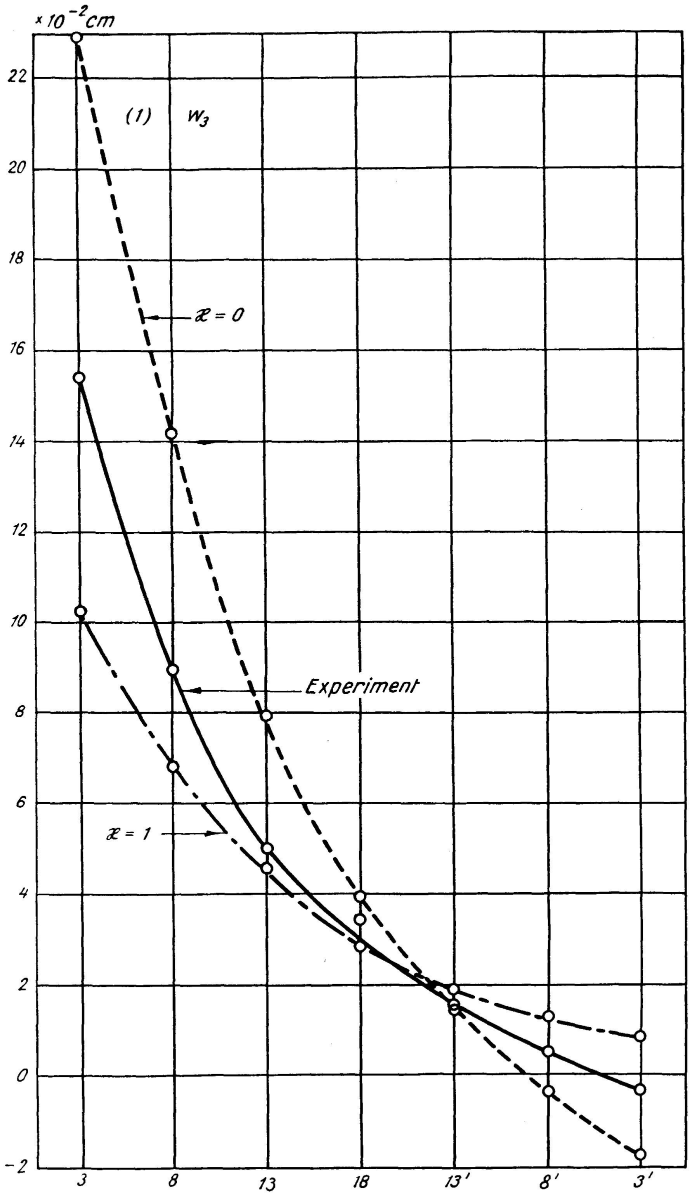


Fig. 8. The Plan of the Skew Grillage Girder Bridge ($A=4$, $B=0.5$) and the Cross Section of the Main and Cross Girders.



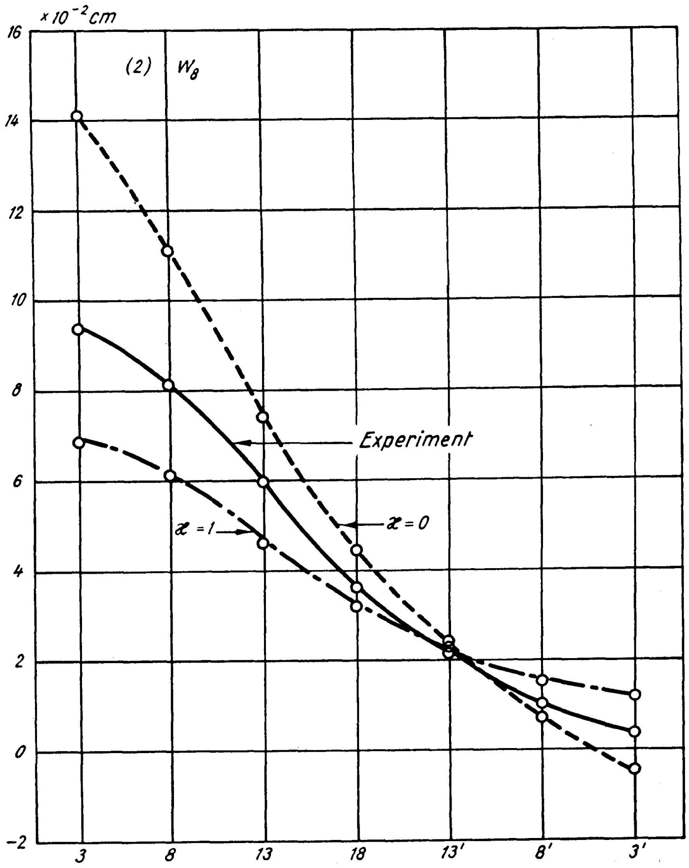
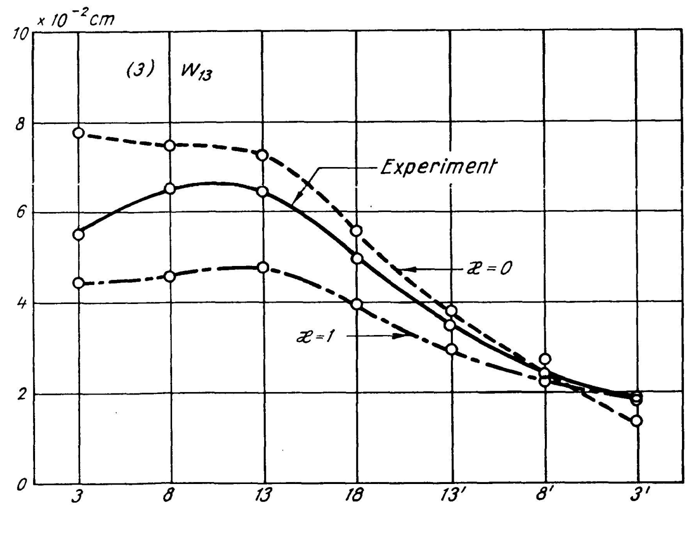
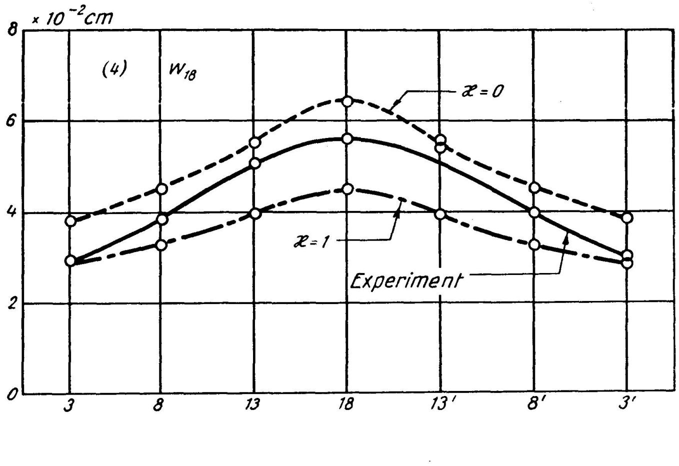


Fig. 9. Comparison of the Deflections Calculated from the Measured Dial Gauge Readings and from the Theory for the Model c)$_1$ ($A=1$, $B=0.5$).





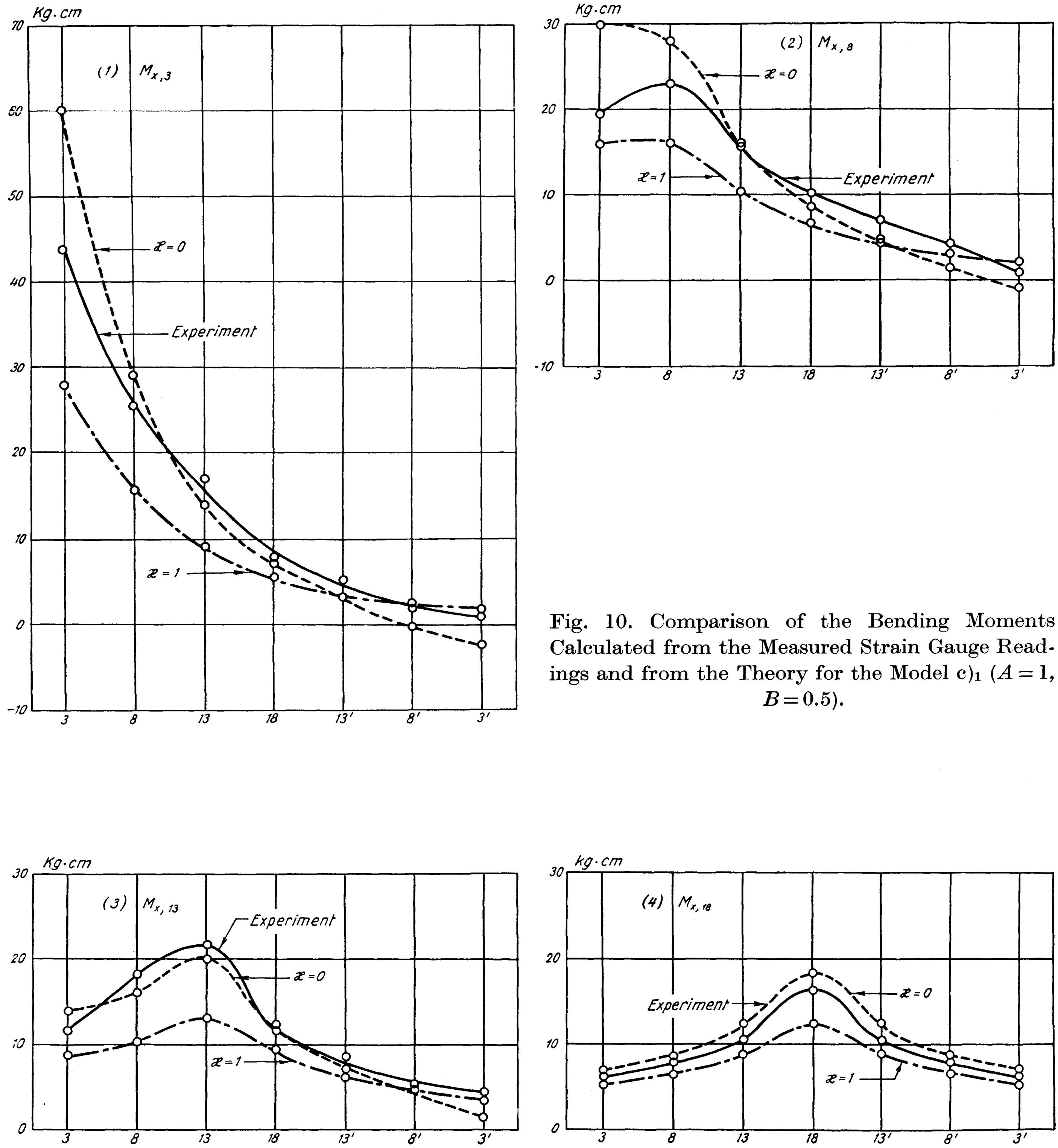
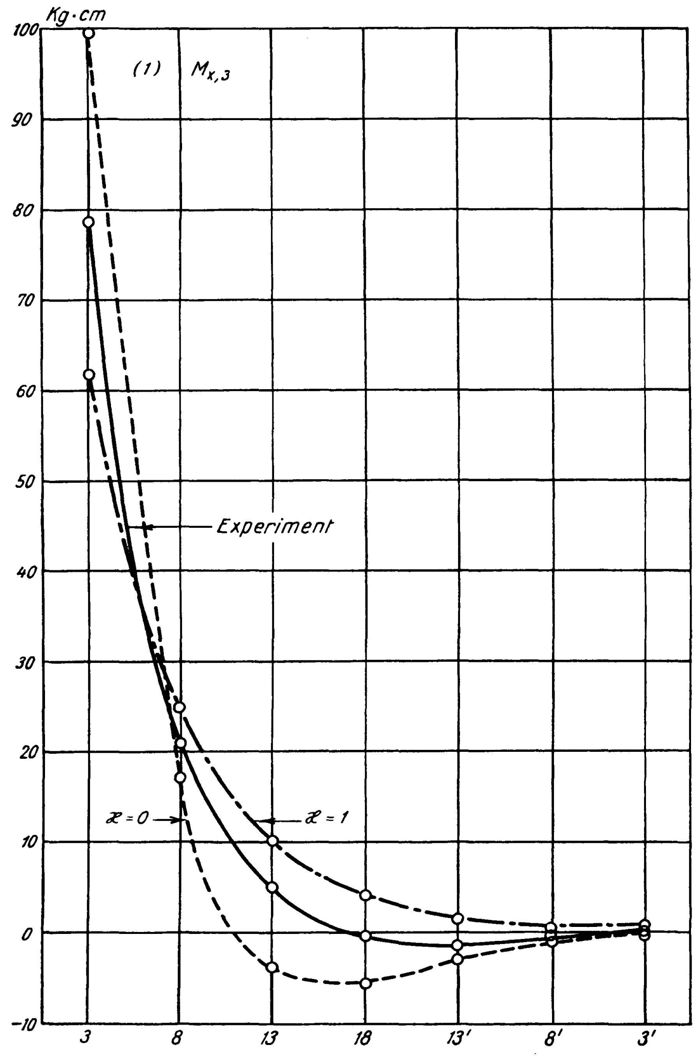


Fig. 10. Comparison of the Bending Moments Calculated from the Measured Strain Gauge Readings and from the Theory for the Model c)$_1$ ($A=1$, $B=0.5$).



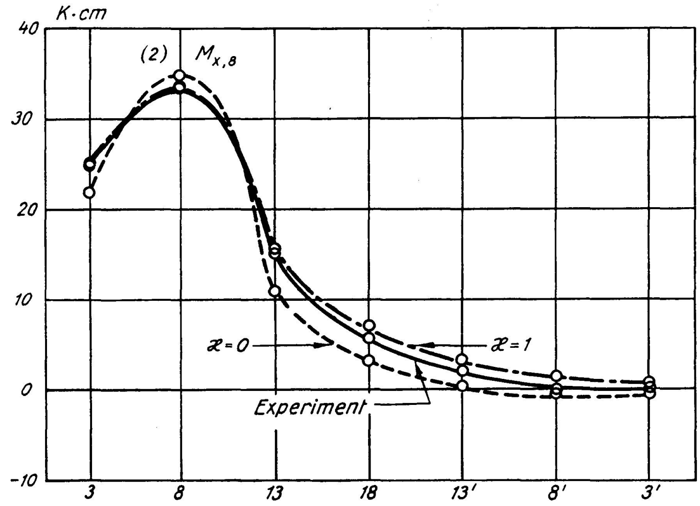
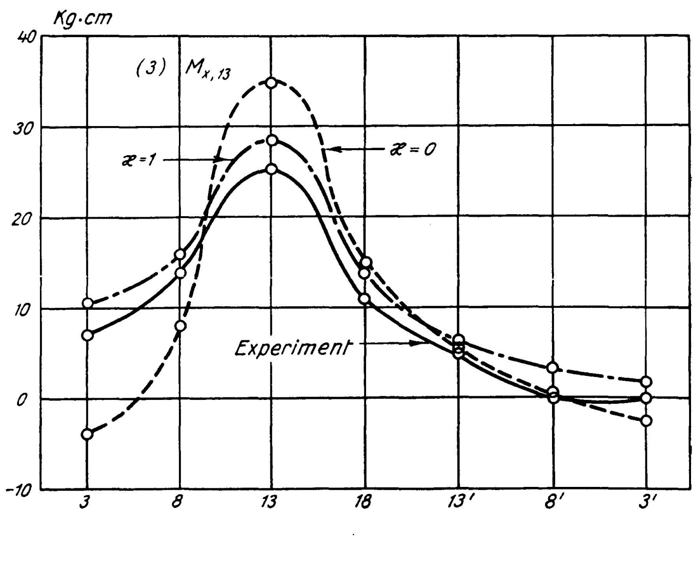
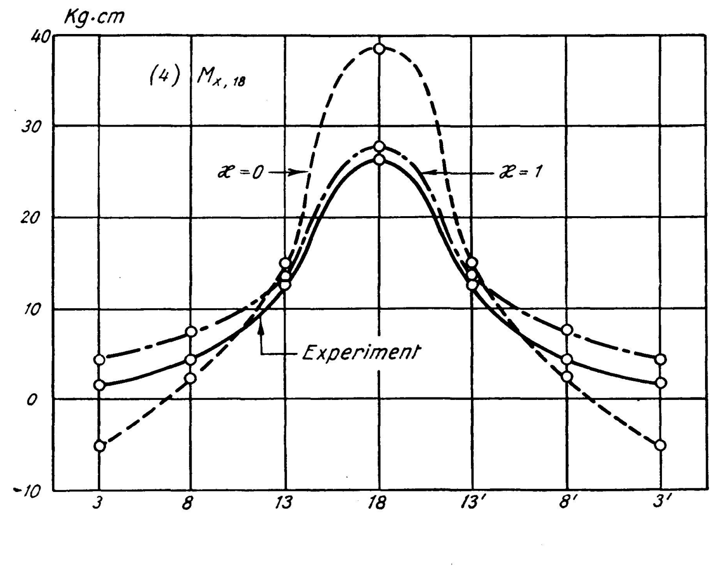


Fig. 11. Comparison of the Bending Moments Calculated from the Measured Strain Gauge Readings and from the Theory for the Model c)$_2$ ($A=4$, $B=0.5$).





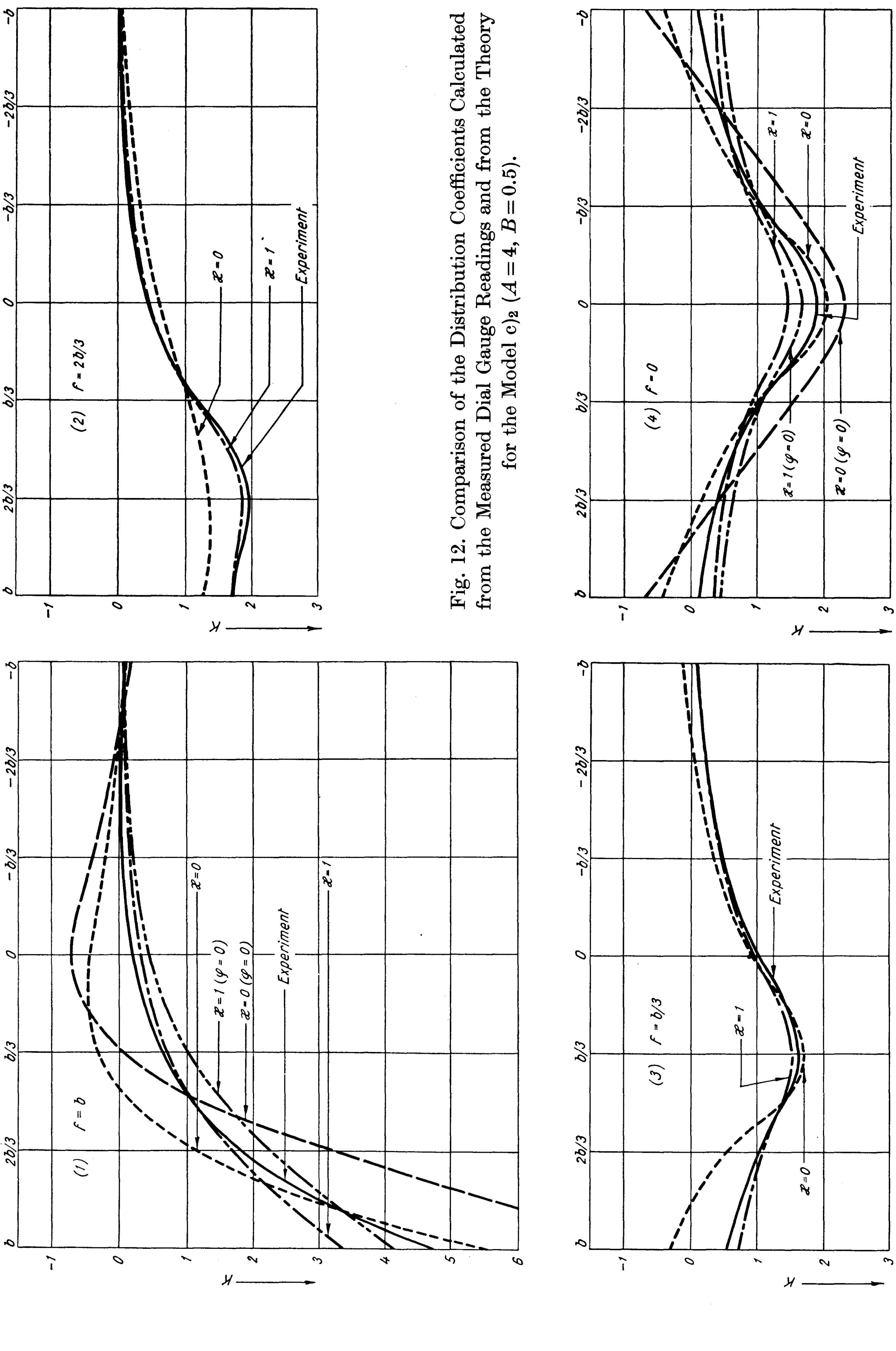


Fig. 12. Comparison of the Distribution Coefficients Calculated from the Measured Dial Gauge Readings and from the Theory for the Model c)$_2$ ($A=4$, $B=0.5$).

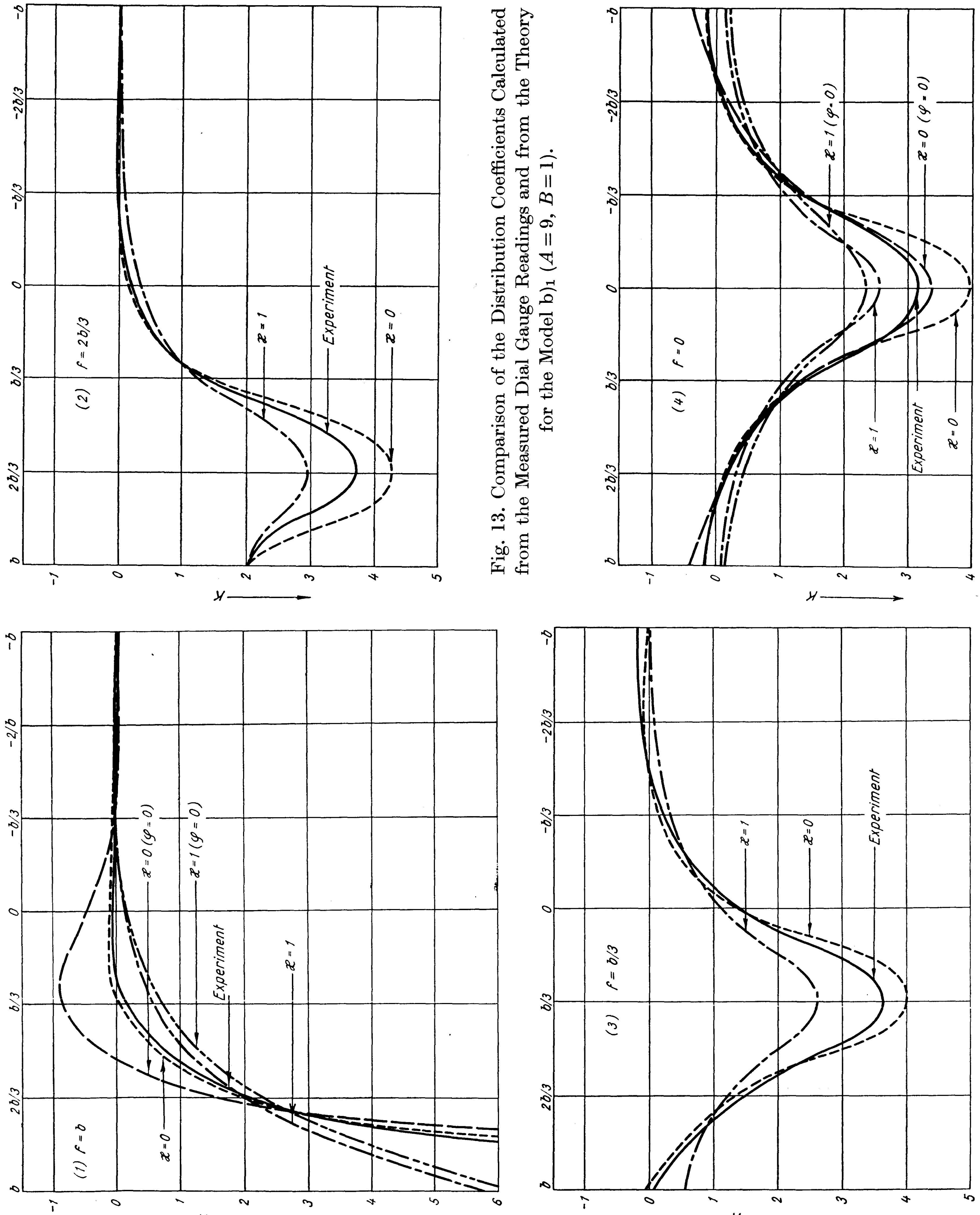


Fig. 13. Comparison of the Distribution Coefficients Calculated from the Measured Dial Gauge Readings and from the Theory for the Model b)$_1$ ($A=9$, $B=1$).

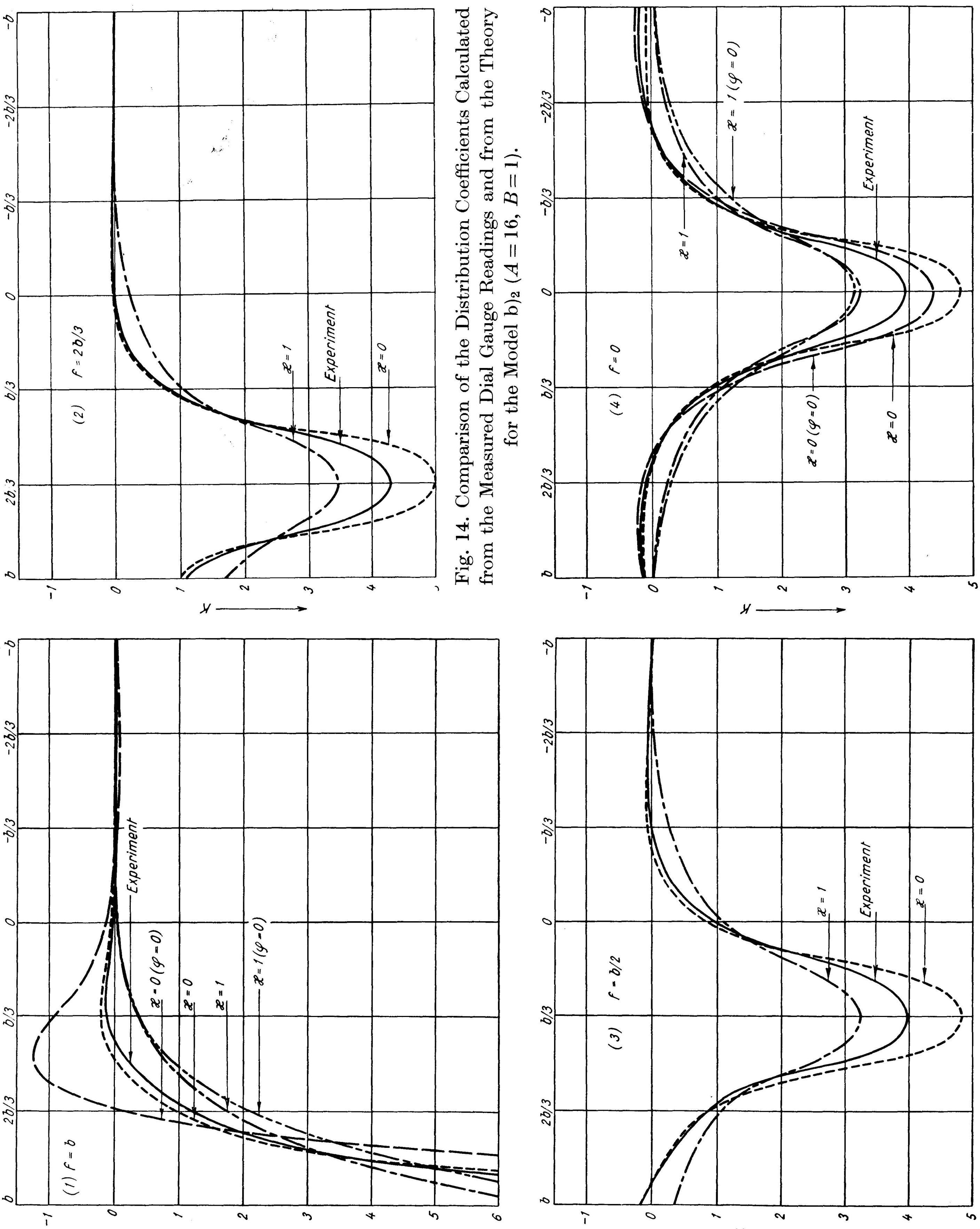


Fig. 14. Comparison of the Distribution Coefficients Calculated from the Measured Dial Gauge Readings and from the Theory for the Model b)$_2$ ($A=16$, $B=1$).

measured values are compared with the theoretical values calculated under the assumption of $\kappa = 1$ and 0 in all these figures.

In the figures for distribution coefficients, the notations used by Y. GUYON and CH. MASSONNET are used, for example, K, b and f. That is, $2b$ is the effective width instead of b used in fig. 4.

The results of experimental researches for the model $c)_1$ and $c)_2$ are shown in figs. 9 and 10, plotting the measured values of the deflection and bending moment for model $c)_1$ and in fig. 11, plotting the measured values of the bending moment for model $c)_2$, but the results for model $c)_2$ are shown in fig. 12, not with the values of deflection itself, but with the values of distribution coefficients K defined by Y. GUYON and CH. MASSONNET. For the purpose of comparison, the distribution coefficients calculated from the dial gauge readings are shown in figs. 13 and 14 with those calculated from the theory for the specimen models $b)_1$ and $b)_2$.

IV. *Consideration.* From above experimental researches, it was clarified that the authors' theory can explain well the experimental results and the measured values seem to be in satisfactory agreement with the theoretical values.

Generally speaking, the measured values check favorably with the theoretical values calculated under the assumption $\kappa = 0$ in these models b), while the measured values do with the theoretical values under the assumption $\kappa = 1$ in the models c). These differences can be well understood from the fact that the former model of aluminium round bars has no slab and the torsional rigidity is small, and that, on the contrary, the latter models have the slab so that the torsional rigidity is larger than that of the former models.

The values of influence coefficients of the deflection and bending moment of the orthotropic parallelogram plate for any arbitrary values of κ between 1.0 and 0 are not yet completed, and therefore, the measured values can not be compared directly with the theoretical c values corresponding to the torsional rigidity of the models themselves. However, the experimental results can be said to be satisfactory practically.

5. Conclusion

From the above numerical analyses on the orthotropic parallelogram plates and the experimental researches on the models of a skew grillage girder, it was clarified that the theory of the orthotropic parallelogram plates is very effective in the analyses of the skew girder bridges.

The authors are now calculating the influence coefficients of deflection and bending moment for the cases of various combinations of characteristic values of the plate, using the skew network shown on in fig. 4, but wish to calculate those in the case of a finer skew network to obtain more accurate values.

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2. M. Naruoka und H. Ohmura, Stahlbau, 28 (1959), not yet published.
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Summary

The authors derived the skew network finite difference equation for the orthotropic parallelogram plate, simply supported on the opposite two skew sides and supported by flexible girders at the other two sides, and calculated the influence coefficients of deflection and bending moment for the special characteristic values of the plates. These values of influence coefficients were verified by the experimental researches on models of skew grillage girder bridges.

Résumé

Les auteurs établissent l'équation limite aux différences dans le cas du treillis oblique, pour la dalle orthotrope en forme de parallélogramme, portant simplement sur les bords obliques opposés et soutenue sur les deux autres par des poutres de flexion.

Il a été ainsi possible de déterminer les coefficients d'influence de la flèche et du moment fléchissant pour les valeurs caractéristiques particulières de la dalle.

Ces coefficients d'influence calculés ont été confirmés par des essais sur modèles de ponts constitués par des grilles portantes obliques.

Zusammenfassung

Die Autoren geben hier die Ableitung der begrenzten Differenzengleichung bei schiefem Netzwerk für die orthotrope Parallelogrammplatte, die an den gegenüberliegenden schiefen Rändern einfach gelagert und an den anderen beiden durch Biegeträger gestützt ist.

Damit wurden die Einflußzahlen der Durchbiegung und des Biegemomentes für die besonderen charakteristischen Werte der Platte bestimmt.

Diese berechneten Einflußzahlen wurden durch Versuche an Modellen von schiefen Trägerrostbrücken bestätigt.