Transformation of the anisotropic cylindrical shell into the isotropic cylindrical shell

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Transformation of the Anisotropic Cylindrical Shell into the Isotropic Cylindrical Shell

Transformation d'un voile cylindrique anisotrope en voile isotrope

Transformation einer anisotropen Zylinderschale in eine isotrope Zylinderschale

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1. Introduction

When the dimensions of a cylindrical shell are such that it cannot resist bending in the circumferential direction, a considerable practical advantage can be obtained by increasing the shell's flexural rigidity through the provision of reinforcing ribs in its circumferential direction. When a cylindrical shell has ribs in the circumferential direction and/or longitudinal direction which are spaced at equal and appropriate distances, an accurate elasticity solution can be derived by regarding it as an anisotropic shell. An equation for the relation between stress and displacement was given by W. FLÜGGE [1] and DISCHINGER [2] established a solution for it. However, his solution is too complicated to be adopted in practical numerical calculations.

The approximation which is presented in this paper has been deduced on the basis of the following assumptions:

1. That Poisson's ratio is zero.
2. That the eccentricity of the reinforcing ribs is negligible.
3. That in the equations of equilibrium for the forces in the circumferential direction acting upon an infinitesimal element of the shell, Q_φ (see Fig. 1), the radial shear force may be neglected. [3], [4], [5].

The solution thus obtained can be expressed in extremely simple forms. The characteristic values are, besides being a function for parameter $\sqrt{t\,r}/l$, a function for longitudinal tensile rigidity D_x and circumferential flexural rigidity K_φ, and have no relation to longitudinal flexural rigidity K_x and circumferential tensile rigidity D_φ.

From this approximation the theory of the transformation of an anisotropic cylindrical shell to an isotropic cylindrical shell is derived. Considering an isotropic cylindrical shell having the same characteristic values as an anisotropic cylindrical shell (regarding the former as the equivalent isotropic cylindrical shell of the latter) we can determine the relation between this pair of shells in terms of dimensions, sectional force and displacement. As a consequence, the required numerical values can be obtained simply by performing computations for the equivalent isotropic cylindrical shell by making use of the published numerical tables for isotropic cylindrical shells, instead of resorting to the complicated and tedious calculations for the theoretical solution of the anisotropic shell's bending.

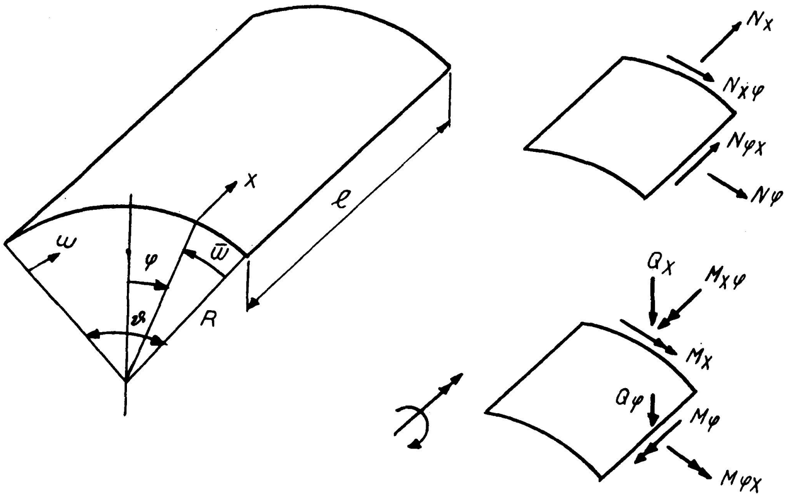

Fig. 1.

2. Symbols

The co-ordinates x and φ of the shell's middle surface are taken as shown in Fig. 1. The co-ordinate z is taken in the radial direction. The sectional forces and displacement are as illustrated in Fig. 1. (Their positive directions are as indicated in the illustration.)

Sectional rigidity (see Fig. 2):

Tensile rigidity $\quad D_x = \frac{E}{1-\nu^2} F_x, \qquad D_\varphi = \frac{E}{1-\nu^2} F_\varphi.$

Shear rigidity $\quad D_{x\varphi} = D_{\varphi x} = \frac{E}{1-\nu^2} F_{x\varphi}.$

Flexural rigidity $\quad K_x = \frac{E}{1-\nu^2} I_x, \qquad K_\varphi = \frac{E}{1-\nu^2} I_\varphi.$

Torsional rigidity $\quad K_{x\varphi} = \frac{E}{1-\nu^2} I_{x\varphi}.$

where
$$F_x = \frac{1}{b_{x0}} \int b_x dz, \qquad F_\varphi = \frac{1}{b_{\varphi 0}} \int b_\varphi dz, \qquad F_{x\varphi} = \int dz = t,$$
$$S_x = \frac{1}{b_{x0}} \int b_x z dz, \qquad S_\varphi = \frac{1}{b_{\varphi 0}} \int b_\varphi z dz,$$
$$I_x = \frac{1}{b_{x0}} \int b_x z^2 dz - F_x \eta_x^2 = \frac{1}{b_{x0}} \int b_x (z^2 - \eta_x^2) dz,$$
$$I_\varphi = \frac{1}{b_{\varphi 0}} \int b_\varphi (z^2 - \eta_x^2) dz,$$
$$\eta_x = \frac{S_x}{F_x}, \qquad \eta_\varphi = \frac{S_\varphi}{F_\varphi},$$
ν: Poisson's ratio.

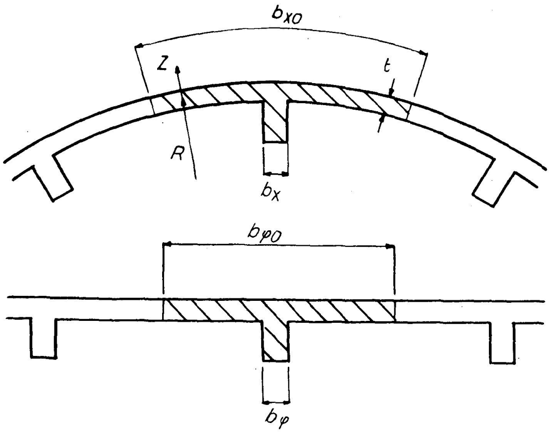

Fig. 2.

The integral of the equations above must be carried out over the entire section along width b_{x0} or $b_{\varphi 0}$. An asterisk given to an integral mark indicates that the integral has to be performed only for the shell and excluding the ribs.

$$d_x = \frac{D_x}{D_{x\varphi}}, \qquad d_\varphi = \frac{D_\varphi}{D_{x\varphi}}, \qquad k_x = \frac{K_x}{K_{x\varphi}}, \qquad k_\varphi = \frac{K_\varphi}{K_{x\varphi}},$$
$$\lambda = n\pi\frac{R}{l}, \qquad \xi = \frac{x}{R}, \qquad f' = \frac{\partial f}{\partial \xi} = \frac{1}{R}\frac{\partial f}{\partial x}, \qquad f^{\cdot} = \frac{\partial f}{\partial \varphi},$$
$$\eta = \frac{\varphi}{\vartheta}, \qquad k = \frac{K_{x\varphi}}{R^2 D_{x\varphi}} = \frac{t^2}{12 R^2}, \qquad K = \frac{\sqrt{12}}{E}\left(\frac{R}{t}\right)^2, \qquad \gamma = n\vartheta\frac{R}{l}.$$

3. Basic Equations

The relations between strain and displacement of the neutral surface are

$$\epsilon_x = \frac{u'}{R}, \qquad \epsilon_\varphi = \frac{v^{\cdot} + w}{R}, \qquad \gamma_{x\varphi} = \frac{u^{\cdot} + v'}{R}. \tag{1}$$

The relations between sectional forces and strain can be expressed in the following manner if we assume that Poisson's ratio is zero and neglect the effect of the ribs' eccentricity.

$$N_x = D_x \epsilon_x, \qquad N_\varphi = D_\varphi \epsilon_\varphi, \qquad N_{x\varphi} = \tfrac{1}{2} D_{x\varphi} \gamma_{x\varphi}, \tag{2}$$

$$M_x = \frac{K_x}{R^2} w'', \qquad M_\varphi = \frac{K_\varphi}{R^2} w^{\cdot\cdot}, \qquad M_{x\varphi} = \frac{K_{x\varphi}}{R^2} w^{\cdot\prime}. \tag{3}$$

The equations of equilibrium of sectional forces on an infinitesimal element of the shell are:

$$N'_x + N^{\cdot}_{\varphi x} = 0, \tag{4a}$$

$$N^{\cdot}_\varphi + N'_{x\varphi} - Q_\varphi = 0, \tag{4b}$$

$$Q^{\cdot}_\varphi + Q'_x + N_\varphi = 0, \tag{4c}$$

$$Q_\varphi = \frac{1}{R}(M^{\cdot}_\varphi + M'_{x\varphi}), \tag{4d}$$

$$Q_x = \frac{1}{R}(M'_x + M^{\cdot}_{\varphi x}). \tag{4e}$$

4. Approximation for the Bending Theory

If we neglect the effect of Q_φ in Eq. (4b) as being small we can introduce a stress function

$$N_x = \Phi^{\cdot\cdot}, \qquad N_\varphi = \Phi'', \qquad N_{x\varphi} = -\Phi^{\cdot\prime}. \tag{5}$$

From Eqs. (4c, d, e) we get

$$R N_\varphi + M^{\cdot\cdot}_\varphi + 2 M^{\cdot\prime}_{x\varphi} + M''_x = 0 \tag{6}$$

and from Eq. (1)

$$\epsilon''_\varphi + \epsilon^{\cdot\cdot}_x + \gamma^{\cdot\prime}_{x\varphi} - \frac{w''}{R} = 0. \tag{7}$$

Introducing Eqs. (3), (2) and (5) in Eqs. (6) and (7), the following two equations can be obtained:

$$\begin{aligned} &\frac{\Phi^{\cdot\cdot\cdot\cdot}}{d_x} + 2\Phi^{\cdot\cdot\prime\prime} + \frac{\Phi''''}{d_\varphi} - \frac{D_{x\varphi}}{R} w'' = 0, \\ &k_\varphi w^{\cdot\cdot\cdot\cdot} + 2 w^{\cdot\cdot\prime\prime} + k_x w'''' + \frac{R^3}{K_{x\varphi}} \Phi'' = 0. \end{aligned} \tag{8}$$

By eliminating Φ or w, we can deduce the following differential equations respectively either relating to w or Φ

$$\begin{aligned} &\frac{k_\varphi}{d_x} w^{\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot} + 2\left(\frac{1}{d_x} + k_\varphi\right) w^{\cdot\cdot\cdot\cdot\cdot\cdot\prime\prime} + \left(\frac{k_x}{d_x} + 4 + \frac{k_\varphi}{d_\varphi}\right) w^{\cdot\cdot\cdot\cdot\prime\prime\prime\prime} \\ &+ 2\left(\frac{1}{d_\varphi} + k_x\right) w^{\cdot\cdot\prime\prime\prime\prime\prime\prime} + \frac{k_x}{d_\varphi} w'''''''' + \frac{R^2 D_{x\varphi}}{K_{x\varphi}} w'''' = 0, \end{aligned} \tag{9a}$$

$$\begin{aligned} &\frac{k_\varphi}{d_x}\Phi^{\cdot\cdot\cdot\cdot\cdot\cdot\cdot\cdot}+2\left(\frac{1}{d_x}+k_\varphi\right)\Phi^{\cdot\cdot\cdot\cdot\cdot\cdot\prime\prime}+\left(\frac{k_x}{d_x}+4+\frac{k_\varphi}{d_\varphi}\right)\Phi^{\cdot\cdot\cdot\cdot\prime\prime\prime\prime} \\ &\quad+2\left(\frac{1}{d_\varphi}+k_x\right)\Phi^{\cdot\cdot\prime\prime\prime\prime\prime\prime}+\frac{k_x}{d_\varphi}\Phi^{\prime\prime\prime\prime\prime\prime\prime\prime}+\frac{R^2 D_{x\varphi}}{K_{x\varphi}}\Phi^{\prime\prime\prime\prime}=0. \end{aligned} \tag{9b}$$

As a solution for Eq. (9a) we assume:

$$w = H e^{m\eta}\sin\lambda\xi. \tag{10}$$

Substituting Eq. (10) into Eq. (9a) we find the characteristic equation to be

$$\begin{aligned} &m^8-2\frac{d_x}{k_\varphi}\left(\frac{1}{d_x}+k_\varphi\right)\vartheta^2\lambda^2 m^6+\frac{d_x}{k_\varphi}\left(\frac{k_x}{d_x}+4+\frac{k_\varphi}{d_\varphi}\right)\vartheta^4\lambda^4 m^4 \\ &\quad-2\frac{d_x}{k_\varphi}\left(k_x+\frac{1}{d_\varphi}\right)\vartheta^6\lambda^6 m^2+\frac{k_x}{k_\varphi}\frac{d_x}{d_\varphi}\vartheta^8\lambda^8+\frac{d_x}{k_\varphi}\frac{R^2 D_{x\varphi}}{K_{x\varphi}}\vartheta^8\lambda^4=0. \end{aligned} \tag{11}$$

Since the value of m is fairly large, we can take an approximate value for Eq. (11), in which the coefficients of m^8 and m^6 and the last term are the same as those in the above equation (6).

$$\left\{m^2-\frac{1}{2}\left(d_x+\frac{1}{k_\varphi}\right)\vartheta^2\lambda^2\right\}^4+\frac{d_x}{k_\varphi}\frac{1}{k}\vartheta^8\lambda^4=0. \tag{12}$$

As the solution for Eq. (12) eight "m"s can be expressed by the equations given below.

$$\begin{aligned} m_{\substack{1\\2}} &= \alpha_1\pm\beta_1, & m_{\substack{3\\4}} &= \alpha_2\pm i\beta_2, \\ m_{\substack{5\\6}} &= -m_{\substack{1\\2}}, & m_{\substack{7\\8}} &= -m_{\substack{3\\4}}, \end{aligned} \tag{13}$$

where

$$\begin{aligned} \alpha_{\substack{1\\2}} &= \sqrt{\frac{1}{2}\left(d_x+\frac{1}{k_\varphi}\right)}\,\gamma\, p_{\substack{1\\2}}, \qquad \beta_{\substack{1\\2}} = \sqrt{\frac{1}{2}\left(d_x+\frac{1}{k_\varphi}\right)}\,\gamma\, q_{\substack{1\\2}}, \\ p_{\substack{1\\2}} &= \sqrt{\frac{\pi}{2}\left\{\sqrt{\pi^2\pm\frac{\sqrt{2}\pi}{\chi_0}+\frac{1}{\chi_0^2}}+\left(\pi\pm\frac{1}{\sqrt{2}\chi_0}\right)\right\}}, \\ q_{\substack{1\\2}} &= \sqrt{\frac{\pi}{2}\left\{\sqrt{\pi^2\pm\frac{\sqrt{2}\pi}{\chi_0}+\frac{1}{\chi_0^2}}-\left(\pi\pm\frac{1}{\sqrt{2}\chi_0}\right)\right\}}, \\ \gamma &= nR\frac{\vartheta}{l}, \\ \chi_0 &= \frac{1}{2}\left(d_x+\frac{1}{k_\varphi}\right)\sqrt[4]{\frac{k_\varphi}{d_\varphi}}\frac{n}{l}\sqrt{\frac{Rt}{\sqrt{12}}}. \end{aligned} \tag{14}$$

If we formulate only the damping waves occurring from an edge of the shell and omit those occurring from the other edge, then we can write:

$$\begin{aligned} w = &\left[e^{-\alpha_1\bar{\omega}}\left(B_1\cos\beta_1\bar{\omega}+B_2\sin\beta_1\bar{\omega}\right)\right. \\ &\left.+e^{-\alpha_2\bar{\omega}}\left(B_3\cos\beta_2\bar{\omega}+B_4\sin\beta_2\bar{\omega}\right)\right]\sin\lambda\xi \end{aligned} \tag{15}$$

and similarly from Eq. (9b)

$$\begin{aligned}\Phi = \ & [e^{-\alpha_1 \bar{\omega}}(C_1 \cos\beta_1 \bar{\omega} + C_2 \sin\beta_1 \bar{\omega}) \\ & + e^{-\alpha_2 \bar{\omega}}(C_3 \cos\beta_2 \bar{\omega} + C_4 \sin\beta_2 \bar{\omega})] \sin\lambda\xi .\end{aligned} \tag{16}$$

Substituting these into Eq. (8b) we obtain the relation between C_i in Eq. (16) and B_i in Eq. (15)

$$\begin{aligned}&\frac{k_\varphi}{\vartheta^4}[e^{-\alpha_1 \bar{\omega}}(B_1^4 \cos\beta_1 \bar{\omega} + B_2^4 \sin\beta_1 \bar{\omega}) + e^{-\alpha_2 \bar{\omega}}(B_3^4 \cos\beta_2 \bar{\omega} + B_4^4 \sin\beta_2 \bar{\omega})] \\ &-\frac{2\lambda^2}{\vartheta^2}[e^{-\alpha_1 \bar{\omega}}(B_1^2 \cos\beta_1 \bar{\omega} + B_2^2 \sin\beta_1 \bar{\omega}) + e^{-\alpha_2 \bar{\omega}}(B_3^2 \cos\beta_2 \bar{\omega} + B_4^2 \sin\beta_2 \bar{\omega})] \\ &+k_x \lambda^4 [e^{-\alpha_1 \bar{\omega}}(B_1 \cos\beta_1 \bar{\omega} + B_2 \sin\beta_1 \bar{\omega}) + e^{-\alpha_2 \bar{\omega}}(B_3 \cos\beta_2 \bar{\omega} + B_4 \sin\beta_2 \bar{\omega})] \\ &-\frac{R^3 \lambda^2}{K_{x\varphi}}[e^{-\alpha_1 \bar{\omega}}(C_1 \cos\beta_1 \bar{\omega} + C_2 \sin\beta_1 \bar{\omega}) + e^{-\alpha_2 \bar{\omega}}(C_3 \cos\beta_2 \bar{\omega} + C_4 \sin\beta_2 \bar{\omega})] = 0,\end{aligned} \tag{17}$$

Because the above equation is always possible it follows that the following equations must also be possible at the same time.

$$\begin{aligned}&\frac{k_\varphi}{\vartheta^4} B_1^4 - 2\frac{\lambda^2}{\vartheta^2} B_1^2 + k_x \lambda^4 B_1 - \frac{R^3}{K_{x\varphi}} \lambda^2 C_1 = 0, \\ &\frac{k_\varphi}{\vartheta^4} B_2^4 - 2\frac{\lambda^2}{\vartheta^2} B_2^2 + k_x \lambda^4 B_2 - \frac{R^3}{K_{x\varphi}} \lambda^2 C_2 = 0, \\ &\frac{k_\varphi}{\vartheta^4} B_3^4 - 2\frac{\lambda^2}{\vartheta^2} B_3^2 + k_x \lambda^4 B_3 - \frac{R^3}{K_{x\varphi}} \lambda^2 C_3 = 0, \\ &\frac{k_\varphi}{\vartheta^4} B_4^4 - 2\frac{\lambda^2}{\vartheta^2} B_4^2 + k_x \lambda^4 B_4 - \frac{R^3}{K_{x\varphi}} \lambda^2 C_4 = 0.\end{aligned} \tag{18}$$

Solving Eq. (18) the relation between integral constants C_i and B_i can be expressed by the following equations:

$$\begin{aligned}C_{\frac{1}{2}} &= \frac{\sqrt{d_x/k_\varphi}}{K}\left[\left\{\left(\frac{k_\varphi}{2} - \frac{2}{d_x + 1/k_\varphi} + \frac{2k_x}{(d_x + 1/k_\varphi)^2}\right)\bar{\chi}^2 + \left(k_\varphi - \frac{2}{d_x + 1/k_\varphi}\right)\bar{\chi}\right\} B_{\frac{1}{2}}\right. \\ &\left.\pm\left\{\left(\frac{2}{d_x + 1/k_\varphi} - k_\varphi\right)\bar{\chi} - k_\varphi\right\} B_{\frac{2}{1}}\right] \equiv \frac{\sqrt{d_x/k_\varphi}}{K} D_{\frac{1}{2}}, \\ C_{\frac{3}{4}} &= \frac{\sqrt{d_x/k_\varphi}}{K}\left[\left\{\left(\frac{k_\varphi}{2} - \frac{2}{d_x + 1/k_\varphi} + \frac{2k_x}{(d_x + 1/k_\varphi)^2}\right)\bar{\chi}^2 - \left(k_\varphi - \frac{2}{d_x + 1/k_\varphi}\right)\bar{\chi}\right\} B_{\frac{3}{4}}\right. \\ &\left.\pm\left\{\left(\frac{2}{d_x + 1/k_\varphi} - k_\varphi\right)\bar{\chi} + k_\varphi\right\} B_{\frac{4}{3}}\right] \equiv \frac{\sqrt{d_x/k_\varphi}}{K} D_{\frac{3}{4}},\end{aligned} \tag{19}$$

where $\bar{\chi} = 2\pi\chi_0$

Φ and w are now to be expressed as under:

$$\begin{aligned}w &= K\vartheta^2 \phi \sin\lambda\xi, \\ \Phi &= \sqrt{\frac{d_x}{k_\varphi}}\vartheta^2 \psi \sin\lambda\xi,\end{aligned} \tag{20}$$

in which

$$\begin{aligned}\phi^{(n)} &= e^{-\alpha_1 \bar{\omega}} (\bar{B}_1^{(n)} \cos \beta_1 \bar{\omega} + \bar{B}_2^{(n)} \sin \beta_1 \bar{\omega}) + e^{-\alpha_2 \bar{\omega}} (\bar{B}_3^{(n)} \cos \beta_2 \bar{\omega} + \bar{B}_4^{(n)} \sin \beta_2 \bar{\omega}),\\ \psi^{(n)} &= e^{-\alpha_1 \bar{\omega}} (\bar{D}_1^{(n)} \cos \beta_1 \bar{\omega} + \bar{D}_2^{(n)} \sin \beta_1 \bar{\omega}) + e^{-\alpha_2 \bar{\omega}} (\bar{D}_3^{(n)} \cos \beta_2 \bar{\omega} + \bar{D}_4^{(n)} \sin \beta_2 \bar{\omega}).\end{aligned} \tag{21}$$

$$\bar{B}_i^{(n)} = \frac{1}{K\vartheta^2} B_i^{(n)}, \qquad \bar{D}_i^{(n)} = \frac{1}{K\vartheta^2} D_i^{(n)}. \tag{22}$$

$$\begin{aligned} B_{\substack{1\\2}}^{(n)} &= \alpha_1 B_{\substack{1\\2}}^{(n-1)} \mp \beta_1 B_{\substack{2\\1}}^{(n-1)}, & B_{\substack{3\\4}}^{(n)} &= \alpha_2 B_{\substack{3\\4}}^{(n-1)} \mp \beta_2 B_{\substack{4\\3}}^{(n-1)}, & B_i^{(0)} &= B_i,\\ D_{\substack{1\\2}}^{(n)} &= \alpha_1 D_{\substack{1\\2}}^{(n-1)} \mp \beta_1 D_{\substack{2\\1}}^{(n-1)}, & D_{\substack{3\\4}}^{(n)} &= \alpha_2 D_{\substack{3\\4}}^{(n-1)} \mp \beta_2 D_{\substack{4\\3}}^{(n-1)}, & D_i^{(0)} &= D_i.\end{aligned} \tag{23}$$

From Eqs. (5), (3) and (4) we find that sectional forces and displacement are:

$$\begin{aligned} N_x &= \sqrt{\frac{d_x}{k_\varphi}}\, \psi^{(2)} \sin \lambda \xi,\\ N_\varphi &= -\sqrt{\frac{d_x}{k_\varphi}}\, (\pi\gamma)^2 \psi \sin \lambda \xi,\\ N_{x\varphi} &= -\sqrt{\frac{d_x}{k_\varphi}}\, (\pi\gamma)\, \psi^{(1)} \cos \lambda \xi,\\ M_x &= -k_x \frac{t}{\sqrt{12}} (\pi\gamma)^2 \phi \sin \lambda \xi,\\ M_\varphi &= -k_\varphi \frac{t}{\sqrt{12}} \phi^{(2)} \sin \lambda \xi,\\ M_{x\varphi} &= \frac{t}{\sqrt{12}} (\pi\gamma)\, \phi^{(1)} \cos \lambda \xi,\\ Q_\varphi &= \frac{t}{\sqrt{12}\,\vartheta R} \{k_\varphi \phi^{(3)} - (\pi\gamma)^2 \phi^{(1)}\} \sin \lambda \xi,\\ Q_x &= \frac{t\pi\gamma}{\sqrt{12}\,\vartheta R} \{\phi^{(2)} - k_x (\pi\gamma)^2 \phi\} \cos \lambda \xi.\end{aligned} \tag{24}$$

$$\begin{aligned} D_{x\varphi} u &= -\frac{R}{\lambda \sqrt{d_x k_\varphi}} \psi^{(2)} \cos \lambda \xi,\\ D_{x\varphi} v &= -\vartheta R \left\{ 2\sqrt{\frac{d_x}{k_\varphi}}\, \psi^{(1)} - \frac{1}{(\pi\gamma)^2 \sqrt{d_x k_\varphi}} \phi^{(3)} \right\} \sin \lambda \xi,\\ D_{x\varphi} w &= \frac{\sqrt{12}\,(R\vartheta)^2}{t} \phi \sin \lambda \xi,\\ D_{x\varphi} \frac{\partial w}{\partial \varphi} &= \frac{\sqrt{12}\,\vartheta R^2}{t} \phi^{(1)} \sin \lambda \xi,\\ D_{x\varphi} \frac{\partial u}{\partial x} &= \frac{1}{\sqrt{d_x k_\varphi}} \psi^{(2)} \sin \lambda \xi,\\ D_{x\varphi} \theta &= -\vartheta \left[2\sqrt{\frac{d_x}{k_\varphi}}\, \psi^{(1)} - \frac{1}{(\pi\gamma)^2 \sqrt{d_x k_\varphi}} \psi^{(3)} + \frac{\sqrt{12}}{t} R\, \phi^{(1)} \right] \sin \lambda \xi.\end{aligned} \tag{25}$$

5. An Equivalent Isotropic Cylindrical Shell for an Anisotropic Cylindrical Shell

5.1. The relation in dimensions between these two shells:

An isotropic cylindrical shell which has the same characteristic values as an anisotropic cylindrical shell can be taken as the equivalent isotropic shell of the latter. When the central angles (angle subtended by the edges of the shell), radii, thicknesses and longitudinal spans of an anisotropic cylindrical shell and its equivalent isotropic shell are represented respectively by ϑ, R, t, l; ϑ_0, R_0, t_0, l_0, we can write, from Eq. (12) the characteristic equations of these shells as

$$\begin{aligned} &\left\{m^2-\frac{1}{2}\left(d_x+\frac{1}{k_\varphi}\right)\vartheta^2\lambda^2\right\}^4+\left\{\frac{d_x}{k_\varphi k}\right\}\vartheta^8\lambda^4=0, \\ &(m_0^2-\vartheta_0^2\lambda_0^2)^4+\frac{1}{k_0}\vartheta_0^8\lambda_0^4=0, \end{aligned} \tag{26}$$

where $k=\dfrac{t^2}{12R^2}$, $\quad k_0=\dfrac{t_0^2}{12R_0^2}$, $\quad \lambda=\dfrac{n\pi R}{l}$, $\quad \lambda_0=\dfrac{n\pi R_0}{l_0}$.

In order that these two shells have identical characteristic values we must find that:

$$\frac{1}{2}\left(d_x+\frac{1}{k_\varphi}\right)\vartheta^2\lambda^2=\vartheta_0^2\lambda_0^2, \qquad \frac{d_x}{k_\varphi}\frac{\vartheta^8\lambda^4}{k}=\frac{\vartheta_0^8\lambda_0^4}{k_0}. \tag{27}$$

Since these equations have three variables of ϑ/ϑ_0, λ/λ_0 and k/k_0 an anisotropic cylindrical shell will have an unlimited number of its equivalent isotropic shells. If one of these three variables is determined, the remaining two can also be calculated. Ribs in the longitudinal direction have no practical effect because they do not reinforce the shell in its flexural rigidity even though they serve to increase the shell's tensile strength. Therefore, we shall deal with the case where the shell has ribs only in the circumferential direction which are effective in increasing the shell's flexural rigidity in the same direction.

For the sake of simplicity, we can assume that on an equivalent isotropic shell $\vartheta_0=\vartheta$ and $R_0=R$. Now if we neglect $1/k_\varphi$ in comparison to 1 since k_φ has a fairly large value, we find from Eq. (27) the following relation between the dimensions of the two shells:

$$t=\frac{2}{\sqrt{k_\varphi}}t_0, \qquad l=\frac{l_0}{\sqrt{2}} \tag{28a}$$

and employing this equation

$$\gamma=\sqrt{2}\gamma_0, \qquad \lambda=\sqrt{2}\lambda_0 \tag{28b}$$

5.2. The relation in sectional forces and displacement between the two shells:

The sectional forces and displacement of an anisotropic cylindrical shell with ribs in the circumferential direction are given by Eqs. (24) and (25). To express them as the functions of ϕ alone we can write

$$
\begin{aligned}
M_\varphi &= \frac{k_\varphi t}{\sqrt{12}} \phi^{(2)} \sin\lambda\xi, \\
M_x &= -\frac{t}{\sqrt{12}} (\pi\gamma)^2 \phi \sin\lambda\xi, \\
M_{x\varphi} &= \frac{t}{\sqrt{12}} (\pi\gamma) \phi^{(1)} \cos\lambda\xi, \\
Q_\varphi &= \frac{t}{\sqrt{12}\,\vartheta R} \{k_\varphi \phi^{(3)} - (\pi\gamma)^2 \phi^{(1)}\} \sin\lambda\xi, \\
R_\varphi &= \frac{t}{\sqrt{12}\,\vartheta R} \{k_\varphi \phi^{(3)} - 2(\pi\gamma)^2 \phi^{(1)}\} \sin\lambda\xi, \\
Q_x &= \frac{t\pi\gamma}{\sqrt{12}\,\vartheta R} \{\phi^{(2)} - (\pi\gamma)^2 \phi\} \cos\lambda\xi, \\
N_\varphi &= -\frac{t}{\sqrt{12}\,\vartheta^2 R} \{k_\varphi \phi^{(4)} - 2(\pi\gamma)^2 \phi^{(2)} + (\pi\gamma)^2 \phi\} \sin\lambda\xi, \\
N_{x\varphi} &= -\frac{t}{\sqrt{12}\,R\vartheta^2 (\pi\gamma)} [k_\varphi \phi^{(5)} - 2(\pi\gamma)^2 \phi^{(3)} + (\pi\gamma)^4 \phi^{(1)}] \cos\lambda\xi, \\
N_x &= \frac{t}{\sqrt{12}\,R\vartheta^2 (\pi\gamma)^2} [k_\varphi \phi^{(6)} - 2(\pi\gamma)^2 \phi^{(4)} + (\pi\gamma)^4 \phi^{(2)}] \sin\lambda\xi, \\
E w &= \frac{\sqrt{12}\,\vartheta^2 R^2}{t^2} \phi \sin\lambda\xi, \\
E\frac{\partial w}{\partial\varphi} &= \frac{\sqrt{12}\,\vartheta R^2}{t^2} \phi^{(1)} \sin\lambda\xi, \\
E v &= \frac{1}{\sqrt{12}\,\vartheta (\pi\gamma)^4} [k_\varphi \phi^{(7)} - 2(k_\varphi + 1)(\pi\gamma)^2 \phi^{(5)} \\
&\qquad + 5(\pi\gamma)^4 \phi^{(3)} - 2(\pi\gamma)^6 \phi^{(1)}] \sin\lambda\xi.
\end{aligned}
\tag{29}
$$

The sectional forces and displacement of the equivalent isotropic shell can be obtained by getting $k_\varphi = 1$ in Eq. (29)

$$
\begin{aligned}
(M_\varphi) &= \frac{t_0}{\sqrt{12}} \phi^{(2)} \sin\lambda\xi, \\
(M_{x\varphi}) &= \frac{t_0}{\sqrt{12}} \pi\gamma_0 \phi^{(1)} \cos\lambda\xi, \\
(M_x) &= -\frac{t_0}{\sqrt{12}} (\pi\gamma_0)^2 \phi \sin\lambda\xi, \\
(Q_\varphi) &= \frac{t_0}{\sqrt{12}\,\vartheta_0 R_0} \{\phi^{(3)} - (\pi\gamma_0)^2 \phi^{(1)}\} \sin\lambda\xi, \\
(R_\varphi) &= \frac{t_0}{\sqrt{12}\,\vartheta_0 R_0} \{\phi^{(3)} - 2(\pi\gamma_0)^2 \phi^{(1)}\} \sin\lambda\xi,
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
(N_\varphi) &= \frac{t_0}{\sqrt{12}\,\vartheta_0^2 R_0}\{\phi^{(4)} - 2(\pi\gamma_0)^2\phi^{(2)} + (\pi\gamma_0)^4\phi\}\sin\lambda\xi, \\
(N_{x\varphi}) &= -\frac{t_0}{\sqrt{12}\,\vartheta_0^2 R_0(\pi\gamma_0)}\{\phi^{(5)} - 2(\pi\gamma_0)^2\phi^{(3)} + (\pi\gamma_0)^4\phi^{(1)}\}\sin\lambda\xi, \\
(N_x) &= \frac{t_0}{\sqrt{12}\,\vartheta_0^2 R_0(\pi\gamma_0)^2}\{\phi^{(6)} - 2(\pi\gamma_0)^2\phi^{(4)} + (\pi\gamma_0)^4\phi^{(2)}\}\sin\lambda\xi, \qquad (30) \\
E(w) &= \frac{\sqrt{12}\,\vartheta_0^2 R_0^2}{t_0^2}\phi\sin\lambda\xi, \\
E\left(\frac{\partial w}{\partial\varphi}\right) &= \sqrt{12}\frac{\vartheta_0 R_0^2}{t_0^2}\phi^{(1)}\sin\lambda\xi.
\end{aligned}
$$

In the above equation the sectional forces and displacements in parentheses are those of the isotropic shell.

When the value of $\phi^{(n)}$ in Eq. (30) representing the sectional forces and displacement of the isotropic shell, and the relation in Eq. (28) are substituted into Eq. (29) which expresses the sectional forces of the anisotropic shell, we find that:

$$
\begin{aligned}
M_\varphi &= 2\sqrt{k_\varphi}(M_\varphi), \\
M_{x\varphi} &= \frac{2\sqrt{2}}{\sqrt{k_\varphi}}(M_{\varphi x}), \\
M_x &= \frac{4}{k_\varphi}(M_x), \\
R_\varphi &= 2\sqrt{k_\varphi}\left\{(\hat{R}_\varphi) + \left(2+\frac{4}{k_\varphi}\right)\frac{\lambda_0}{R_0}(\hat{M}_{x\varphi})\right\}\sin\lambda\xi, \\
N_\varphi &= 2\sqrt{k_\varphi}\left\{(N_\varphi) - \left(2-\frac{4}{k_\varphi}\right)\frac{\lambda_0^2}{R_0}(M_\varphi) - \left(1-\frac{4}{k_\varphi}\right)\frac{\lambda_0^2}{R_0}(M_x)\right\}, \\
N_{x\varphi} &= \sqrt{2k_\varphi}\left\{(\hat{N}_{x\varphi}) - \left(2-\frac{4}{k_\varphi}\right)\lambda_0(\hat{R}_\varphi) - \left(3-\frac{4}{k_\varphi}\right)\frac{\lambda_0^2}{R}(\hat{M}_{x\varphi})\right\}\cos\lambda\xi, \qquad (31) \\
N_x &= \sqrt{k_\varphi}\left\{(\hat{N}_x) - \left(2-\frac{4}{k_\varphi}\right)(\hat{N}_\varphi) + \left(3-\frac{4}{k_\varphi}\right)\frac{\lambda_0^2}{R_0}(\hat{M}_\varphi) + \left(2-\frac{4}{k_\varphi}\right)\frac{\lambda_0^2}{R_0}(\hat{M}_x)\right\}\sin\lambda\xi, \\
w &= \frac{k_\varphi}{4}(w), \\
Ev &= \frac{k_\varphi}{4}\left\{E(\hat{v}) - \frac{4}{k_\varphi}\frac{R_0}{\lambda_0^3\vartheta_0^3 t_0}(\hat{N}_{x\varphi}) - \frac{4}{k_\varphi}\left(7-\frac{20}{k_\varphi}\right)\frac{R_0}{t_0}(\hat{R}_\varphi)\right. \\
&\qquad \left. -2\left(1-\frac{28}{k_\varphi}+\frac{80}{k_\varphi^2}\right)\frac{\lambda_0}{t_0}(\hat{M}_{x\varphi})\right\}\sin\lambda\xi,
\end{aligned}
$$

assuming that $\theta \doteqdot \dfrac{1}{R}\dfrac{\partial w}{\partial\varphi}$ we may write $\theta = \dfrac{k_\varphi}{4}(\theta)$.

Eqs. (31) express the relation between the sectional forces and displacement of the anisotropic shell and those of the equivalent isotropic shell. In these equations the mark $\wedge$ above the symbols of the sectional forces or the displacement indicate that those are the maximum values either at both ends of the shell or at the midspan.

6. The Relation in Sectional Forces between an Anisotropic Cylindrical Shell and Its Equivalent Isotropic Cylindrical Shell, Both Subjected to Edge Loads

When both an anisotropic cylindrical shell and its corresponding equivalent shell are subjected along the longitudinal edges to loads $\bar{N}_\varphi$, $\bar{N}_{x\varphi}$, $\bar{R}_\varphi$ and $\bar{M}_\varphi$, the relation between the sectional forces of the two shells and also the displacements thereof can be expressed respectively, from Eq. (31), by the following equations.

$$
\begin{aligned}
[N_\varphi]_i &= 2\sqrt{k_\varphi}\left\{(\hat{N}_\varphi)_i - \left(2-\frac{4}{k_\varphi}\right)\frac{\lambda_0^2}{R_0}(\hat{M}_\varphi)_i - \left(1-\frac{4}{k_\varphi}\right)\frac{\lambda_0^2}{R_0}(\hat{M}_x)_i\right\},\\
[N_x]_i &= \sqrt{k_\varphi}\left\{(\hat{N}_x)_i - \left(2-\frac{4}{k_\varphi}\right)(\hat{N}_\varphi)_i + \left(3-\frac{4}{k_\varphi}\right)\frac{\lambda_0^2}{R_0}(\hat{M}_\varphi)_i - \left(2-\frac{4}{k_\varphi}\right)\frac{\lambda_0^2}{R_0}(\hat{M}_x)_i\right\},\\
[N_{\varphi x}]_i &= \sqrt{2k_\varphi}\left\{(\hat{N}_{\varphi x})_i - \left(2-\frac{4}{k_\varphi}\right)\lambda_0(\hat{R}_\varphi)_i - \left(3-\frac{4}{k_\varphi}\right)\frac{\lambda_0^2}{R_0}(\hat{M}_{\varphi x})_i\right\},\\
[R_\varphi]_i &= 2\sqrt{k_\varphi}\left\{(\hat{R}_\varphi)_i + \left(2-\frac{4}{k_\varphi}\right)\frac{\lambda_0}{R_0}(\hat{M}_{\varphi x})_i\right\},\\
[M_\varphi]_i &= 2\sqrt{k_\varphi}(M_\varphi)_i,\\
[w]_i &= \frac{k_\varphi}{4}(w)_i,\\
E[v]_i &= \frac{k_\varphi}{4}\left\{E(\hat{v})_i - \frac{4}{k_\varphi}\frac{R_0}{\lambda_0^3\vartheta_0^2 t_0}(\hat{N}_{\varphi x}) - \frac{4}{k_\varphi}\left(7-\frac{20}{k_\varphi}\right)\frac{R_0}{t_0}(\hat{R}_\varphi)_i\right.\\
&\qquad\left. -2\left(1-\frac{28}{k_\varphi}+\frac{80}{k_\varphi^2}\right)\frac{\lambda_0}{t_0}(\hat{M}_{\varphi x})_i\right\}.
\end{aligned}
\tag{32}
$$

in which $i=\bar{N}_\varphi$, $\bar{N}_{x\varphi}$, $\bar{R}_\varphi$ and $\bar{M}_\varphi$, and $(F)_i$ represents the sectional forces and displacement working on the equivalent isotropic cylindrical shell and $[F]_i$ represents those working on the anisotropic cylindrical shell, when either cylindrical shell has been subjected to the edge loads i.

These edge loads are denoted by:

$$X_1 = \bar{N}_\varphi,\quad X_2 = \bar{N}_{\varphi x},\quad X_3 = \bar{R}_\varphi,\quad X_4 = \bar{M}_\varphi.$$

When a unit edge load acts separately upon the longitudinal edge of the anisotropic shell and that of its corresponding equivalent shell we find from Eq. (32) that the relation between the sectional forces of the two shells and the displacement thereof can be expressed by Eq. (33).

	i	
$[F]_i$	$X_1=\overline{N}_\varphi=1$	$X_2=\overline{N}_{\varphi x}=1$
$[N_\varphi]_i^* =$	$2\sqrt{k_\varphi}\left\{1-\left(1-\frac{4}{k_\varphi}\right)\frac{\lambda_0^2}{R_0}(M_x)_1^*\right\}$	$-2\sqrt{k_\varphi}\left(1-\frac{4}{k_\varphi}\right)\frac{\lambda_0^2}{R_0}(M_x)_2^*$
$[N_{\varphi x}]_i^* =$	$-\sqrt{2k_\varphi}\left(3-\frac{4}{k_\varphi}\right)\frac{\lambda_0^2}{R_0}(M_{x\varphi})_1^*$	$\sqrt{2k_\varphi}\left\{1-\left(3-\frac{4}{k_\varphi}\right)\frac{\lambda_0^2}{R_0}(M_{x\varphi})_2^*\right\}$
$[R_\varphi]_i^* =$	$2\sqrt{k_\varphi}\left(2-\frac{4}{k_\varphi}\right)\frac{\lambda_0}{R_0}(M_{x\varphi})_1^*$	$2\sqrt{k_\varphi}\left(2-\frac{4}{k_\varphi}\right)\frac{\lambda_0}{R_0}(M_{x\varphi})_2^*$
$[M_\varphi]_i^* =$	0	0
$[w]_i^* =$	$\frac{k_\varphi}{4}(w)_1^*$	$\frac{k_\varphi}{4}(w)_2^*$
$E[v]_i^* =$	$\frac{k_\varphi}{4}\left\{E(v)_1^*-2\left(1-\frac{28}{k_\varphi}+\frac{80}{k_\varphi^2}\right)\cdot\frac{\lambda_0}{t_0}(M_{x\varphi})_1^*\right\}$	$\frac{k_\varphi}{4}\left\{E(v)_2^*-\frac{4}{k_\varphi}\frac{R_0}{\lambda_0^3\vartheta_0^3 t_0}-2\left(1-\frac{28}{k_\varphi}+\frac{80}{k_\varphi^2}\right)\frac{\lambda_0}{t_0}(M_{x\varphi})_2^*\right\}$
$[\theta]_i^* =$	$\frac{k_\varphi}{4}(\theta)_1^*$	$\frac{k_\varphi}{4}(\theta)_2^*$
$[F]_i$	$X_3=\overline{R}_\varphi=1$	$X_4=\overline{M}_\varphi=1$
$[N_\varphi]_i^* =$	$-2\sqrt{k_\varphi}\left(1-\frac{4}{k_\varphi}\right)\frac{\lambda_0^2}{R_0}(M_x)_3^*$	$-2\sqrt{k_\varphi}\frac{\lambda_0^2}{R_0}\left\{\left(2-\frac{4}{k_\varphi}\right)+\left(1-\frac{4}{k_\varphi}\right)(M_x)_4^*\right\}$
$[N_{\varphi x}]_i^* =$	$-\sqrt{2k_\varphi}\left\{\left(2-\frac{4}{k_\varphi}\right)\lambda_0+\left(3-\frac{4}{k_\varphi}\right)\frac{\lambda_0^2}{R_0}(M_{x\varphi})_3^*\right\}$	$-\sqrt{2k_\varphi}\left(3-\frac{4}{k_\varphi}\right)\frac{\lambda_0^2}{R_0}(M_{x\varphi})_4^*$
$[R_\varphi]_i^* =$	$2\sqrt{k_\varphi}\left\{1+\left(2-\frac{4}{k_\varphi}\right)\frac{\lambda_0}{R_0}(M_{x\varphi})_3^*\right\}$	$2\sqrt{k_\varphi}\left(2-\frac{4}{k_\varphi}\right)\frac{\lambda_0}{R_0}(M_{x\varphi})_4^*$
$[M_\varphi]_i^* =$	0	$2\sqrt{k_\varphi}$
$[w]_i^* =$	$\frac{k_\varphi}{4}(w)_3^*$	$\frac{k_\varphi}{4}(w)_4^*$
$E[v]_i^* =$	$\frac{k_\varphi}{4}\left\{E(v)_3^*-\frac{4}{k_\varphi}\left(7-\frac{20}{k_\varphi}\right)\frac{R_0}{t_0}-2\left(1-\frac{28}{k_\varphi}+\frac{80}{k_\varphi^2}\right)\frac{\lambda_0}{t_0}(M_{x\varphi})_3^*\right\}$	$\frac{k_\varphi}{4}\left\{E(v)_4^*-2\left(1-\frac{28}{k_\varphi}+\frac{80}{k_\varphi^2}\right)\cdot\frac{\lambda_0}{t_0}(M_{x\varphi})_4^*\right\}$
$[\theta]_i^* =$	$\frac{k_\varphi}{4}(\theta)_3^*$	$\frac{k_\varphi}{4}(\theta)_4^*$

(33)

In Eq. (33) mark * denotes either the sectional forces or displacement of the shell at the edge.

When the anisotropic shell is subjected simultaneously to edge loads X_1, X_2, X_3, X_4 the sectional forces working on it are expressed by the following equations:

$$\begin{aligned} N_\varphi &= [N_\varphi]_1 X_1 + [N_\varphi]_2 X_2 + [N_\varphi]_3 X_3 + [N_\varphi]_4 X_4, \\ N_{\varphi x} &= [N_{\varphi x}]_1 X_1 + [N_{\varphi x}]_2 X_2 + [N_{\varphi x}]_3 X_3 + [N_{\varphi x}]_4 X_4, \\ R_\varphi &= [R_\varphi]_1 X_1 + [R_\varphi]_2 X_2 + [R_\varphi]_3 X_3 + [R_\varphi]_4 X_4, \\ M_\varphi &= [M_\varphi]_1 X_1 + [M_\varphi]_2 X_2 + [M_\varphi]_3 X_3 + [M_\varphi]_4 X_4. \end{aligned} \tag{34}$$

Similar expressions can be found for the sectional forces and displacement other than those expressed by the above equations.

From these equations we also find for the edges of these two shells that:

$$\begin{aligned} N_\varphi^* &= [N_\varphi]_1^* X_1 + [N_\varphi]_2^* X_2 + [N_\varphi]_3^* X_3 + [N_\varphi]_4^* X_4, \\ N_{\varphi x}^* &= [N_{\varphi x}]_1^* X_1 + [N_{\varphi x}]_2^* X_2 + [N_{\varphi x}]_3^* X_3 + [N_{\varphi x}]_4^* X_4, \\ R_\varphi^* &= [R_\varphi]_1^* X_1 + [R_\varphi]_2^* X_2 + [R_\varphi]_3^* X_3 + [R_\varphi]_4^* X_4, \\ M_\varphi^* &= [M_\varphi]_4^* X_4, \\ w^* &= [w]_1^* X_1 + [w]_2^* X_2 + [w]_3^* X_3 + [w]_4^* X_4, \\ v^* &= [v]_1^* X_1 + [v]_2^* X_2 + [v]_3^* X_3 + [v]_4^* X_4, \\ \theta^* &= [\theta]_i^* X_i + [\theta]_2^* X_2 + [\theta]_3^* X_3 + [\theta]_4^* X_4. \end{aligned} \tag{35}$$

When the boundary conditions of the anisotropic shell are given, the edge loads X_i can be determined by selecting four suitable equations from (35). The employment of Eq. (34) then produces the sectional forces and displacement.

For the cases where the anisotropic shell is subjected separately to each of the edge loads $\bar{N}_\varphi = 1$, $\bar{N}_{\varphi x} = 1$, $\bar{R}_\varphi = 1$, $\bar{M}_\varphi = 1$, obtain first the edge loads X_i, $(i = 1, 2, 3, 4)$ by solving simultaneous Eq. (36) and then the sectional forces and displacement by employing Eq. (34).

X_1	X_2	X_3	X_4	R. H. S.
$[N_\varphi]_1^*$	$[N_\varphi]_2^*$	$[N_\varphi]_3^*$	$[N_\varphi]_4^*$	1, 0, 0, 0
$[N_{\varphi x}]_1^*$	$[N_{\varphi x}]_2^*$	$[N_{\varphi x}]_3^*$	$[N_{\varphi x}]_4^*$	0, 1, 0, 0
$[R_\varphi]_1^*$	$[R_\varphi]_2^*$	$[R_\varphi]_3^*$	$[R_\varphi]_4^*$	0, 0, 1, 0
0	0	0	$[M_\varphi]_4^*$	0, 0, 0, 1

(36)

7. The Transformation of an Isotropic Cylindrical Shell to Another Isotropic Cylindrical Shell

We shall consider the transformation of one isotropic cylindrical shell to another isotropic cylindrical shell having the same characteristic values as the former. The radii, angles, spans and thicknesses of these two shells are denoted respectively by R, ϑ, l, t; $R_0, \vartheta_0, l_0, t_0$.

Then we find from Eq. (27) the relation

$$\frac{\vartheta_0}{\vartheta} = \frac{\lambda}{\lambda_0} = \sqrt[4]{\frac{k_0}{k}}, \tag{37}$$

where
$$\lambda = \frac{n\pi R}{l}, \qquad \lambda_0 = \frac{n\pi R_0}{l_0},$$
$$k = \frac{t^2}{12R^2}, \qquad k_0 = \frac{t_0^2}{12R_0^2},$$

by rewriting this equation we get

$$\frac{\vartheta_0}{\vartheta} = \frac{R\sqrt{l}}{R_0/l_0} = \frac{\sqrt{t_0/R_0}}{\sqrt{t/R}} \tag{38}$$

and by expressing the value of the above equation as c we find

$$\vartheta = \frac{1}{c}\vartheta_0, \qquad \frac{R}{l} = c\frac{R_0}{l_0}, \qquad \frac{t}{R} = \frac{1}{c^2}\frac{t_0}{R_0}. \tag{39}$$

From Eq. (37) we find that:

$$\gamma = \gamma_0, \tag{40}$$

in which
$$\gamma = \frac{nR\vartheta}{l}, \qquad \gamma_0 = \frac{nR_0\vartheta_0}{l_0}.$$

The sectional forces and displacement of the isotropic shell can be obtained by substituting, $d_x = k_x = d_\varphi = k_\varphi = 1$ in Eqs. (24) and (25). By using Eqs. (39) and (40) the following equations to express the relation between the sectional forces and displacement of the first shell and those of the second are produced.

$$\begin{aligned}
N_x &= (N_x), & M_x &= \frac{1}{c^2}\frac{R}{R_0}(M_x), \\
N_\varphi &= (N_\varphi), & M_\varphi &= \frac{1}{c^2}\frac{R}{R_0}(M_\varphi), \\
N_{x\varphi} &= (N_{x\varphi}), & M_{x\varphi} &= \frac{1}{c^2}\frac{R}{R_0}(M_{x\varphi}), \\
Q_x &= \frac{1}{c}(Q_x), & Q_\varphi &= \frac{1}{c}(Q_\varphi), \\
w &= c^2(w), & u &= \frac{1}{c}(u), \\
v &= c(v), & \theta &= c^3\frac{R}{R_0}(\theta).
\end{aligned} \tag{41}$$

In this equations the sectional forces and displacements in parentheses are those of the second shell. When $\vartheta = \vartheta_0$ we find that $c = 1$ and $R/R_0 = l/l_0 = t/t_0$. That is, with respect to a pair of cylindrical shells which are similar in shape but different in size, as for instance, is the case with an actual cylindrical shell and its model, the relation between their respective stresses and displacements can be obtained by substituting $c=1$ in Eq. (41).

8. Accuracy

In the proposed approximation for the anisotropic cylindrical shell it has been assumed that (1) Poisson's ratio is zero and (2) the eccentricity of the ribs is negligible. These assumptions will produce an element of error. With respect to concrete cylindrical shells, however, the assumption that Poisson's ratio is zero is not considered to have any appreciable effect. Neither does the negligence of the rib's eccentricity seem to cause any perceptible error in practical application so long as this eccentricity does not become large in proportion to the radius of the shell.

In this solution, in which an anisotropic cylindrical shell is transformed into an equivalent isotropic cylindrical shell, we write $1-1/k_\varphi \doteqdot 1$, because the circumferential flexural rigidity ratio k_φ is very large in comparison to 1. The error due to this assumption will be approximately $1/k_\varphi$ in comparison to the results obtained on the basis of the first two of the assumptions mentioned above.

9. Examples for Calculation

Cylindrical shell with ribs: $R=28\ 031$ m, $l=40.0$ m, $t=0.10$ m, $\vartheta=60°$.
Dimensions of rib, as illustrated in Fig. 3.
Distance between ribs, 5 m.

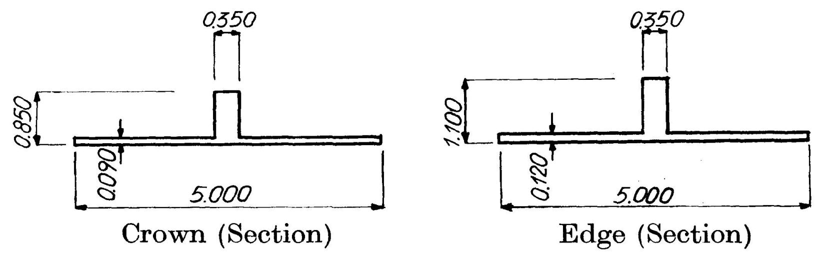

Fig. 3.

$$
\begin{aligned}
D_x &= D_{x\varphi} = 0.1\,E, & K_x &= K_{x\varphi} = 0.00008333\,E,\\
D_\varphi &= 0.1659\,E, & K_\varphi &= 0.013805\,E,\\
\therefore d_\varphi &= 1.659, & k_\varphi &= 165.664.
\end{aligned}
$$

For the thickness and I_φ the average of the values at the top and lower end of the shell has been taken.

Dimensions of the equivalent cylindrical shell:

$$R_0 = R, \quad t_0 = 0.6474395, \quad l_0 = 56.3986.$$

Table 1

Edge Load	Sectional Forces	Solution*	Angle $\bar{\omega}$ 0	0.2	0.4	0.6	0.8	1.0
$\bar{N}_\varphi=1$	N_x	A	14.1605	-1.2844	-3.6957	-1.7665	0.2598	1.0573
		B	14.0693	-1.2762	-3.6718	-1.7543	0.2586	1.0505
	N_φ	A	1.0000	0.2165	-0.5189	-0.5418	-0.1881	0.1335
		B	1.0000	0.2163	-0.5187	-0.5414	-0.1878	0.1334
	$N_{\varphi x}$	A	0	2.2302	0.7857	-0.5333	-0.8434	-0.4901
		B	0	2.2235	0.7833	-0.5316	-0.8403	-0.4883
	M_φ	A	0	-0.4963	-1.2779	-1.5003	-1.0993	-0.4652
		B	0	-0.4960	-1.2772	-1.4994	-1.0985	-0.4648
$\bar{R}_\varphi=1$	N_x	A	45.957	-9.223	-12.682	-2.720	4.379	5.439
		B	45.741	-9.166	-12.605	-2.701	4.353	5.428
	N_φ	A	0	-2.2005	-3.3288	-1.9771	0.0146	1.1865
		B	0	-2.1994	-3.3271	-1.9764	0.0147	1.1864
	$N_{\varphi x}$	A	0	5.3917	-0.7210	-4.3526	-3.7352	-1.2691
		B	0	5.3753	-0.7188	-4.3994	-3.7338	-1.2652
	M_φ	A	0	-5.4844	-8.4341	-7.5780	-4.3729	-1.1071
		B	0	-5.4842	-8.4335	-7.5773	-4.3725	-1.1071
$\bar{N}_{\varphi x}=1$	N_x	A	-5.5043	-0.7812	0.5244	0.4537	0.0845	-0.1540
		B	-5.4878	-0.7789	0.5229	0.4523	0.0841	-0.1535
	N_φ	A	0	-0.08748	-0.05075	0.10096	0.05932	-0.00311
		B	0	-0.08763	-0.05090	0.10116	0.05940	-0.00312
	$N_{\varphi x}$	A	1.00000	-0.26280	-0.23816	0.01404	0.13762	0.11528
		B	1.00000	-0.26277	-0.23814	0.01404	0.13758	0.11528
	M_φ	A	0	0.04616	0.17997	0.26208	0.23047	0.12856
		B	0	0.04630	0.18053	0.26288	0.23116	0.12894
$\bar{M}_\varphi=1$	N_x	A	-2.6554	0.7041	0.6298	-0.0436	-0.3695	-0.3065
		B	-2.6429	0.7002	0.6263	-0.0431	-0.3662	-0.3048
	N_φ	A	0	0.11058	0.12937	0.02327	-0.08050	-0.11280
		B	0	0.11062	0.12938	0.02331	-0.08154	-0.11279
	$N_{\varphi x}$	A	0	-0.22900	0.13830	0.26906	0.15594	-0.01075
		B	0	-0.22829	0.13787	0.26824	0.15553	-0.01065
	M_φ	A	1.0000	0.9762	0.8315	0.5429	0.2266	0.0023
		B	1.0000	0.9762	0.8315	0.5430	0.2268	0.0023

* *A:* The values obtained by the approximation discussed in Section 4.
B: The values calculated by the transformation of an anisotropic shell to its equivalent isotropic shell.

Table 1 shows the results of the calculations made for an anisotropic cylindrical shell which has one of its edges subjected to loads $\bar{N}_\varphi$, $\bar{R}_\varphi$, $\bar{N}_{x\varphi}$, $\bar{M}_\varphi = 1$. Of the figures given for each type of sectional force, those in line with A are the values obtained by the approximation discussed in Section 4, and those in line with B are the values obtained by the transformation of an anisotropic shell to its equivalent isotropic shell described in Section 6. The errors in both are less than 1%. W. ZERNA's method [5] was employed for the calculation of the equivalent isotropic shell transformed from an anisotropic shell.

10. Conclusion

The approximation proposed has been deduced from the assumptions that (1) Poisson's ratio is zero, (2) the eccentricity of reinforcing ribs may be neglected, (3) the radial shear force Q_φ in the equation of equilibrium for an infinitesimal element in the circumferential direction is negligible. The characteristic equation, the characteristic values, and the sectional forces etc. have been reduced to simple expressions. These considerations allow this approximation to be employed for practical calculations.

Denoting $d_x = d_\varphi = 1$, $k_x = k_\varphi = 1$ in the various equations in this method will yield a solution for an isotropic cylindrical shell, which is in complete agreement with the results obtained by W. ZERNA's method [5]. The observation of the equations in the proposed method will clarify the relationship between an anisotropic cylindrical shell and an isotropic cylindrical shell.

The characteristic values of an anisotropic shell are determined, according to Eq. (14), by the longitudinal tensile rigidity ratio d_x and circumferential flexural rigidity ratio k_φ, but they do not refer to the longitudinal flexural rigidity ratio k_x. Therefore the provision of reinforcing ribs in the longitudinal direction has no practical effect. It would be the same if the thickness of the shell were increased by an amount equivalent to the volume of material in the ribs being spread over the entire surface of the shell.

The addition to the shell's flexural rigidity by means of circumferential ribs will produce a considerable effect in practical use.

For any anisotropic cylindrical shell (with span l and thickness t) and having reinforcing ribs, there will be innumerable equivalent isotropic cylindrical shells which have the same characteristic values as the anisotropic shell. If an equivalent isotropic shell were to be provided with the same radius and central angle as those of the corresponding anisotropic shell, its span l_0 and thickness t_0 would be expressed by

$$l_0 = \sqrt{2}\, l, \qquad t_0 = \frac{\sqrt{k_\varphi}}{2}\, t.$$

That is, the sectional forces will damp off from the edges of an anisotropic cylindrical shell with reinforcing ribs in the same way as from the edges of its equivalent shell, whose span and thickness are expressed by the above equation. Between the sectional forces and displacements of these two shells there is a certain fixed relationship as denoted by Eq. (31).

The numerical calculations for an anisotropic shell is a very simple procedure if the proposed method for the replacement of the anisotropic shell by its equivalent isotropic shell is followed, because the readily available numerical tables for isotropic shells can be used.

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Summary

An isotropic cylindrical shell is considered which has a central angle and characteristic values both identical with those of an anisotropic cylindrical shell having ribs in a transverse direction. This isotropic cylindrical shell is called the equivalent isotropic cylindrical shell of the latter. The relations between these two cylindrical shells are defined in terms of the dimensions, sectional forces and displacements. Consequently the sectional forces and displacements working on any anisotropic cylindrical shell can be found by calculating the sectional forces and displacements of its equivalent isotropic shell. With the use of the available numerical tables this is a much simpler procedure than solving the equations of the anisotropic cylindrical shell.

Résumé

L'auteur considère un voile cylindrique isotrope dont les valeurs caractéristiques et l'angle au centre sont identiques à ceux d'un voile cylindrique anisotrope, présentant des nervures transversales. Ce voile isotrope est appelé voile cylindrique isotrope équivalent. Les relations entre ces deux voiles cylindriques sont exprimées en fonction des dimensions, des efforts intérieurs et des déformations. Les efforts intérieurs et les déformations d'un voile cylindrique anisotrope quelconque se trouveront donc en résolvant le problème pour le voile isotrope équivalent; grâce aux tables numériques existantes, ce procédé est bien plus simple qu'une étude directe du voile cylindrique anisotrope.

Zusammenfassung

Betrachten wir eine isotrope Zylinderschale, deren Zentriwinkel und maßgebende Größen identisch denjenigen einer gegebenen anisotropen Zylinderschale mit querverlaufenden Rippen seien. Diese betrachtete Schale bezeichnen wir als entsprechende isotrope Zylinderschale. Die Beziehungen zwischen diesen beiden Schalen werden wir in Funktion der Abmessungen, Schnittkräfte und Deformationen erhalten. So kann denn die Bestimmung der Schnittkräfte und Deformationen einer anisotropen Zylinderschale auf diejenige einer isotropen, entsprechenden Schale zurückgeführt werden, was dank den zur Verfügung stehenden Tabellen viel einfacher ist als die direkte Behandlung der anisotropen Zylinderschale.

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