The influence surface for the indeterminate clamping moment of slab-bridge-type skewed plates

The Influence Surface for the Indeterminate Clamping Moment of Slab-Bridge-Type Skewed Plates

Surfaces d'influence des moments d'encastrement hyperstatiques dans les plaques biaises des ponts dalles

Einflußflächen der Einspannmomente bei statisch unbestimmten, schiefen Plattenbrücken

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1. Introduction

It is somewhat unusual for modern roads to cross over obstacles at a right angle. Most bridges must therefore be designed as skew bridges. Skew bridges of small span are built as skew slabs. Then it is necessary that such skew plates should be designed for a line load as live load. For this purpose the use of influence surfaces is most advantageous.

However, at present it would seem that there are not sufficient design data available relating to skew plates with various boundary conditions. Since analytical methods generally require a considerable amount of troublesome numerical work in order to construct the influence surfaces of these skew plates, then the model analysis may be preferable if the accuracy is maintained at a sufficiently high level for practical purposes.

Many influence surfaces have already been obtained by experimental methods. These surfaces relate to slab-bridge-type skew plates having two opposite edges simply supported. But influence surfaces for skew plates with two opposite edges clamped, which appear, for example, as the slabs of rigid-frame-type bridges are not yet available.

In this paper, the influence surfaces for the clamping moment of the above-mentioned skew plates, which are particularly required for design purposes, are presented.

In addition, the effects due to the angles of skew and the ratio of the length of clamped edge to the span length perpendicular to the clamped edge on the clamping moment have been investigated under a uniformly distributed load or line live load.

2. Apparatus and Measurement for Model Analysis

The apparatus for supporting the test plate is constructed with a pair of upper and lower frames. The clamping is effected by pressing the specimen from both the upper and lower sides by additional steel blocks placed along the edges, as shown in Fig. 1, and the clamping pressure is applied by means of a screw-clamp.

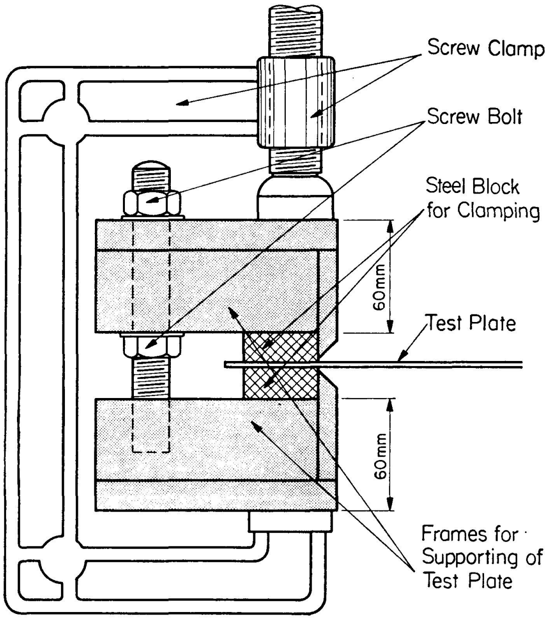

Fig. 1.

A lever, 250 cm in length, with a balance weight, was used as the test load.

For the measurement of the strain of a plate surface in the vicinity of the clamped edge, wire resistance gauges, bonded as nearly as possible to the boundary edges, were used.

In order to maintain a high degree of accuracy when making the above-mentioned measurement, it is desirable that the base area of the gauges should be as small as possible.

In the present model analysis, gauges measuring $3\,\text{mm} \times 3\,\text{mm}$ in base area were used, and the results of preliminary tests showed that a satisfactory degree of accuracy was attained (see Fig. 2).

Aluminium plates: 3.0 mm in thickness and 300 mm in width perpendicular to the clamped edges, angles of skew from 45° to 90° by stages of 15°, the ratios of clamped edge length to free edge length being approx. 1 to 2, Young's modulus = 690 000 kg/cm², Poisson's ratio $\nu = 0.27 \sim 0.31$, were used as test plates.

3. Correction of the Data at the Singularity in the Influence Surface

It is a theoretical determination that a singularity occurs in the influence surface of the plate. However, we cannot evaluate this characteristic by means of the experimental method.

To take an example. Fig. 2 illustrates this fact by showing the discrepancies between the theoretical and the experimental values of the ordinates of the influence surface for the moments of the square plate with all edges clamped.

Since, in the practical design, the load is usually distributed over a certain area, it can be considered that the parts where there is a difference, shown by the hatched area in Fig. 2, do not make much contribution. Such parts may therefore be neglected without serious error. Now, Fig. 2 shows that the greater part of the discrepancy occurs in the section $A-B$ which is normal

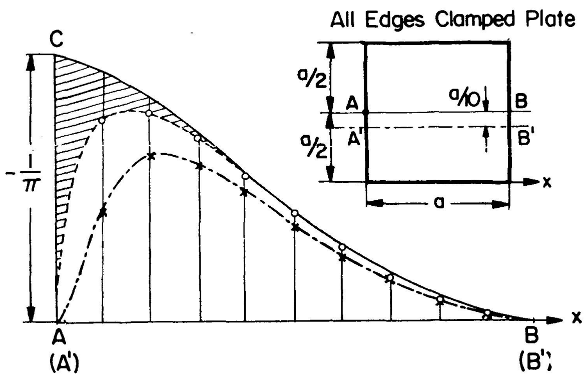

Fig. 2. The ordinates of the influence surface in the section A—B and A'—B' for the clamping moment at a point A.

Curve ———— : Theoretical values in section A—B (by PUCHER).

Curve —·—·— : Theoretical values in section A'—B'.

Curve – – – – : Mark o: Experimental values in section A—B.
Mark ×: Experimental values in section A'—B'.

to the fixed edge at the point A. The theoretical value of the ordinate of the influence surface at the edge point A is given by $M=-\frac{1}{\pi}$.

Thus we can correct the obtained curve experimentally by extending such a curve to the point C which gives the value $-\frac{1}{\pi}$ for the ordinate at A. By the above procedure, the influence surfaces can for practical purposes, be constructed with greater accuracy.

4. Influence Surfaces for the Clamping Moments of Skew Plates with Two Opposite Edges Clamped and the Other Two Edges Free

The typical influence surfaces are shown in Fig. 4 to 21 for the case when the length of the clamped edge is the same as that of the free edge. Influence surfaces are given for the clamping moments induced at such the points, as shown in Fig. 3.

The values in these figures are multiplied by 10π, as an example, in the case of the plates subjected to a uniformly distributed load, and the clamping moments are given by following expression:

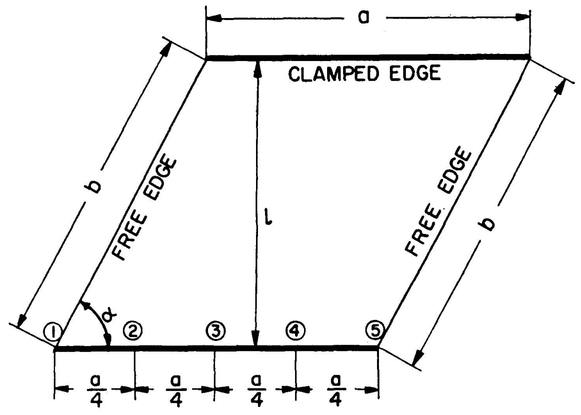

Fig. 3.

α: Angle of skew.

a, b: Length of a clamped and a free edge, respectively.

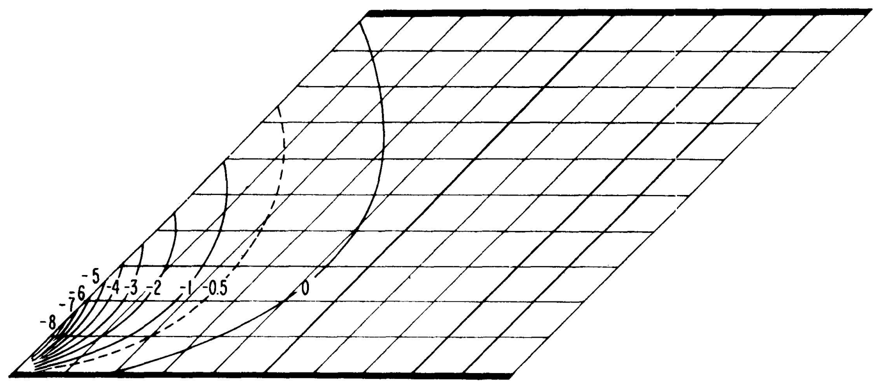

Fig. 4. Influence surface for clamping moment at pt. 1 ($\alpha=45°$, $b/a=1$).

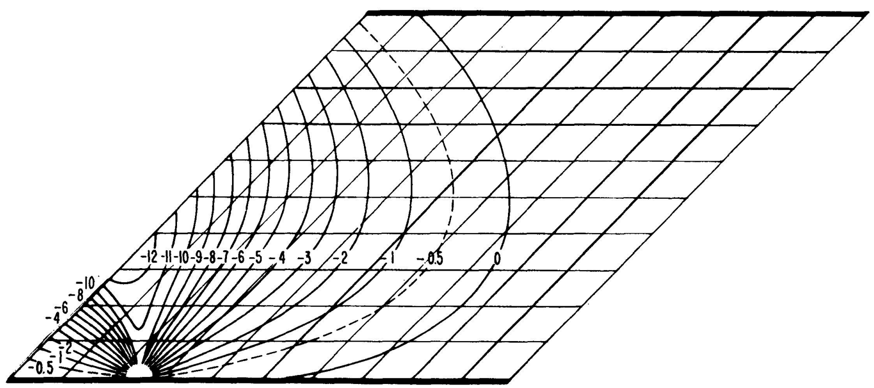

Fig. 5. Influence surface for clamping moment at pt. 2 ($\alpha=45°$, $b/a=1$).

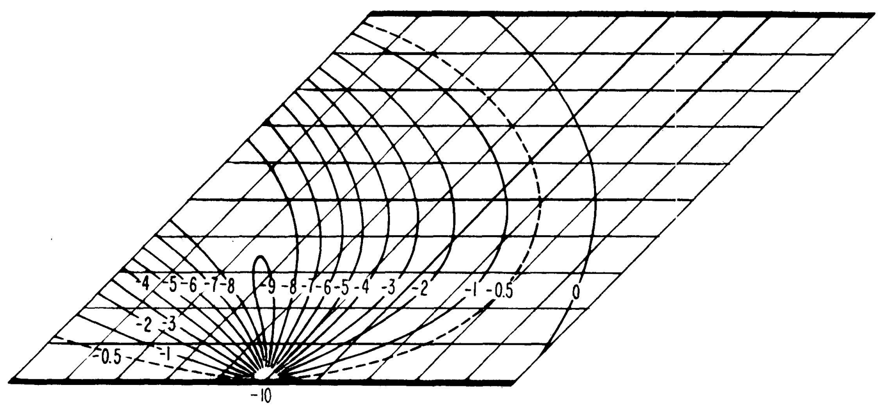

Fig. 6. Influence surface for clamping moment at pt. 3 ($\alpha=45°$, $b/a=1$).

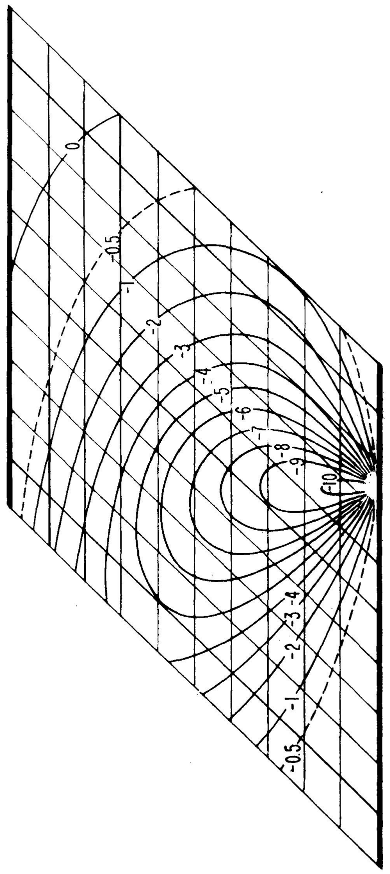

Fig. 7. Influence surface for clamping moment at pt. 4 ($\alpha=45°$, $b/a=1$).

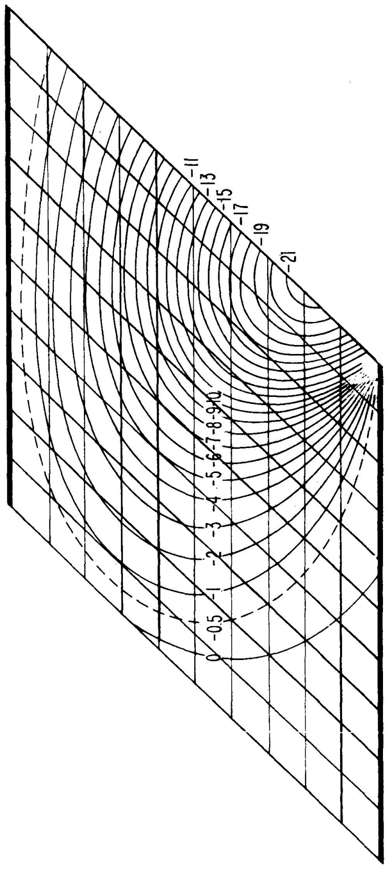

Fig. 8. Influence surface for clamping moment at pt. 5 ($\alpha=45°$, $b/a=1$).

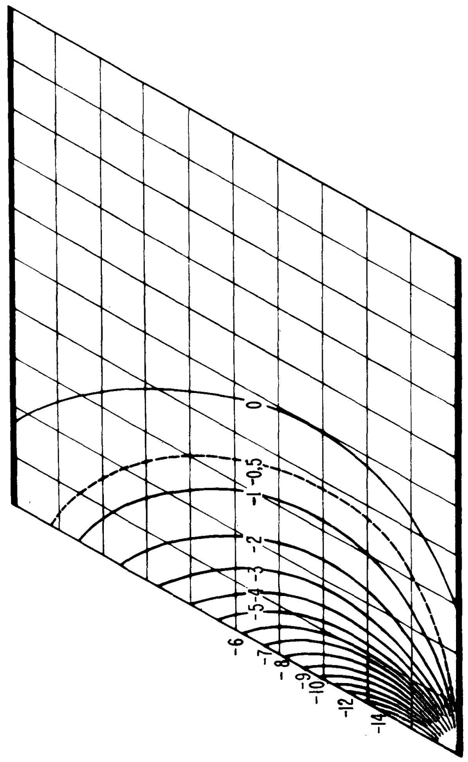

Fig. 9. Influence surface for clamping moment at pt. 1 ($\alpha=60°$, $b/a=1$).

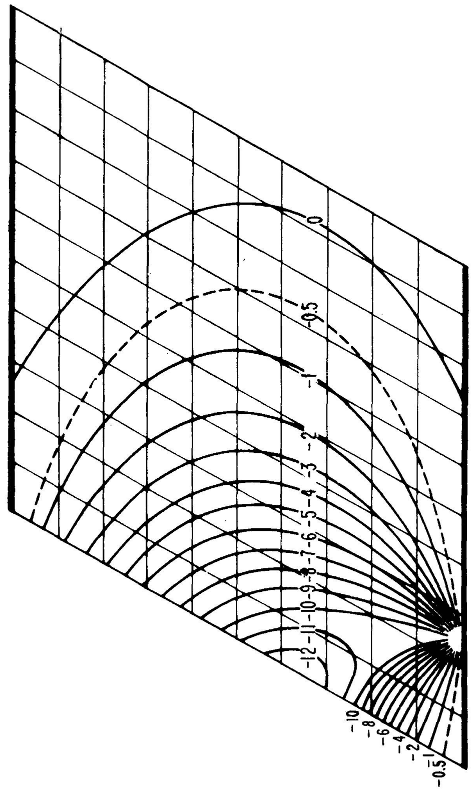

Fig. 10. Influence surface for clamping moment at pt. 2 ($\alpha=60°$, $b/a=1$).

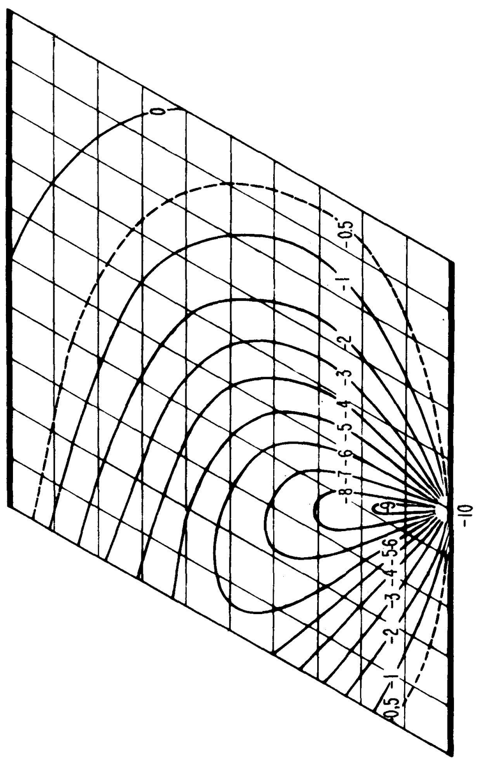

Fig. 11. Influence surface for clamping moment at pt. 3 ($\alpha = 60°$, $b/a = 1$).

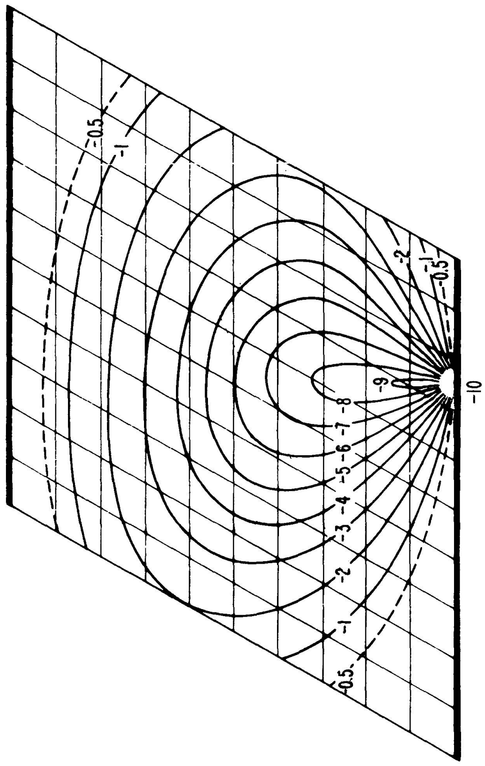

Fig. 12. Influence surface for clamping moment at pt. 4 ($\alpha = 60°$, $b/a = 1$).

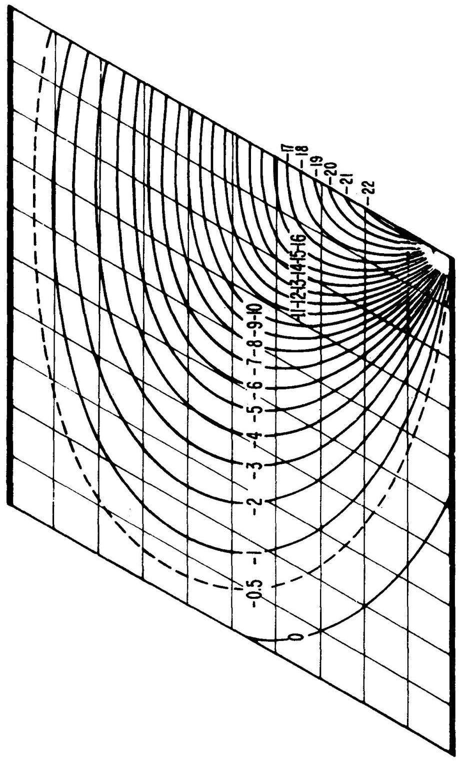

Fig. 13. Influence surface for clamping moment at pt. 5 ($\alpha = 60°$, $b/a = 1$).

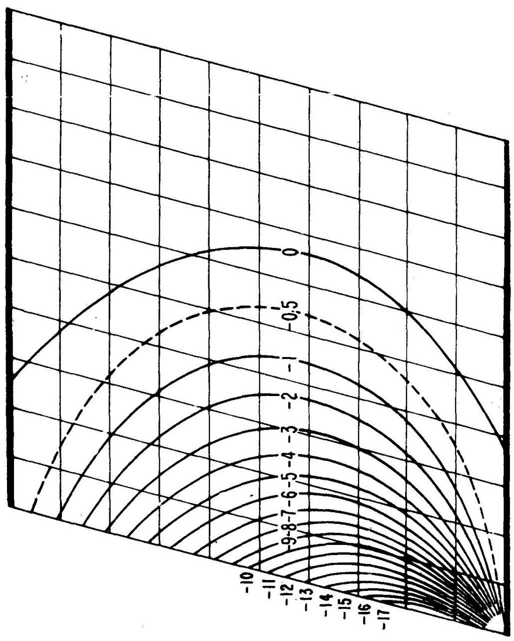

Fig. 14. Influence surface for clamping moment at pt. 1 ($\alpha = 75°$, $b/a = 1$).

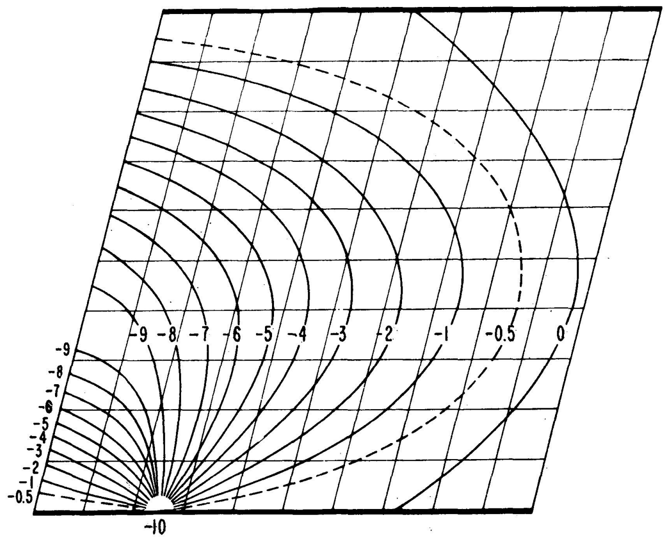

Fig. 15. Influence surface for clamping moment at pt. 2 ($\alpha = 75°$, $b/a = 1$).

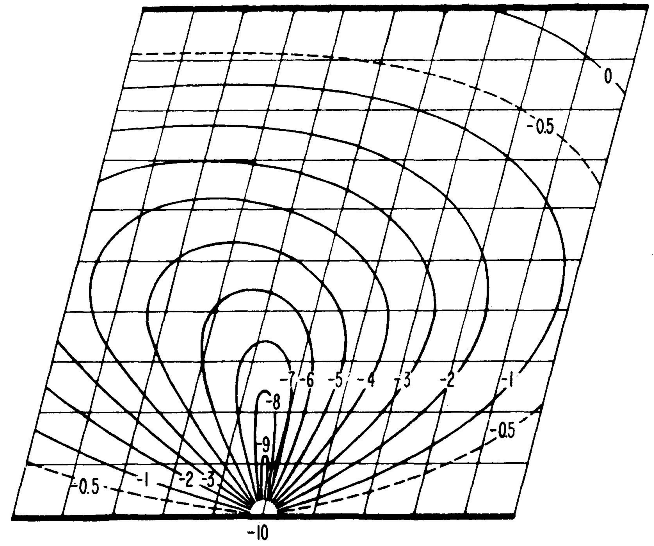

Fig. 16. Influence surface for clamping moment at pt. 3 ($\alpha = 75°$, $b/a = 1$).

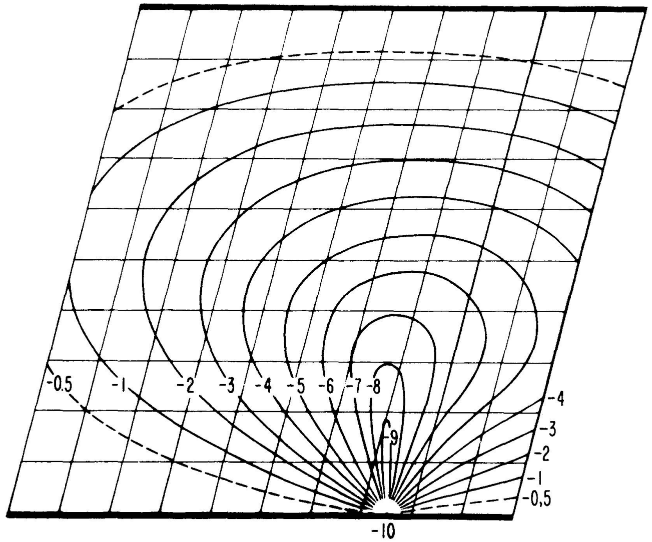

Fig. 17. Influence surface for clamping moment at pt. 4 ($\alpha = 75°$, $b/a = 1$).

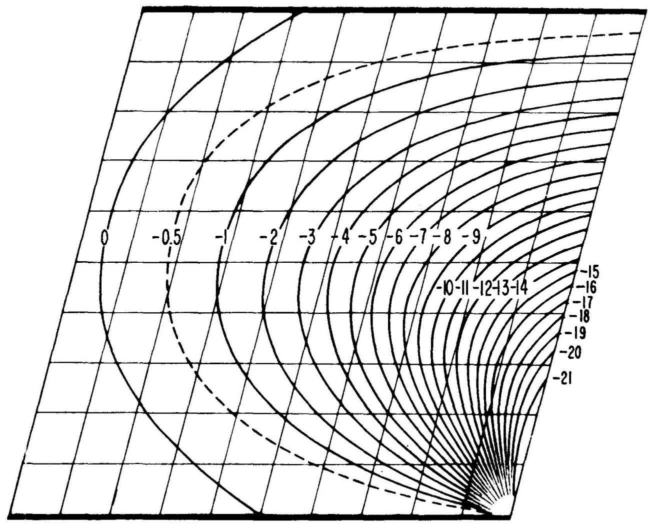

Fig. 18. Influence surface for clamping moment at pt. 5 ($\alpha = 75°$, $b/a = 1$).

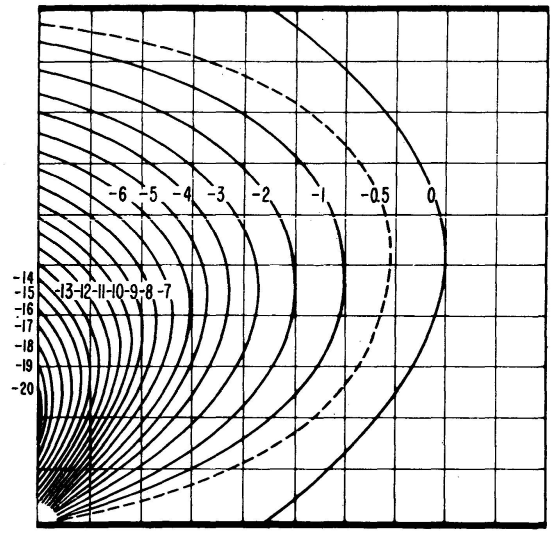

Fig. 19. Influence surface for clamping moment at pt. 1 ($\alpha = 90°$, $b/a = 1$).

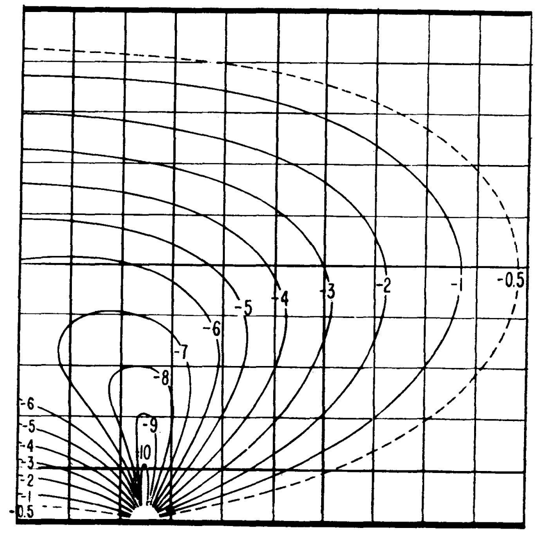

Fig. 20. Influence surface for clamping moment at pt. 2 ($\alpha = 90°$, $b/a = 1$).

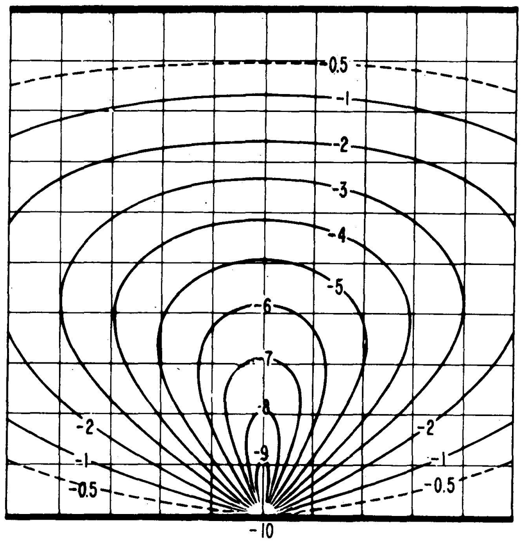

Fig. 21. Influence surface for clamping moment at pt. 3 ($\alpha = 90°$, $b/a = 1$).

$$M = \frac{1}{10\pi} V q, \tag{1}$$

Where M: Clamping moment;
q: Intensity of load;
V: Volume, which is obtained from the influence surface.

5. Accuracy of the Data

A comparison of the theoretical and experimental values of the clamping moment at pt. 3 of a square plate subjected to a uniformly distributed load is given in table 1, where the experimental value has been obtained by using the influence surface shown in Fig. 21. For the other experimental data we have no theoretical values for comparison, but it may be assumed that the same order of accuracy has been retained.

Table 1

	Moment at pt. (3)	Error (%)
Theoretical	$-0.084\ qa^2$	
Experimental	$-0.082\ qa^2$	-2.4

6. Relationships Between the Clamping Moments and the Angle of Skew of the Plates

The relationships between the clamping moments and the skew angle of the plates subjected to a uniformly distributed load or to a line live load perpendicular to the free edges are shown in Fig. 22 and 23, respectively. In Fig. 23, the variation of the maximum clamping moments under a line live load at each of the relevant points are plotted against the angles of skew. The values of the clamping moments to which Fig. 22 and 23 refer are obtained by the following procedure:

$$M = k q a, \tag{2}$$

where M: Clamping moment;
q: Intensity of a uniformly distributed load or line load;
a: Area under a uniformly distributed load or length under a line load;
k: Coefficients to be obtained from Fig. 22 and 23.

The other indications in Fig. 22 and 23 are as same as in Fig. 3.

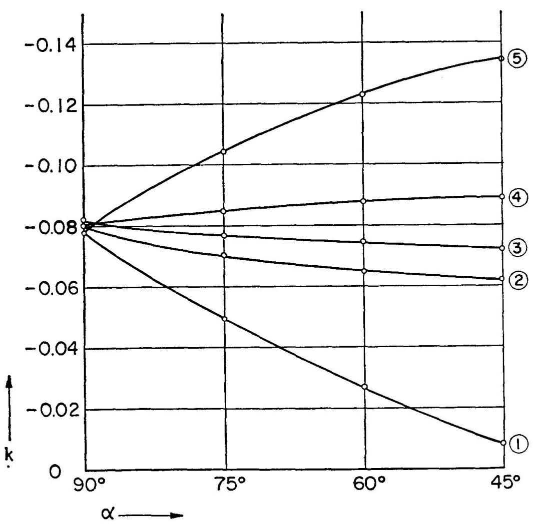

Fig. 22. Relationships between clamping moments and angles of skew of the plates subjected to a uniformly distributed load.

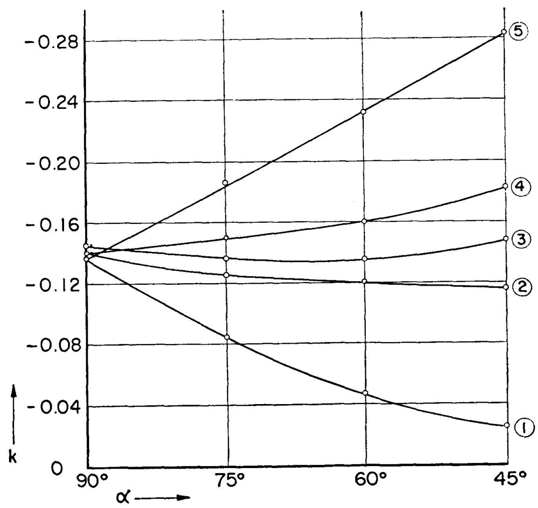

Fig. 23. Relationships between maximum clamping moments and angles of skew of the plates subjected to a line live load perpendicular to free edges.

7. Relationships Between the Clamping Moments and the Ratio of the Length of Clamped Edge to the Span Length Perpendicular to the Clamped Edge

A typical example of such a relationship is given in Fig. 24 for the case of a plate which has a skew angle of 60° and is loaded uniformly. In the figure, a denotes the length of the clamped edge, l: the span length perpendicular to the clamped edge, q: the intensity of a uniformly distributed load, and M: the clamping moment. The curves (1), (2), (3), (4), (5), correspond to the respective points as shown in Fig. 3.

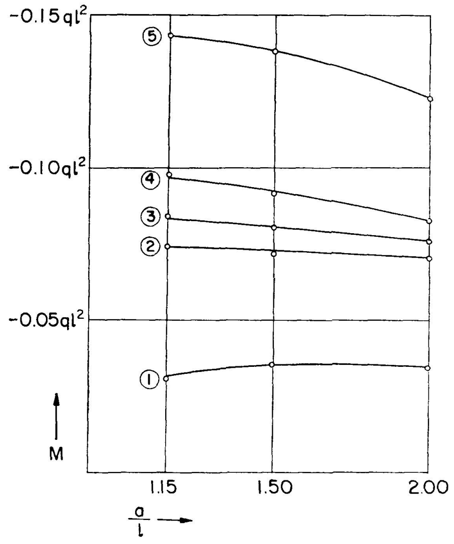

Fig. 24.

8. Conclusions

The conclusions are as follows: as the angle of skew of the plate becomes more acute, the value of the clamping moment in the vicinity of the obtuse angle corner increases considerably. For example, when the skew angle is 45°, the value of the clamping moment near this corner increases by about 80% as compared with that of the clamping moment of a square plate. Therefore, in the design of a skew slab bridge with clamped edges, particular consideration must be given to this clamping moment.

Furthermore, we can say that the value of such a clamping moment at the central portion of the edge becomes almost the same as that of an equivalent fixed beam of the same span length under a uniform load.

References

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Summary

The influence surfaces for the indeterminate clamping moments along the built in edges of skewed plates, with two opposite edges clamped and the other two edges free, have been obtained by a method of model analysis.

Plates having angles of skew of 45° to 90° and ratios of the length of the clamped edge to that of the free edge of approx. 1 to 2 were studied. The effects due to the angles of skew and the ratios of the length of the clamped edge to the span length perpendicular to the clamped edge, on the above-mentioned clamping moment, were investigated under a uniformly distributed load or a movable line load.

The data obtained in the form of influence surfaces may be of use in the design of concrete slab bridges.

Résumé

C'est par des essais sur modèles qu'on a obtenu les surfaces d'influence des moments d'encastrement hyperstatiques le long des bords de dalles biaises ayant deux bords opposés encastrés et les deux autres libres.

Pour les essais, on a pris des dalles ayant une obliquité de 45° à 90° et un rapport de la longueur du bord encastré à celle du bord libre approximativement égal à 1 ou à 2.

Pour étudier les effets de l'angle du biais et du rapport de la longueur du bord encastré à celle de la portée comptée perpendiculairement à ce bord, on a déterminé le moment d'encastrement précité sous une charge uniformément répartie et sous une charge linéaire mobile.

Les résultats obtenus à la faveur de l'étude des surfaces d'influence pourront être utiles dans le calcul des ponts dalles en béton.

Zusammenfassung

Es wurden die Einflußflächen der Einspannmomente bei statisch unbestimmten, schiefen Platten durch Modellversuche aufgenommen; dabei waren zwei gegenüberliegende Ränder eingespannt und die zwei anderen frei.

Es wurden Platten verwendet, deren Schiefe 45° bis 90° und deren Verhältnis zwischen der Länge des eingespannten und derjenigen des freien Randes 1 bis 2 betrug. Um die Wirkung der Schiefe und des Verhältnisses zwischen der Länge des eingespannten Randes und der zu ihm senkrecht liegenden Spannweite zu untersuchen, wurden die erwähnten Einspannmomente unter einer gleichmäßig verteilten Belastung und einer beweglichen Linienlast bestimmt.

Die in Form von Einflußflächen erhaltenen Ergebnisse können bei der Berechnung von Betonplattenbrücken Verwendung finden.