# Optimum reinforcement in a concrete slab subjected to multiple loadings

Autor(en): **Kemp, K.O.**

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# Optimum Reinforcement in a Concrete Slab Subjected to Multiple Loadings

*Armature optimale dans une dalle en béton soumise à des charges multiples*

*Optimale Armierung in einer mehrfachen Belastungen ausgesetzten Betonplatte*

K. O. KEMP
B. Sc. (Eng.)., Ph. D., C. Eng., M.I.C.E., M. I. Struct. E., Professor and Head of Department of Civil and Municipal Engineering, University College London

## Introduction

A procedure for proportioning orthogonal reinforcement in a concrete slab subjected to a single moment triad $(M_x, M_y, M_{xy})$ has been suggested by HILLERBORG [2] and NIELSEN [3]. The method has been expounded by WOOD [4] and extended to skew reinforcement by MORLEY [5].

The procedure rests on the concept of perfect plasticity and leads in general to a lower bound on the collapse load of the slab. The yield moments of the slab $(m_x, m_y)$ in the two chosen reinforcement directions are defined by satisfying the yield equation for the slab and by minimising the total amount of reinforcement, or moment $(m_x + m_y)$ at each point. In certain situations, such as where the applied principal moments are of opposite sign, it may not be possible to minimise the sum moment $(m_x + m_y)$. The yield moments $(m_x, m_y)$ are then defined by setting one equal to a specified minimum value $m_c$ and determining the second by satisfying the yield equation. The method is quite general and can be readily incorporated into an automatic computer programme [6].

In practice, however, many slabs and particularly bridge decks are subjected to multiple loadings. The reinforcement must then be proportioned to satisfy the multiple moment triads $(M_{x_i}, M_{y_i}, M_{xy_i})$, $i = 1$ to $n$ produced by the multiple loadings. The designer must now provide the least reinforcement in the chosen directions such that the yield criterion for the slab is not exceeded by any of the multiple moment triads. This is the problem to be considered in this paper and is essentially an exercise in non-linear programming, with a linear optimisation function and non-linear constraints.

**Orthogonal Reinforcement for a Single Moment Triad**

The yield function for an orthotropically reinforced concrete slab has been shown by several workers [3, 7, 8] to be:

$$(m_x - M_x)(m_y - M_y) \geqq M_{xy}^2, \tag{1}$$

where $m_x, m_y$ are the yield moments of the slab per unit length in the $y$ and $x$ directions respectively and $(M_x, M_y, M_{xy})$ is the applied moment triad at a point. If the equality sign is introduced, this is the equation to a rectangular hyperbola with asymptotes $m_x = M_x$, $m_y = M_y$ as shown in Fig. 1.

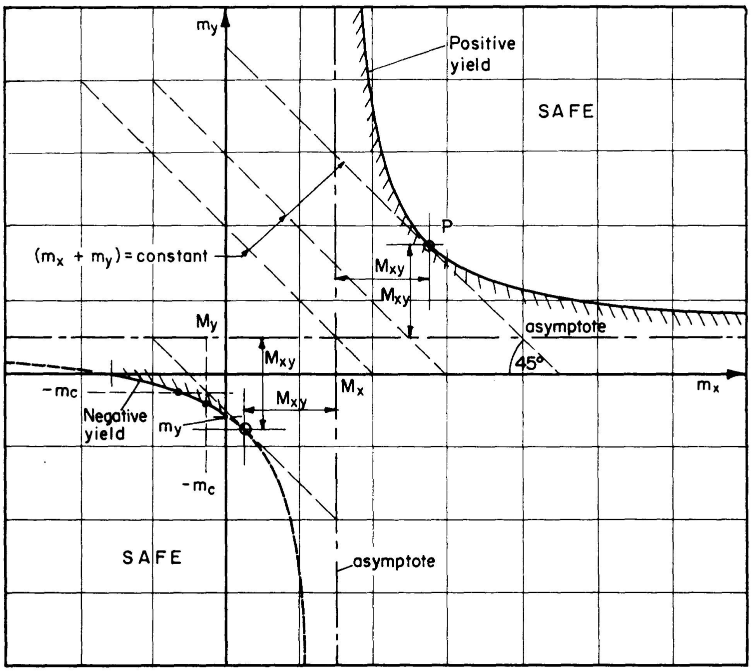


Fig. 1. The yield curve for orthogonal reinforcement.

There are the further conditions that for positive yield (bottom reinforcement) $m_x$ and $m_y$ must both be positive and also $m_x \geqq M_x$ and $m_y \geqq M_y$. Similarly for negative yield (top reinforcement), $m_x$, $m_y$ must both be negative and also $m_x \leqq M_x$ and $m_y \leqq M_y$. Thus the only parts of the rectangular hyperbola which define the yield equation are those in the first quadrant, $(m_x, m_y) + ve$, on the positive side of the asymptotes and in the third quadrant $(m_x, m_y) - ve$, on the negative side of the asymptotes. The first quadrant defines positive yield and the third quadrant negative yield. If both applied principal moments are of the same sign, the real parts of the rectangular hyperbola will lie only in the first quadrant for positive principal moments and in the third quadrant for negative principal moments.

The question whether top and bottom reinforcement is required is therefore answered by putting $m_x = m_y = 0$ in Eq. (1), from which the governing condition is whether $M_x M_y$ is greater or less than $M_{xy}^2$. If $M_x$ and $M_y$ are both positive and $M_x M_y \geqq M_{xy}^2$ there will be no negative moments and no need of top reinforcement. Similarly if $M_x$ and $M_y$ are both negative and $M_x M_y \geqq M_{xy}^2$ there will be no positive moments and no requirement for bottom steel. If $M_x M_y < M_{xy}^2$ there will be both positive and negative moments in both cases. If $M_x$ and $M_y$ are of opposite sign $M_x M_y$ is necessarily less than $M_{xy}^2$ and there will be positive and negative moments and both top and bottom steel are required.

Any point $(m_x, m_y)$ lying on the real parts of the curve in Fig. 1, defines the yield moments for which yield will just occur under the applied moment triad. Any point $(m_x, m_y)$ on the positive side of the positive yield curve defines a safe solution or upper bound on the required positive yield moments. Similarly any point on the negative side of the negative yield curve defines a safe solution or upper bound on the required negative yield moments.

## Minimum Yield Moments

To minimise the reinforcement provided the point $P$, $(m_x, m_y)_{min}$ must be selected which lies in the safe region and minimises the function $(m_x + m_y)$. This optimisation function $(m_x + m_y)$ = constant defines a family of straight lines at 45° to the axes as shown in Fig. 1, and for the least value of the constant, the line must be as close to the origin as possible. This point $P$, $(m_x, m_y)_{min}$ is clearly the point where a $(m_x + m_y)$ = constant line is tangential to the yield curve as shown in Fig. 1. The coordinates of this point can be easily determined since from the yield Eq. (1)

$$m_y = \frac{M_{xy}^2}{(m_x - M_x)} + M_y \qquad \therefore \; (m_x + m_y) = m_x + \frac{M_{xy}^2}{(m_x - M_x)} + M_y .$$

For a stationary minimum value of $(m_x + m_y)$,

$$\frac{d\,(m_x + m_y)}{dm_x} = 1 - \frac{M_{xy}^2}{(m_x - M_x)^2} = 0$$

or $m_x = M_x \pm M_{xy}$.

When $m_x$ and $m_y$ are both positive, the minimum value corresponds with the positive sign, but in any case the positive yield equation is only valied for $m_x \geqq M_x$, $m_y \geqq M_y$.

For positive yield the coordinates of the point $(m_x m_y)_{min}$ are then;

$$\begin{aligned} m_x &= M_x + |M_{xy}|, \\ m_y &= M_y + |M_{xy}|, \end{aligned} \tag{2}$$

which means that it lies on the positive side of the asymptotes at $M_{xy}$ from each.

Similarly for negative yield, the corresponding stationary minimum point can be shown to have the coordinates,

$$\begin{aligned} m_x &= M_x - |M_{xy}|, \\ m_y &= M_y - |M_{xy}|, \end{aligned} \tag{3}$$

so that it lies on the negative side of the asymptotes at $M_{xy}$ from each.

If the point $(m_x, m_y)_{min}$ does not lie on the real parts of the yield curves, there is not a stationary minimum for the required sum reinforcement. For example in Fig. 1, the point $(m_x, m_y)_{min}$ on the negative yield curve, does not lie in the third quadrant i. e. $m_x, m_y$ are not both negative and there is not a stationary minimum value for $(m_x + m_y)$ for negative moments. A *least* value of $(m_x + m_y)$ for negative yield will however be provided by the point $(0, m_y^0)$ where the negative yield curve cuts the $m_y$ axis. Substituting $m_x = 0$ into the yield Eq. (1) gives the required value of $m_y^0$ as

$$m_y^0 = M_y - \frac{M_{xy}^2}{M_x}.$$

Frequently, however, a minimum amount of reinforcement will be specified in each direction corresponding say to yield moments $\pm m_c$. In Fig. 1, the required negative yield moments would then be specified by the point where the line $m_x = -m_c$ cuts the negative yield curve whose coordinates are

$$m_x = -m_c, \qquad m_y' = M_y - \frac{M_{xy}^2}{(m_c + M_x)},$$

where $m_y'$ must be negative and less than $-m_c$.

In certain cases the points corresponding to minimum permissible reinforcement in each direction, $(+m_c, +m_c)(-m_c, -m_c)$ will lie on the safe side of the yield curve. This minimum permissible reinforcement must then be provided in each direction and is in excess of that just required for yielding.

For a single moment triad then, in which a minimum yield moment $m_c$ is specified in each direction, three situations can arise. One is when the stationary minimum value of $(m_x + m_y)$ lies on the real part of the yield curve and $m_x$ and $m_y$ are numerically greater than $m_c$. The optimum yield moments are then $m_x = M_x \pm |M_{xy}|$, $m_y = M_y \pm |M_{xy}|$ where the positive and negative signs apply respectively to positive and negative yield. The second is when the stationary minimum point does not lie on the real part of the curve and then the yield moments are specified by the intersection of one of the lines $m_x = \pm m_c$, $m_y = \pm m_c$ with the appropriate yield curve again with the condition that $m_x$ and $m_y$ must be numerically greater than or equal to $m_c$. The third situation is when the points $(m_c m_c)$ or $(-m_c, -m_c)$ lie off the real parts of the curve in the safe region and then the specified yield moments $\pm m_c$ are required in each direction.

Algebraic expressions for the required yield moments in the various situations have been derived by HILLERBORG [2], NIELSEN [3], WOOD [4] and MORLEY [5]. The graphical presentation of the problem given in Fig. 1 is very helpful in visualising the different situations arising but will be found to be even more useful when the problem of multiple moment triads is considered.

## Orthogonal Reinforcement for Multiple Moment Triads

Suppose a slab is to be designed to withstand $n$ different loadings and that each loading produces at each point of the slab, a different moment triad. $(M_{x_i}, M_{y_i}, M_{xy_i})$ $i=1$ to $n$. The selection of the optimum reinforcement in these circumstances is a problem in non-linear programming. The non-linear constraints are that the yield criterion shall not be exceeded by any of the moment triads. The optimisation function is linear and is that $(m_x+m_y)$ shall be minimised. Expressed in mathematical form, the problem is to select $(m_x m_y)$ such that,

$$(m_x - M_x)(m_y - M_y) \geqq M_{xy_i}^2, \qquad i = 1 \text{ to } n,$$
$$(m_x + m_y) \text{ to be a minimum}.$$

The nature of the problem is readily visualised by the graphical presentation used for the single moment triads. In Fig. 2, the yield curves for three different moment triads $(M_{x_i}, M_{y_i}, M_{xy_i})$ $i=1$ to 3 are shown plotted. Both principal moments are assumed to be positive for each triad so the rectangular hyperolae are real in the first quadrant only.

The points $P_1, P_2, P_3$ give stationary minimum values of $(m_x+m_y)$ for each of the individual moment triads. They are the points where the $(m_x+m_y) =$

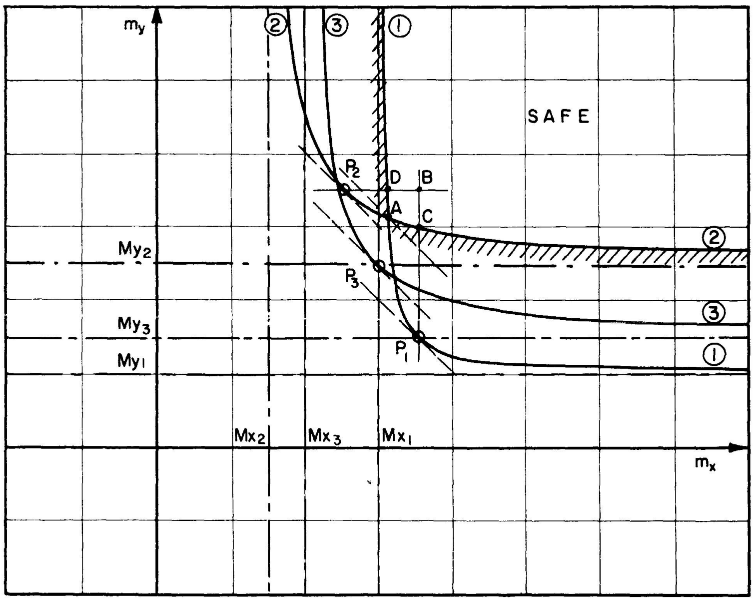


Fig. 2. Optimum yield moments for multiple moment triads.

constant lines are tangential to the yield curves and the coordinates of $P_i$ are again given by $(m_x)_i = M_{x_i} + |M_{xy_i}|$ and $(m_y)_i = M_{y_i} + |M_{xy_i}|$. The safe region for $(m_x m_y)$ for all three moment triads is shown hatched in Fig. 2 and is here defined by portions of the yield curves (1) and (2) only. None of the stationary minimum points $P_1, P_2, P_3$ lie within the safe region in this example so that none of them provide sufficient reinforcement.

Any point on the boundary of the safe region will satisfy all the yield functions but the point which gives the least value of $(m_x + m_y)$ will be point $A$ where the yield curves (1) and (2) intersect, since the $(m_x + m_y)$ = constant line through $A$ is closest to the origin. The optimum solution to this problem therefore is obtained by satisfying the equality condition of the yield functions (1) and (2), i.e.

$$m_y = \frac{(M_{xy1}^2)}{(m_x - M_{x_1})} + M_{y_1}, \qquad m_y = \frac{(M_{xy2}^2)}{(m_x - M_{x_2})} + M_{y_2}. \tag{4}$$

The solution leads to a quadratic equation for $m_x$ and $m_y$. The equation for $m_x$ is

$$\begin{aligned}(M_{y_1} - M_{y_2})\, m_x^2 + m_x\,[(M_{xy_1}^2 - M_{xy_2}^2) - (M_{y_1} - M_{y_2})(M_{x_1} + M_{x_2})] \\ + [M_{x_2}(M_{x_1} M_{y_1} - M_{xy_1}^2) - M_{x_1}(M_{x_2} M_{y_2} - M_{xy_2}^2)] = 0.\end{aligned} \tag{5}$$

For positive yield, the root giving the real solution must be positive and $m_x \geqq M_{x_1}, M_{x_2}$. The corresponding value of $m_y$ can then be found by back substitution in Eq. (4). If these yield moments $(m_x^A, m_y^A)$ are provided in the slab, yield will occur under moment triads (1) and (2) but not under (3).

A special case occurs when the moment triads are such that one stationary minimum point $P_i$ has coordinates $(m_x m_y)$ which are both numerically greater

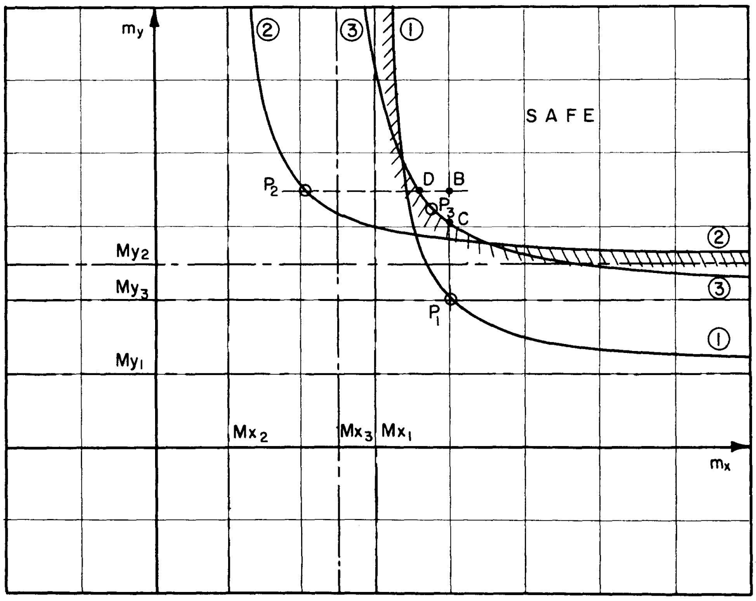


Fig. 3. Upper Bounds on the optimum yield moments for multiple triads.

than that of any of the other stationary minimum points i.e. $(m_x)_j \geqq M_{x_i} + |M_{xy_i}|)$, $(m_y)_j \geqq M_{y_i} + |M_{xy_i}|)$. Then the point $P_i$ defines the optimum yield moments for the slab since all the yield inequalities will be satisfied and the value of $(m_x + m_y)$ will be a stationary minimum.

In general the optimum yield moments will be given by either an intersection point of two yield curves or a stationary minimum point of the type just described. The latter is always clearly identifiable from its coordinates but when there are numerous loading cases, the correct intersection point cannot be easily identified, see Fig. 3, unless a graphical output is used. In such circumstances it is simple and should usually be sufficiently economic to compute a close upper bound to the optimum yield moments.

## Upper Bounds on the Optimum Yield Moments

In Fig. 2 and 3 point $B$ is defined by the largest $m_x$ and $m_y$ coordinates obtained from all the stationary minimum points. This point will always lie in the safe region and will give an upper bound on the required yield moments.

Closer upper bounds are given by the points $C$ and $D$ which are the intersection points of the lines

$$m_x = [M_{x_i} + |M_{xy_i}|]_{max}, \qquad m_y = [M_{y_i} + |M_{xy_i}|]_{max}$$

with the yield curves bounding the safe region.

The $m_y$ coordinate of $C$ can be found by substituting $m_x = [M_{x_i} + |M_{xy_i}|]_{max}$ into each of the yield equations and selecting the largest positive value of $m_y$ obtained. Similarly the $m_x$ coordinate of $D$ can be found by substituting $m_y = [M_{y_i} + |M_{xy_i}|]_{max}$ into all the yield equations and selecting the largest positive value of $m_x$ so obtained. Whichever of the two points $C$ or $D$ give the least value of $(m_x + m_y)$ could then be adopted as a suitable upper bound on the optimum yield moments.

This procedure for determining upper bounds on $(m_x + m_y)$ could be readily programmed for automatic computation. If graphical procedures are employed, the intersection point on the boundary of the safe region giving the least value of $(m_x + m_y)$, point $A$ in Fig. 2 or the stationary minimum point $P_3$ in Fig. 3 could be selected by inspection.

## Negative Yield Moments

When the multiple moment triads produce negative moments in the slab, similar procedures can be used to determine the optimum or upper bounds on the optimum negative yield moments. The only difference is that $m_x$ and

$m_y$ must always be negative and the stationary minimum points have coordinates

$$(m_x)_i = [M_{x_i} - |M_{xy_i}|]\,(m_y)_i = [M_{y_i} - |M_{xy_i}|].$$

If the intersection point of two yield curves is required, the negative root in Eq. (5) must be used where $m_x \leqq M_{x_1}$ and $M_{x_2}$.

## Special Case of Small Applied Moments

When the positive or negative applied moments are small, all the stationary minimum points $(m_x, m_y)$ may lie on the non-real parts of the yield curves. The problem of determining the optimum yield moments for a single moment triad in these circumstances has already been discussed. This problem is not generally going to be so important with multiple moment triads since the required yield moments will always be dictated by the moment triads with the largest values. However it may occasionally arise that all the loading cases give small values of either positive or negative moments.

It will again be assumed that there is a minimum yield moment $\pm m_c$ which must be provided in each orthogonal direction. The problem of small moments with multiple loadings is best considered in three categories, depending on whether the $m_x$ and $m_y$ asymptotes are or are not all less than $m_c$ for positive moments or all greater than $-m_c$ for negative moments.

*Case a. All $m_x$ and $m_y$ asymptotes less than $m_c$ for positive moments or greater than $-m_c$ for negative moments.*

In this case, the optimum positive yield moments can be found by substituting $m_x = m_c$ into each yield equation and selecting the largest positive value of $m_y$ produced say $m_y = m_y'$. Similarly substitute $m_y = m_c$ into each yield equation and select the largest positive value of $m_x$ produced say $m_x = m_x'$. Whichever of the points $(m_c m_y')$ or $(m_x', m_c)$ gives the smallest value of $(m_x + m_y)$ will give the optimum yield moments.

For negative moments the procedure is identical except $m_x = -m_c$ and $m_y = -m_c$ are substituted into the yield equations, and the largest negative moments $m_y'$ and $m_x'$ selected. A graphical illustration of this procedure with negative moments is shown in Fig. 4.

*Case b. All $m_x$ (or $m_y$) asymptotes, but not all $m_y$ (or $m_x$) asymptotes less than $m_c$ for positive yield or greater than $-m_c$ for negative yield.*

For positive moments if all the $m_x$ asymptotes are less than $m_c$ but at least one of the $m_y$ asymptotes is greater than $m_c$, the optimum yield moments can be found directly by substituting $m_x = m_c$ into all the yield equations and

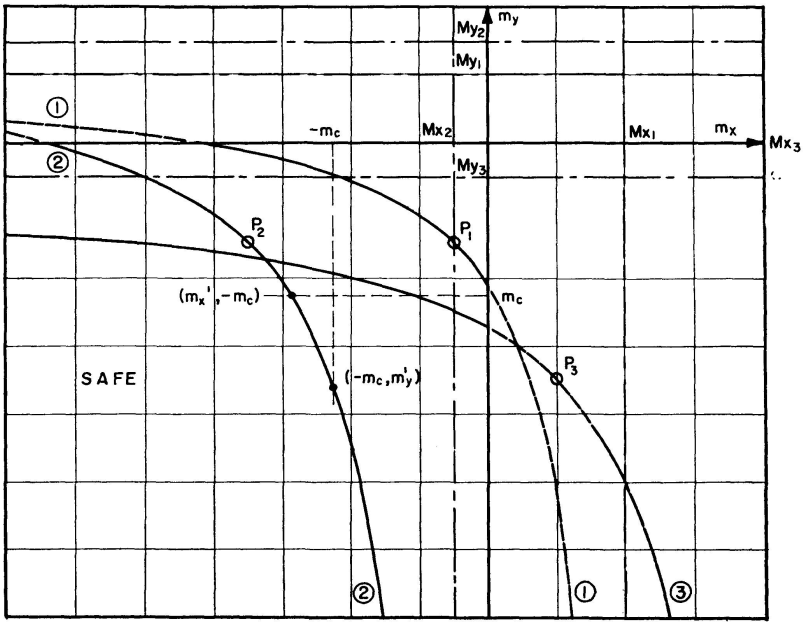


Fig. 4. Optimum negative yield moments when all $m_x$ and $m_y$ asymptotes greater than $-m_c$.

selecting the largest positive value of $m_y$. Alternatively if all the $m_y$ asymptotes are less than $m_c$ but not all the $m_x$ asymptotes are less than $m_c$ substitute $m_y = m_c$ in all the yield equations and select the largest value of $m_x$.

A similar procedure applies to negative moments except that $m_x = -m_c$ or $m_y = -m_c$ is substituted into each yield equation depending on whether all the $m_x$ asymptotes are greater than $-m_c$ or all the $m_y$ asymptotes are greater than $-m_c$. An example of the latter is shown in Fig. 5.

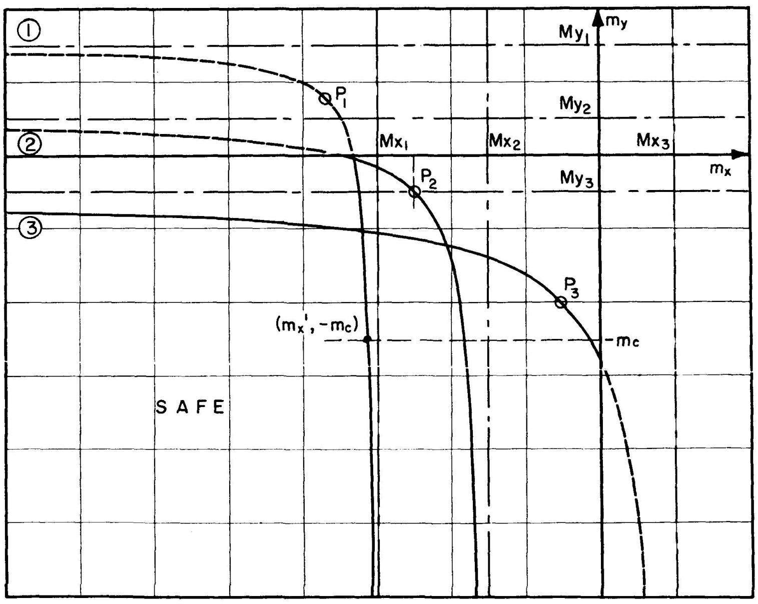


Fig. 5. Optimum negative yield moments when all $m_y$ asymptotes but not all $m_x$ asymptotes greater than $-m_c$.

*Case c. Some $m_x$ and $m_y$ asymptotes greater than $m_c$ for positive moments or less than $-m_c$ for negative moments.*

Where at least one of both the $m_x$ and the $m_y$ asymptotes are greater than $m_c$, the lines $m_x = m_c$ and $m_y = m_c$ will not intersect all the yield curves. An upper bound on the optimum positive yield moments can be found by putting $m_x$ equal to the largest positive $m_x$ coordinate of all the stationary minimum points, i. e. $m_x = [M_{x_i} + |M_{xy_i}|]_{max}$. The required value of $m_y$ is then the largest positive value obtained from all the yield equations. Similarly if $m_y$ is put equal to the largest positive $m_y$ coordinate of all the stationary minimum points i. e. $m_y = [M_{y_i} + |M_{xy_i}|]_{max}$ in each yield equation and the largest positive value of $m_x$ selected a second upper bound is produced. Whichever of these two points gives the least value of $(m_x + m_y)$ could be adopted to specify the yield moments.

The procedure for negative moments is identical except $m_x$ and $m_y$ are respectively set equal to the largest negative $m_x$ and $m_y$ coordinates of the stationary minimum points, and the largest negative values of $m_y$ and $m_x$ obtained from the yield equation are adopted.

## Skew Reinforcement. Optimum Yield Moments for Single and Multiple Moment Triads

For a slab reinforced with skew reinforcement producing yield moments $m_x$ per unit length in the $y$ direction and $m_\alpha$ per unit length in a direction at $\alpha$ to the $x$ axis as in Fig. 6, the yield function is given by

$$[(m_x + m_\alpha \operatorname{Sin}^2 \alpha) - M_x]\,[m_\alpha \operatorname{Cos}^2 \alpha - M_y] \geqq [M_{xy} - m_\alpha \operatorname{Sin} \alpha \operatorname{Cos} \alpha]^2. \tag{6}$$

The yield equality again defines a rectangular hyperbola as shown in Fig. 6 with asymptotes,

$$\begin{aligned} m_x &= M_x + M_y \tan^2 \alpha - 2\, M_{xy} \operatorname{Tan} \alpha, \\ m_\alpha &= M_y \operatorname{Sec}^2 \alpha. \end{aligned} \tag{7}$$

Only those parts of the curve which lie in the first quadrant on the positive side of the asymptotes and in the third quadrant on the negative side of the asymptotes are real. Thus, for positive yield moments

$$\begin{aligned} &m_x \text{ and } m_\alpha \text{ are positive}, \\ &m_x \geqq M_x + M_y \operatorname{Tan}^2 \alpha - 2\, M_{xy} \operatorname{Tan} \alpha, \\ &m_\alpha \geqq M_y \operatorname{Sec}^2 \alpha. \end{aligned}$$

And for negative yield moments,

$$\begin{aligned} &m_x \text{ and } m_\alpha \text{ are negative}, \\ &m_x \leqq M_x + M_y \operatorname{Tan}^2 \alpha - 2\, M_{xy} \operatorname{Tan} \alpha, \\ &m_\alpha \leqq M_y \operatorname{Sec}^2 \alpha. \end{aligned}$$

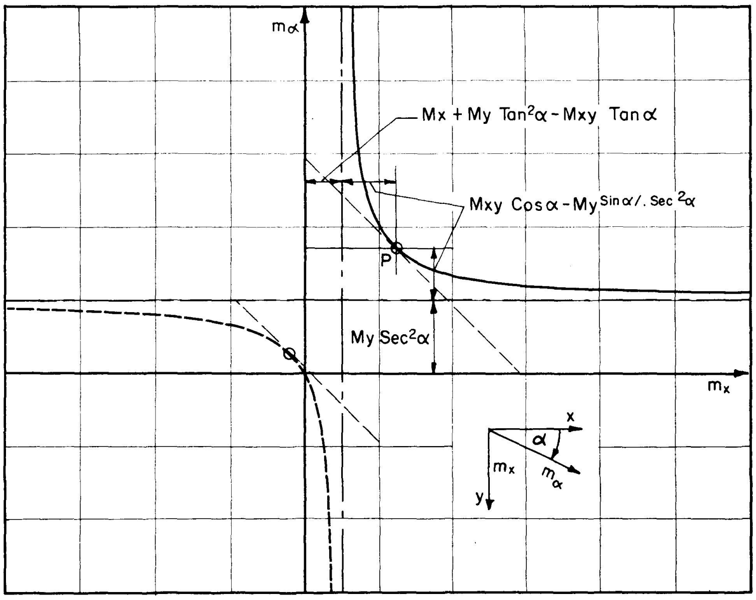


Fig. 6. The yield curve for skew reinforcement.

The points on the yield equation defining the stationary minimum values of $(m_x + m_\alpha)$ are where the lines $(m_x + m_\alpha)$ = constant are tangential to the curve and their coordinates are:

$$\begin{aligned} m_x &= M_x + M_y \operatorname{Tan}^2 \alpha - 2 M_{xy} \operatorname{Tan} \alpha \pm |M_{xy} \operatorname{Cos} \alpha - M_y \operatorname{Sin} \alpha| \operatorname{Sec}^2 \alpha \\ m_\alpha &= M_y \operatorname{Sec}^2 \alpha \pm |M_{xy} \operatorname{Cos} \alpha - M_y \operatorname{Sin} \alpha| \operatorname{Sec}^2 \alpha \end{aligned} \tag{8}$$

the positive and negative signs applying respectively to positive and negative yield moments.

All these expressions, of course, reduce to those derived for orthogonal reinforcement when $\alpha = 0$.

If the appropriate expressions for skew reinforcement are substituted for those used in the treatment of orthogonally reinforced slabs, all the procedures suggested for determining the optimum yield moments for multiple moment triads with orthogonal reinforcement are applicable also to skew reinforcement.

## Conclusions

The problem of selecting the optimum orthogonal or skew reinforcement for a concrete slab subjected to a single moment triad has been considered using a graphical presentation. This provides a clear illustration of the different situations arising for which algebraic expressions for the required yield moments have been previously derived.

The same graphical presentation has then been used to solve the non-linear programming problem of selecting the optimum reinforcement for a slab subjected to multiple moment triads. The procedures suggested are all suitable for use in an automatic computer programme or can be used in a graphical form. They are equally applicable to skew and orthogonal reinforcement and the similarities between the two are emphasized.

## Notation and Sign Convention

| | |
|---|---|
| $i$ | Suffix denoting any single moment triad. |
| $m_x, m_y$ | Yield moments per unit length in the $y$ and $x$ directions respectively. |
| $m_\alpha$ | Yield moment per unit length in a direction at $\alpha$ to the $x$ axis. |
| $m'_x, m'_y$ | Intersection point of the lines $m_y$ = constant and $m_x$ = constant respectively with the yield curve. |
| $m^0_x, m^0_y$ | Intersection point of the lines $m_y = 0$, $m_x = 0$ respectively with the yield curve. |
| $m_c$ | Specified minimum yield moment per unit length in any direction. |
| $M_x, M_y$ | Bending moments per unit length in the $y$ and $x$ directions respectively. |
| $M_{xy}, M_{yx}$ | Twisting moments per unit length in the $y$ and $x$ directions respectively. |
| $n$ | Number of loadings. |
| $P_i$ | Point giving stationary minimum value of $(m_x + m_y)$. |
| $x, y$ | Orthogonal coordinate directions in the plane of the slab. |
| $\alpha$ | Clockwise angle from the $x$ axis to the $m_\alpha$ moment line. |

The sign convention for coordinates $x$, $y$ and moments $M_x$, $M_y$, $M_{xy}$, $M_{yx}$ are those used by TIMOSHENKO S. and WOINOWSKY-KRIEGER S. [1].

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## Summary

Using a graphical presentation, procedures are suggested for determining the optimum reinforcement in a concrete slab subjected to multiple loadings. The procedures can be utilised in an automatic computer programme or in a graphical solution and are applicable to both skew and orthogonal reinforcement layouts.

## Résumé

Utilisant une représentation graphique on propose des méthodes pour la détermination de l'armature optimale dans une dalle en béton soumise à des charges multiples. Les procédés peuvent être utilisés dans un programme automatique pour ordinateur ou dans une solution graphique et sont applicables aux armatures de biais aussi bien qu'orthogonales.

## Zusammenfassung

Unter Benützung einer graphischen Darstellung werden Verfahren zur Bestimmung der optimalen Armierung in einer Betonplatte vorgeschlagen, die mehrfachen Belastungen ausgesetzt ist. Die Verfahren können in einem automatischen Computer-Programm oder in einer graphischen Lösung benützt werden und sind sowohl für schräge wie für rechtwinklige Armierungs-Auslegungen anwendbar.

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