Zeitschrift: IABSE publications = Mémoires AIPC = IVBH Abhandlungen

Band: 33 (1973)

Artikel: Linear program for optimal design of reinforced concrete frames

Autor: Krishnamoorthy, C.S. / Munro, J.

DOI: https://doi.org/10.5169/seals-25622

Linear Program for Optimal Design of Reinforced Concrete Frames

Programme linéaire de programmation pour le projet optimal de charpentes en béton armé

Lineare Programmierung für den optimalen Entwurf von Tragwerken aus Stahlbeton

C. S. KRISHNAMOORTHY
Ph. D., Lecturer, Civil Engineering Dept., Indian Institute of Technology, Madras. (Formerly Commonwealth Scholar, Imperial College)

J. MUNRO
Ph. D., Senior Lecturer, Civil Engineering Dept., Imperial College, London

1. Introduction

In contrast to the design of steel frames the limit design of reinforced concrete frames requires special consideration due to the inelastic behaviour of the members under various stages of loading, the limited ductility of the member sections and the serviceability requirements under working loads. Special methods have, therefore, been proposed (1 to 8) for the limit design of reinforced concrete frames. With the development of mathematical programming techniques and the availability of large computing facilities there has been growing interest in the field of optimum limit design of reinforced concrete frames. COHN and GRIERSON [9, 10, 11] proposed two formulations for optimal design. One is a linear programming formulation [9, 10] with the limit equilibrium of the governing collapse mechanisms and serviceability criteria as the main constraints and the compatibility condition is to be checked separately. The other formulation [11] has constraints associated with compatibility, limited ductility, limit equilibrium of the governing collapse mechanisms and serviceability criteria. This formulation leads, to a highly non-linear programming problem of considerable complexity. However only simple linearised examples were solved.

The present authors [12] have proposed a linear programming model for the optimal design of reinforced concrete frames with compatibility, limited ductility, equilibrium and serviceability criteria as governing constraints. The objective function to be minimised was the total volume of steel reinforcement. The location of plastic hinges was restricted to critical sections in beams.

The present paper extends the linear programming formulation to the more general case where there are no such restrictions. A computer program which formulates and solves the optimal frame design problem is described with the aid of a flow chart. The computational aspects of the problem are also given along with the input and output details of the program.

Practical design examples of multi-storey concrete frames have been presented to illustrate the use of the optimal design program described here. The economical merits of the optimal design compared to the ultimate strength design based on elastic analysis for factored loads is discussed for different cases of hinge pattern.

2. Basic Principles

2.1. Definition of Design Moment

Consider a reinforced concrete frame subjected to a given system of service dead, live and wind loads. When these loads are increased by their over load factors and the structure thus subjected to ultimate load stage, plastic hinges develop at critical sections resulting in a redistribution of bending moments. The design bending moment M_d^j at a critical section of the frame under ultimate load can be expressed in the following form [7, 10, 12]:

$$M_d^j = x_j M_u^j, \tag{1}$$

where x_j is called the yield safety parameter [7].

The optimal design problem is to determine the values of x_j, knowing the values of M_u^j from an elastic analysis for factored loads, that satisfies the desired design objective and other criteria.

2.2. Objective Function and Optimality Criterion

The optimality criterion is expressed through either a maximisation or a minimisation of an objective function of the design variables x_j. The objective function may, for example, represent the area of the moment diagram, cost of concrete and steel or the volume of steel reinforcement [10]. In this formulation the selected optimality criterion is minimisation of the total volume of reinforcement for the frame.

From the design charts [13] the area of reinforcement for the beam and column sections may be expressed as a linear function,

$$A_j = \bar{D}_1 + \bar{D}_2 x_j, \tag{2}$$

where $\bar{D}_1, \bar{D}_2$ are constants.

Assuming the reinforcement for a critical section is extended over an equivalent length, l_j, [10] the volume of reinforcement with respect to a critical section is given by,

$$V_j = (\bar{D}_1 + \bar{D}_2 x_j)\, l_j$$

or

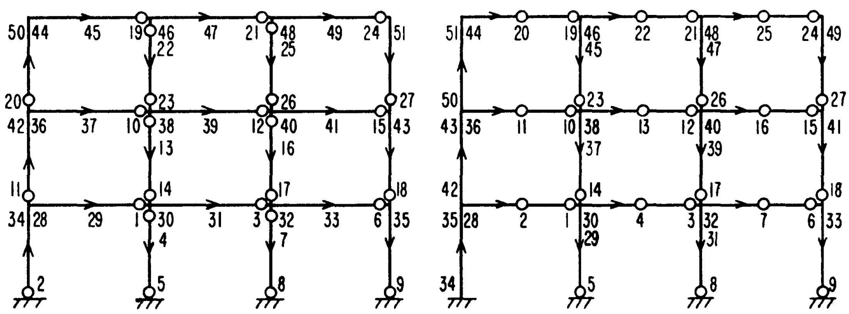
$$V_j = D_1^j + D_2^j x_j.$$

The objective function, V, representing the total volume of reinforcement for the frame is, therefore, given by,

$$V = \sum_{j=1}^{c} D_1^j + D_2^j x_j. \tag{3}$$

2.3. Compatibility and Limited Ductility Criteria

Fig. 1 shows two types of moment releases for a frame which is α times indeterminate. The selected hinge releases will be termed "basic hinges" and the other critical sections will be termed "non-basic hinges". The compatibility



(a) System A (b) System B

Fig. 1. Release systems for a frame.

equation relating the angular discontinuities at the basic and non-basic hinges is given by [6, 14],

$$\sum_{j=1}^{\alpha} g_{ij} m^j + v_{i0} + \sum_{q=\alpha+1}^{c} k_{iq} \theta_n^q + \theta_b^i = 0 \qquad (i = 1, 2, \ldots, \alpha), \tag{4}$$

where θ_n^q and θ_b^i are as shown in Fig. 2.

Positive moment Positive rotation

Fig. 2. Sign conventions.

In order that all stress resultants shall correspond to strain resultants the following parity rule can be applied to all plastic hinges:

$$m^r \theta^r \geqq 0 \qquad (r = 1, 2, \ldots, c). \tag{5}$$

Substituting for m^r from Eq. (1) the above rule can be stated as

$$M_u^r x_r \theta^r \geqq 0 \qquad (r = 1, 2, \ldots, c). \tag{6}$$

Noting that x_r would be restricted to take only positive values Eq. (6) becomes

$$M_u^r \theta^r \geqq 0 \qquad (r = 1, 2, \ldots, c). \tag{7}$$

For the discussion to follow, it was found convenient to express θ^r as

$$\theta^r = \delta_r |\theta^r|, \tag{8}$$

where δ_r will be termed the "parity delta". According to the parity rule expressed by Eq. (7) the value of δ_r will be as follows:

$$\begin{aligned} \delta_r &= +1 \quad \text{for } M_u^r > 0, \\ \delta_r &= -1 \quad \text{for } M_u^r < 0. \end{aligned} \tag{9}$$

Substituting Eqs. (1) and (8) into Eq. (4), yields

$$\sum_{j=1}^{\alpha} g_{ij} M_u^j x_j + v_{i0} + \sum_{q=\alpha+1}^{c} k_{iq} \delta_q |\theta_n^q| + \delta_i |\theta_b^i| = 0 \qquad (i = 1, 2, \ldots, \alpha). \tag{10}$$

The value of δ_r is determined from the sense of moment M_u^r, known from elastic analysis for factored loads, through Eq. (9).

The inelastic rotation θ^r in Eq. (10) can be expressed in terms of the permissible rotation, θ_p^r, as

$$|\theta^r| = |\theta_p^r| - y_{c+r}, \tag{11}$$

where y_{c+r} is a slack variable and is subjected to the following restrictions:

$$\text{(I)} \quad y_{c+r} \geqq 0. \tag{12}$$

This condition will ensure satisfaction of the *limited ductility criterion* i.e., the inelastic rotation at section r shall not exceed the allowable rotation θ_p^r. The lower bound value $y_{c+r} = 0$ will make $|\theta^r| = |\theta_p^r|$. The Simplex algorithm treats both design and slack variables as non-negative. Hence Eq. (12) need not be stated explicitly.

$$\text{(II)} \quad y_{c+r} \leqq |\theta_p^r|. \tag{13}$$

This constraint ensures both the sense invariance of θ^r and the satisfaction of the parity rule. The upper bound value $y_{c+r} = |\theta_p^r|$ indicates zero inelastic rotation and the section, therefore, remains elastic.

Substituting from Eq. (11) into Eq. (10) yields:

$$\sum_{j=1}^{\alpha} g_{ij} M_u^j x_j + v_{i0} + \sum_{q=\alpha+1}^{c} k_{iq} \delta_q \{|\theta_p^q| - y_{c+q}\} + \delta_i \{|\theta_p^i| - y_{c+i}\} = 0 \tag{14}$$
$$(i = 1, 2, \ldots, \alpha).$$

Eqs. (13) and (14) jointly express the compatibility and limited ductility requirements for the frame.

The rotational capacity of reinforced concrete sections can be expressed as a linear function of moment m^r in the following form:

$$\theta_p^r = B_1^r + B_2^r |m^r|. \tag{15}$$

Noting that $m^r = M_u^r x_r$ the Eq. (15) may be expressed as

$$\theta_p^r = B_1^r + B_2^r |M_u^r| x_r. \tag{16}$$

Substituting from Eq. (16) into Eqs. (13) and (14) and after rearrangement the following constraints are obtained.

$$\begin{aligned} \sum_{j=1}^{\alpha} g_{ij} M_u^j x_j + v_{i0} + \sum_{q=\alpha+1}^{c} k_{iq} \delta_q \{(B_1^q + B_2^q |M_u^q| x_q) - y_{c+q}\} & \\ + \delta_i \{(B_1^i + B_2^i |M_u^i| x_i) - y_{c+i}\} = 0 \qquad (i = 1, 2, \ldots, \alpha), & \end{aligned} \tag{17}$$

$$y_{c+r} \leqq B_1^r + B_2^r |M_u^r| x_r \qquad (r = 1, 2, \ldots, c). \tag{18}$$

The Eqs. (17) and (18) are the compatibility and limited ductility criteria for the frame.

2.4. Equilibrium Criteria

The moments at the non-basic hinge sections can be determined from the moment and force equilibrium for the frame and is expressed by,

$$\sum_{j=1}^{\alpha} h_{qj} m^j + m_0^q = m^q \qquad (q = \alpha + 1, \ldots, c), \tag{19}$$

where $h_{qj} = k_{jq}$ from contragredience [6]. Substituting for m^r from Eq. (1) into Eq. (19),

$$\sum_{j=1}^{\alpha} h_{qj} M_u^j x_j + m_0^q = M_u^q x_q \qquad (q = \alpha + 1, \ldots, c). \tag{20}$$

Eq. (20) is the equilibrium criteria for the frame.

2.5. Serviceability Criteria

The optimal design must satisfy the desired serviceability conditions for the behaviour of the members under working loads. It is assumed that the serviceability requirements depend on the yield moment of the critical sections and hence can be specified by stipulating a suitable value of λ_*^j. The serviceability criteria is, therefore, satisfied if

$$x_j \geqq \frac{\lambda_*^j}{\lambda_u},$$

$$\text{i.e.} \quad x_j \geqq L_j. \tag{21}$$

2.6. Other Criteria

It should be noted that the final detailing ensures satisfaction of the yield criterion and, in conjunction with the equilibrium criteria of section 2.4, the procedure can be considered to be "safe" from the plastic limit analysis point of view. Thus it is not possible for a plastic collapse mechanism to form for a load inferior to the ultimate load. However, it is also necessary to exclude inadmissible mechanism deformations from the elasto-plastic compatibility equations (18). In addition, it will be frequently necessary to impose other more arbitrary kinematic conditions such as those introduced previously (12) to partially satisfy certain stability conditions.

These additional kinematic conditions may be treated by judicious use of a device termed the plastic hinge indicator which controls the formation of plastic hinges in the optimal design program. It should be emphasised however that the basic hinge set is, as its name implies, the basis for the analysis but that plastic hinges may form wherever permitted by the plastic hinge indicator and are not confined to members of the basic set.

3. Linear Programming Formulation

The optimal frame design problem with the compatibility and limited ductility, equilibrium and serviceability criteria as the governing constraints can be stated as follows: Minimise:

Objective function, $$V = \sum_{j=1}^{c} D_1^j + D_2^j\, x_j . \tag{22}$$

Subject to: Compatibility and limited ductility constraint:

$$\begin{aligned} \sum_{j=1}^{\alpha} g_{ij}\, M_u^j x_j + v_{i0} + \sum_{q=\alpha+1}^{c} k_{iq}\, \delta_q \{(B_1^q + B_2^q\, |M_u^q|\, x_q) - y_{c+q}\} \\ + \delta_i \{(B_1^i + B_2^i\, |M_u^i|\, x_i) - y_{c+i}\} = 0 \qquad (i = 1, 2, \ldots, \alpha), \end{aligned} \tag{23}$$

$$y_{c+r} \leqq B_1^r + B_2^r\, |M_u^r| x_r \qquad (r = 1, 2, \ldots, c). \tag{24}$$

Equilibrium constraint:

$$\sum_{j=1}^{\alpha} h_{qj}\, M_u^j x_j + m_0^q = M_u^q\, x_q \qquad (q = \alpha + 1, \ldots, c). \tag{25}$$

Serviceability constraint:

$$x_j \geqq L_j \qquad (j = 1, 2, \ldots, c). \tag{26}$$

This is a linear programming problem which can be solved by using the standard Simplex algorithm [15].

Lower Bound Restrictions

If the following substitution is made for the design variable x_j, the lower bound restriction of Eq. (26) may be replaced by the non-negativeness of the transformed variables.

$$x_j = L_j + U_j y_j \qquad (j = 1, 2, \ldots, c). \tag{27}$$

The constant U_j introduced in the above equation will, by suitable stipulation, enable to control the bending moment value at a section with respect to the elastic moment for factored loads.

Substituting for x_j into the Eqs. (23) to (26) and introducing slack variables, the problem can be expressed in the standard form of a linear program:

Min. $$z = \boldsymbol{c}^T \boldsymbol{y}. \tag{28}$$

Subject to $$\boldsymbol{A}\boldsymbol{y} = \boldsymbol{b} \tag{29}$$

and $$\boldsymbol{y} \geqq \boldsymbol{0}, \tag{30}$$

where the cost, structural coefficients and the stipulations are given in Appendix I.

3.1. Additional Design Constraints

It may be necessary from practical considerations to introduce additional design constraints as described below.

It may be desirable to restrict the value of the bending moment with respect to the elastic moment value for factored loads. For example the moments at the ends of the beams may be restricted not to exceed the equivalent elastic value. This may be achieved by assigning $U_j = 1 - L_j$ and introducing a constraint,

$$y_j \leqq 1. \tag{31}$$

Sometimes it may be found desirable to keep symmetry in design and hence may be necessary to restrict the moment at a section not to exceed the value at a corresponding symmetrical section. This may easily be included by incorporating the necessary symmetry constraint. With this type of constraint the cost coefficient, c_j, may correspondingly be weighted to keep the symmetry in design.

The design constraints, some of them discussed above, must form a consistent set with the governing constraint equations.

3.2. Change in Sign of Moment

During the process of calculation of optimal solution for the frame with the inelastic rotations at critical sections, the bending moment at some of the sections may change from positive to negative or vice versa with reference to the elastic solution. This may be treated in a similar fashion to the non-negative constraint of a linear programming problem [16]. The design variable y_j is expressed as a difference of two positive variables noting that in the Simplex algorithm the value of the variables are positive, thus

$$y_j = y'_j - y''_j . \tag{32}$$

It can be shown that no more than one of the complementary variables (y'_j and y''_j) can be non-zero.

It becomes difficult to include an inelastic rotation at the section, where the moment changes in sign, into the compatibility equation satisfying the parity rule as well. However in practical designs, such sections may be treated as having zero rotations (i.e. remained elastic), without significant error from the practical point of view.

4. Computer Program for Optimal Design

4.1. Flexibility Matrix

The release system chosen for the computation of flexibility matrix is shown in Fig. 3. This release system is advantageous in plotting the moment diagrams (m_0^q and h_{qj}) for loads and for unit actions at the releases as shown in Fig. 3. The flexibility matrix, $\boldsymbol{G}$, is automatically generated in the computer program through a set of subroutines, programmed to compute the coefficients using the expressions given in Reference [1], with the modification of a directed graph sign convention [14]. Though the program is written for this particular release system it will be shown later that this can be used for any other release system through a suitable transformation.

4.2. Static and Kinematic Matrices

It has been previously stated that elements of the static and kinematic matrices are related contragrediently. Consequently the coefficients (k_{jq}) of the kinematic matrix are obtained directly from the previously calculated coefficients (h_{qj}) of the static matrix.

$$k_{jq} = h_{qj} .$$

A sub-routine of the program generates these coefficients and assembles the two matrices.

4.3. Transformation of Basis

It may be desired in the analysis for optimal solution to have a different release system. The transformation to another basis (i.e. release system) is done using the procedure explained in Reference [17]. The following is a brief

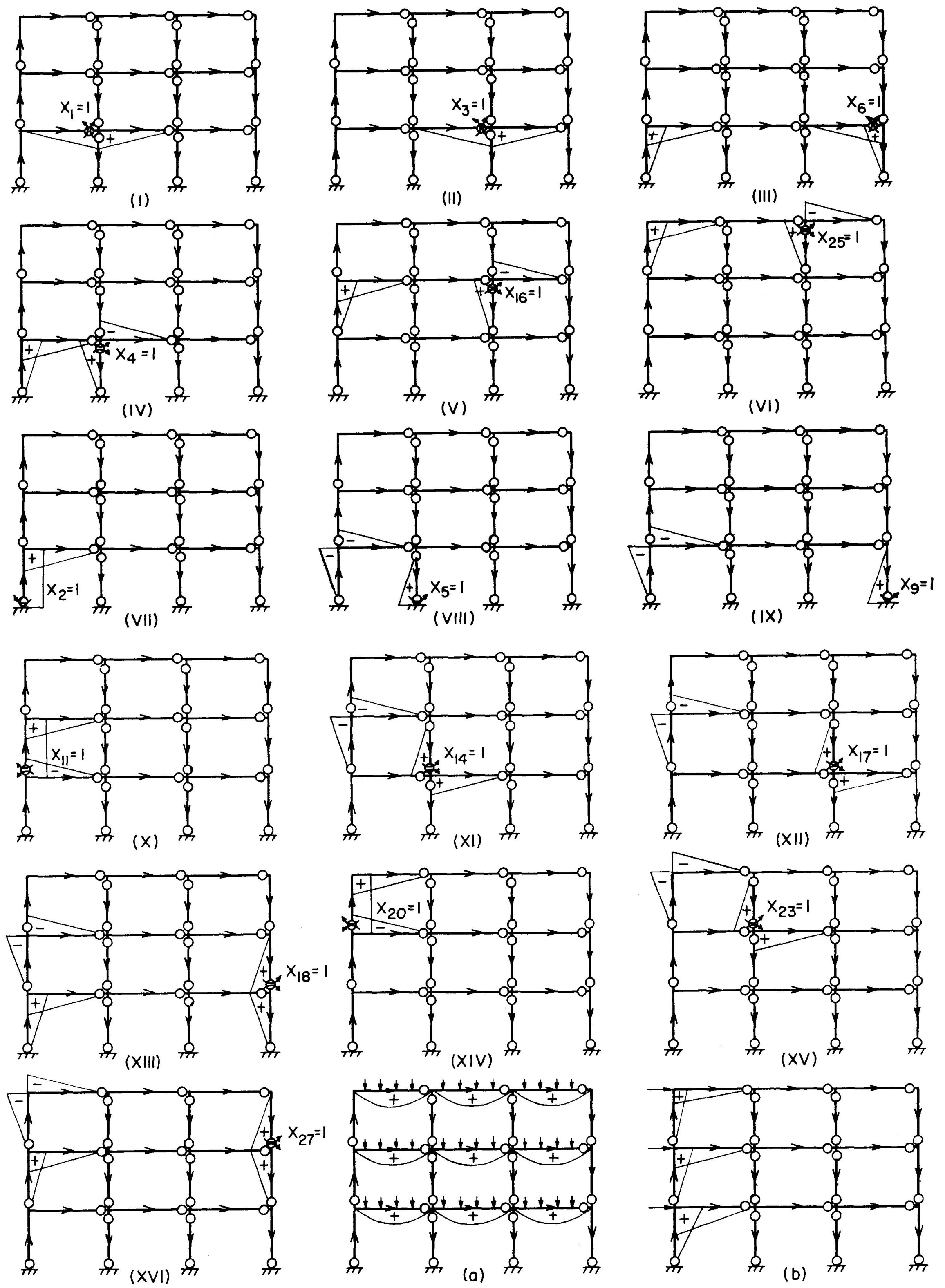


Fig. 3. Typical moment diagrams for unit actions at basic hinges and for loads

summary of the expressions used in connection with the transformation of basis.

The redundant actions $\boldsymbol{m}_b^B$ at the releases of the system B can be expressed in terms of the actions $\boldsymbol{m}_b^A$ at the releases of the system A as,

$$\boldsymbol{m}_b^B = \boldsymbol{Q}\,\boldsymbol{m}_b^A, \tag{33}$$

where $\boldsymbol{Q}$ is called the release transformation matrix of the order $\alpha \times \alpha$.

The flexibility matrix, $\boldsymbol{G}^B$, the displacement vector $\boldsymbol{v}_0^B$, the static matrix $\boldsymbol{H}^B$ and the particular solution due to loads $\boldsymbol{m}_0^B$ for the release system B are given by the following expressions:

$$\boldsymbol{G}^B = (\boldsymbol{Q}^{-1})^T\,\boldsymbol{G}^A\,\boldsymbol{Q}^{-1}, \tag{34}$$

$$\boldsymbol{v}_0^B = (\boldsymbol{Q}^{-1})^T\,\boldsymbol{v}_0^A - \boldsymbol{G}^B\,\boldsymbol{m}_{0B}^A, \tag{35}$$

$$\boldsymbol{H}^B = \boldsymbol{H}^A\,\boldsymbol{Q}^{-1}, \tag{36}$$

$$\boldsymbol{m}_0^B = \boldsymbol{m}_0^A - \boldsymbol{H}^B\,\boldsymbol{m}_{0B}^A, \tag{37}$$

where $\boldsymbol{m}_{0B}^A$ would be the values of $\boldsymbol{m}_0^A$ at the positions of the redundant actions $\boldsymbol{m}_b^B$.

The expressions (34) to (37) show that for a complete transformation from release system A to system B, only the release transformation matrix $\boldsymbol{Q}$ and vector $\boldsymbol{m}_{0B}^A$ are required. The coefficients of $\boldsymbol{Q}$ and $\boldsymbol{m}_{0B}^A$ are easily calculated from h_{qj} and m_0^q diagrams for system A.

A subroutine that will form the matrix $\boldsymbol{Q}$ for transformation to a release system shown in Fig. 1b has been included in the computer program.

4.4. Objective Function and Permissible Rotations

From the design charts given in Reference [13] (for CEB Recommendations [18]) suitable piece-wise linear expressions of the type of Eq. (2) are derived for the calculation of area of reinforcement for beam and column sections. A section of the Program uses these expressions to compute the coefficients D_1^j and D_2^j of the objective function. If it is, however, desired to use a linear approximation to some other design charts this section of the Program could easily be replaced.

These design charts facilitate computation of the neutral axis depth. This in turn permits derivation of linear expressions for the permissible rotations through use of the appropriate graphs in Reference [13].

4.5. Flow Chart, Input and Output Details

A computer program has been developed [19] for the formulation and solution of the optimal design of R.C. frames based on the theory described here. The program deals with two loading conditions: (I) Wind + dead + live load and (II) dead + live load. The flow chart indicating the main aspects of the program is shown in Fig. 4.

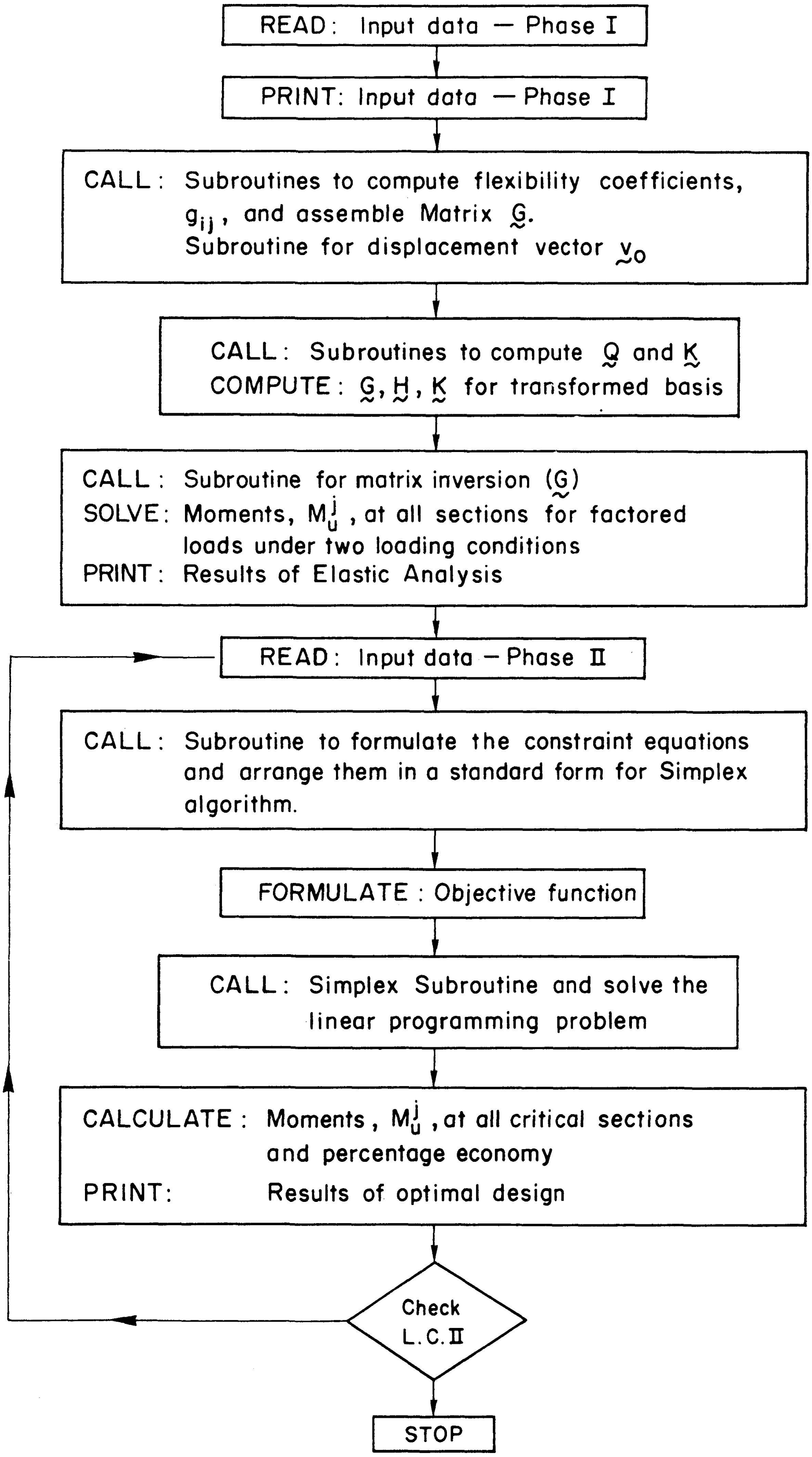


Fig. 4. Flow Chart.

Input data: The input data to be supplied for the program are as follows:

Phase I

(I) Number of storeys and number of bays.

(II) Design strengths of concrete and steel.

(III) Beam properties and Loads:

a) Beam Number, length, effective depth, width of flange near mid span, EI value, equivalent length of steel at mid span and end sections.

b) Simply supported moment values $\left(\frac{\overline{W}L}{8}\right)$ due to dead and live load.

(IV) Column properties:

Column Number, height, breadth, effective depth, total depth and EI value.

(V) Wind load:

Total wind load shear at each storey level.

Phase II

The following set of data is required for each of the two loading conditions:

(I) a) Lower bound values, L_j, for the basic hinges.
b) Lower bound values, L_j, for the non-basic hinges.

(II) a) Upper bound values, U_j, for the basic hinges.
b) Upper bound values, U_j, for the non-basic hinges.

(III) a) Plastic Hinge Indicator for the basic hinges.
b) Plastic Hinge Indicator for the non-basic hinges.
(1 for plastic hinge, 0 for no plastic hinge.)

(IV) a) Moment Change Indicator for the basic hinges.
b) Moment Change Indicator for the non-basic hinges.
(1 for possible change in sign of moment, 0 for no such change.)

Fig. 1b indicates the numbering system required for the preparation of input data.

Output: The output of the program is listed below:

(I) Frame and member properties as given in the data.

(II) Results of elastic analysis for factored loads under two loading conditions (printed separately).
For each storey –
Beams – Moments at the left end, mid span and right end.
Columns – Moments at the top and bottom ends, and axial load.

(III) Results of optimal solution for two loading conditions (printed separately):

a) For each storey –
Beams – Moments at the left end, mid span and right end.
Columns – Moments at the top and bottom ends, and axial load.

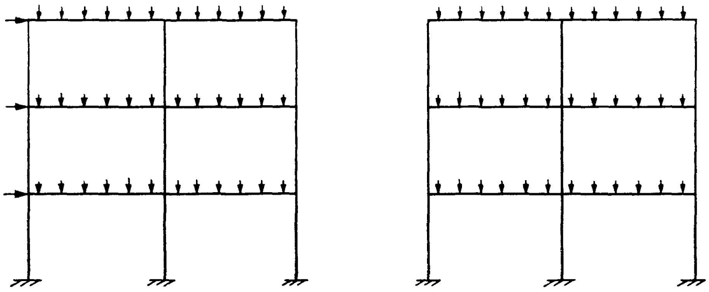
b) Volume of steel for elastic analysis.

d) Percentage economy for optimal design compared to ultimate strength design for elastic analysis for factored loads.

5. Optimal Design Examples

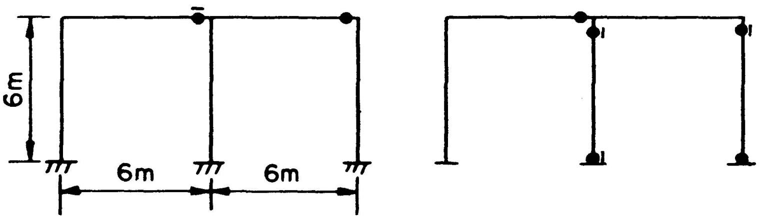
5.1. General Description and Cases Analysed

Six reinforced concrete multi-storey frames, shown in Fig. 5, have been designed using the Optimal Design Program presented here. The loading and other details of the frames are given in Table 1.



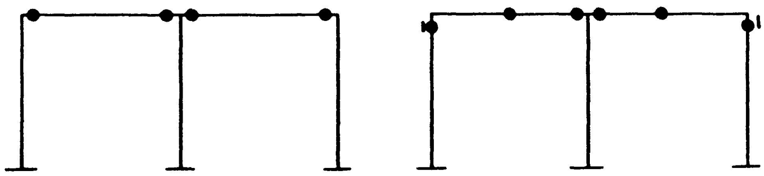
Loading condition I $\lambda_c\ (W+D+L)$ Loading condition II $(\lambda_D D+\lambda_L L)$

Fig. 5a. Loading conditions.



Case 1 Cases 2 and 3

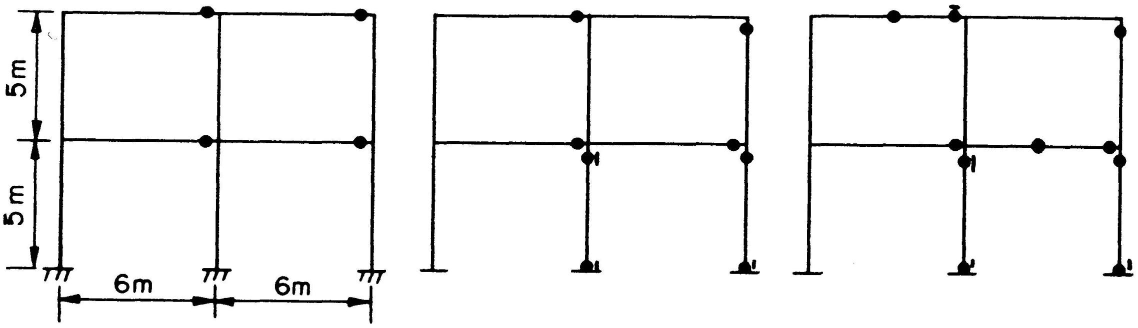
Loading condition I



Cases 1 and 2 Case 3

Loading condition II

Fig. 5b. Example 1.



Case 1 Case 2 Case 3

Loading condition I

Cases 1 and 2
Loading condition II

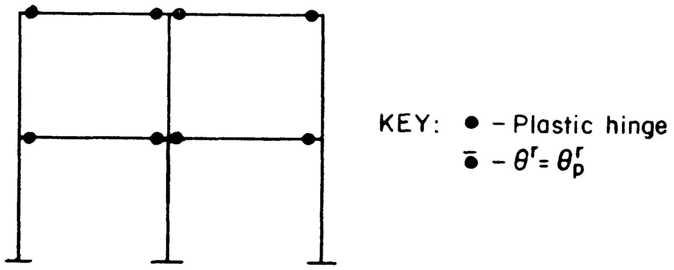
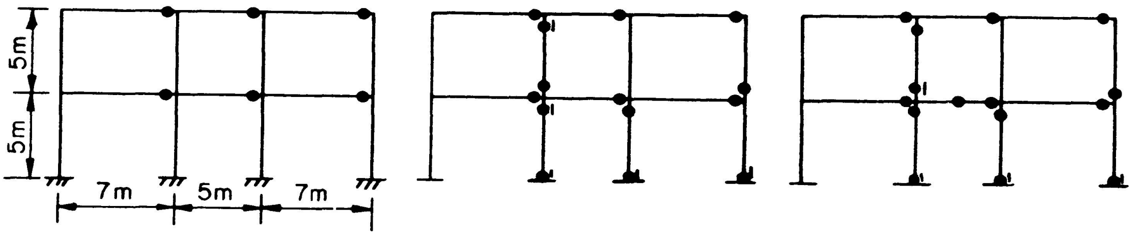


Fig. 5c. Example 2.



Case 1 Case 2 Case 3

Loading condition I

Cases 1 and 2
Loading condition II

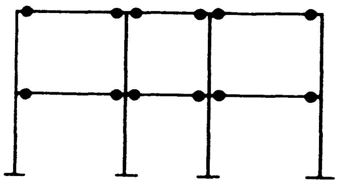
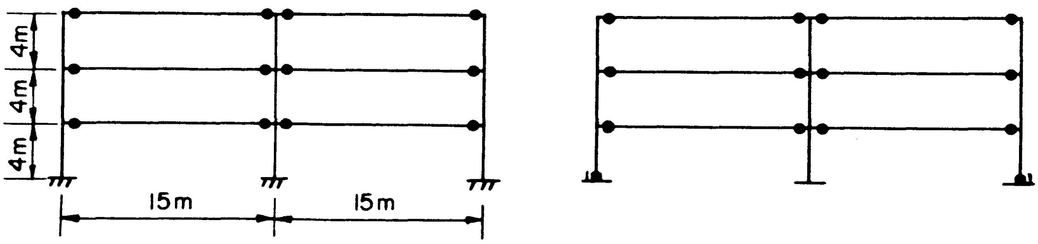


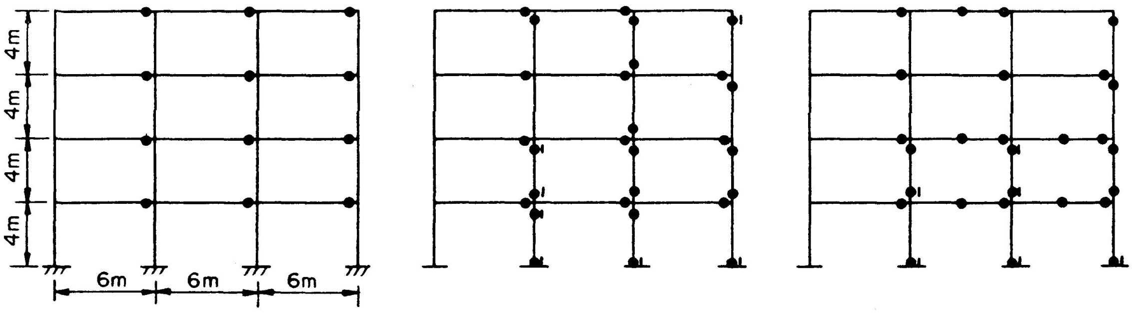
Fig. 5d. Example 3.



Case 1 Case 2

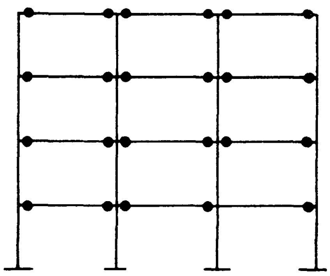
Loading condition II

Fig. 5e. Example 4.



Case 1 Case 2 Case 3

Loading condition I



Cases 1 and 2

Loading Condition II

Fig. 5f. Example 5.

Table 1. Loading, Serviceability and Strength Details of Frames

	Optimal Design Example					
	1	2	3	4	5	6
Loadings:						
Live load, kN/m^2	3.75	5.0	5.0	6.0	2.5	3.0
Dead load, kN/m^2	5.0	6.25	6.0	11.0	4.5	5.0
Wind load, kN/m^2	1.0	1.0	1.2	—	0.7	0.7
Beams:						
Live load per metre run, kN/m	15.0	20.0	25.0	36.0	9.4	12.0
Dead load per metre run, kN/m	20.0	25.0	30.0	66.0	19.5	20.0
Wind force:						
Top floor level, kN	20.0	10.0	15.0	—	8.4	5.6
Other floor levels, kN	—	20.0	30.0	—	16.8	11.2
Load factors:						
λ_D	1.4	1.4	1.4	1.4	1.4	1.4
λ_L	1.6	1.6	1.6	1.6	1.6	1.6
λ_C	1.25	1.25	1.25	—	1.25	1.25
Serviceability factor: λ_*^i						
Loading Condition I	1.05	1.05	1.05	—	1.05	1.05
Loading Condition II	1.10	1.10	1.10	1.10	1.10	1.10
Design strengths:						
Concrete, σ_b, N/mm^2	16.0	16.0	16.0	24.0	17.0	16.0
Steel, σ_{a1}, N/mm^2	415.0	415.0	415.0	415.0	415.0	415.0

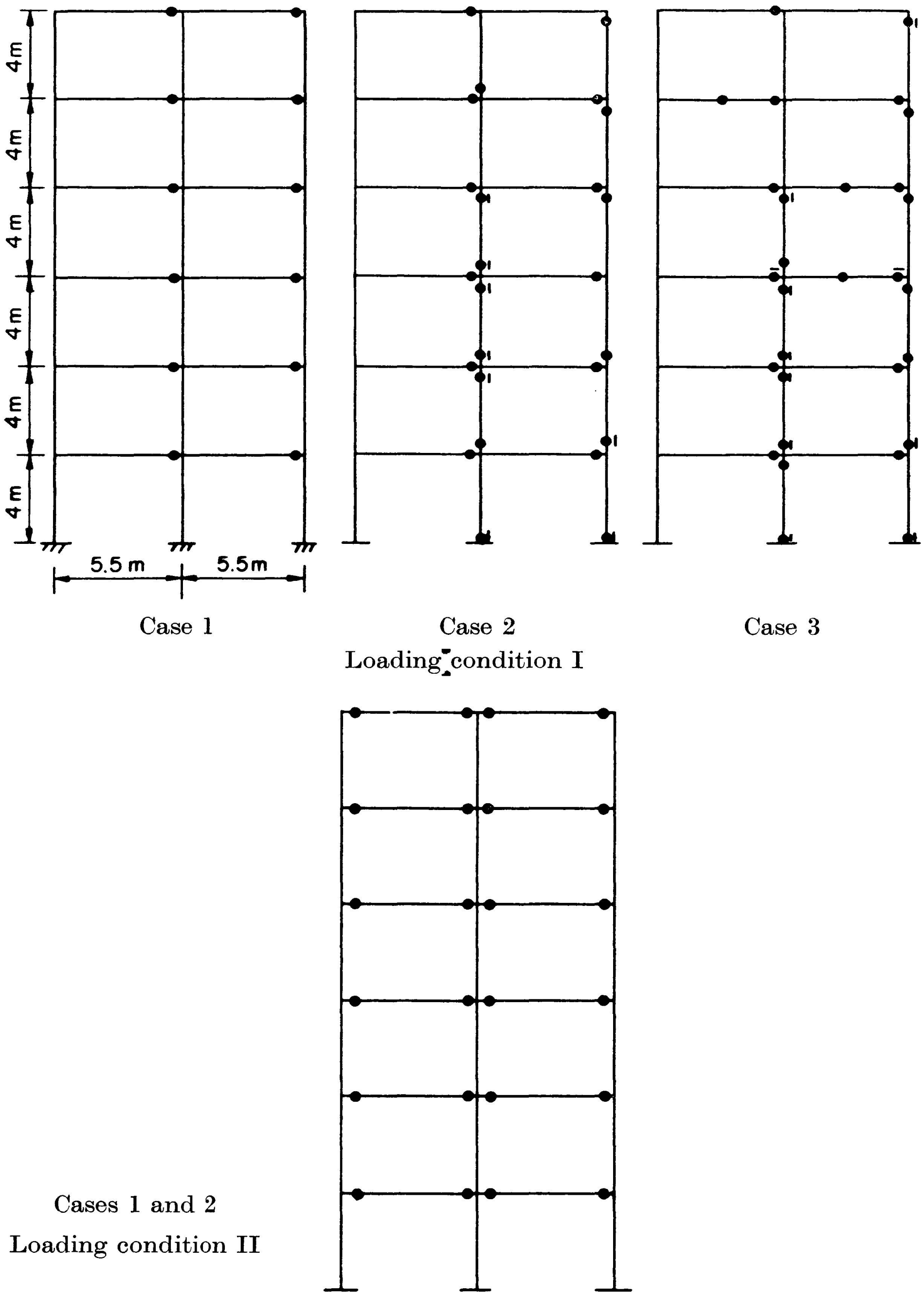


Fig. 5g. Example 6.

The frames have been analysed for optimal solution under two loading conditions: I – $\lambda_C(W+D+L)$ and II – $(\lambda_D D+\lambda_L L)$. In order to study the merits of economy for certain of the specific design requirements, explained below, the following three cases of hinge pattern have been considered for each of the loading conditions.

Case 1: Hinges at ends of beams only.

A recent investigation [20] has shown that for frames which require consideration of instability effects under ultimate load it would be desirable to avoid hinges in columns. Hence in the case of loading condition I the plastic hinges are allowed to form only at the leeward end of the beams and for loading condition II the plastic hinges are allowed to form at both ends of the beams. The hinges at the mid span sections of the beams may be avoided as discussed below.

Case 2: Beam and column hinges but no hinge at mid span sections of beams.

For satisfactory serviceability conditions under working load, especially to avoid excessive deflection, it has been recommended by BAKER [1] that hinges at mid span sections of beams may be avoided. Hence in this case assuming that there are no serious stability problems, beam and column hinges are allowed to form but the hinges at the mid span sections of beams are avoided.

Case 3: No restriction of hinges.

For the cases of frames that satisfy the above two requirements no restriction is placed on the location of hinges and plastic hinges are allowed to form at all the critical sections.

The analysis for the above three cases can be carried out by assigning the corresponding values for the Plastic Hinge Indicator, required for the Input data explained earlier.

Fig. 5 shows the position of plastic hinges as obtained for optimal solution, for the three cases under two loading conditions for the six frames described here.

It should be noted that the minimum weight design of steel frames corresponds to multi-degree of freedom (FOULKES) collapse mechanisms [21]. This result must be modified for the case of reinforced concrete frames whose positive and negative plastic moments of resistance are of different magnitudes [22]. However, the above properties of frames of unlimited ductility are further modified by the inclusion of limited ductility constraints.

5.2. Merits of Economy

Table 2 gives the percentage economy of optimal design for the six frames for the three cases under two loading conditions. The percentage economy is calculated with respect to the steel requirement for the ultimate strength design of sections for actions obtained from the elastic analysis for factored loads. It has been observed that the percentage economy depends on the geometry, loading condition and choice of hinge pattern. Savings of steel of up to 20% have been recorded.

In general it has been noticed that the savings of steel for loading condition II are higher than that for loading condition I. The area of steel, hence the volume, for the sections is proportional to the bending moment at the corresponding sections. In the loading condition I with the development of

Table 2. Comparison of Ultimate Strength and Optimal Designs

Example	Loading Condition	Case	Vol. of Steel (Ultimate Strength Design) cm^3	Vol. of Steel (Optimal Design) cm^3	Percentage economy
1	I	1	19,390	18,162	6.34
		2	19,390	17,096	11.83
		3	19,390	17,096	11.83
	II	1	11,138	9,785	12.14
		2	11,138	9,785	12.14
2	I	1	40,504	37,709	6.90
		2	40,504	36,656	9.50
		3	40,504	36,614	9.60
	II	1	28,752	24,703	14.08
		2	28,752	24,703	14.08
3	I	1	64,860	61,635	4.97
		2	64,860	59,517	8.24
		3	64,860	58,873	9.23
	II	1	703,933	59,403	20.16
		2	703,933	59,403	20.16
4	II	1	74,401	618,652	12.12
		2	74,401	618,652	12.12
5	I	1	76,774	71,914	6.33
		2	76,774	70,038	8.77
		3	76,774	69,225	9.83
	II	1	67,210	56,137	16.48
		2	67,210	56,137	16.48
6	I	1	129,821	120,778	6.97
		2	129,821	117,457	9.52
		3	129,821	116,804	10.03
	II	1	78,129	64,671	17.23
		2	78,129	64,671	17.23

plastic hinges a reduction in the value of moment is found at the ends of beams but this cannot be said of column sections. But for loading condition II, for symmetrical frames, there is an overall redistribution of moments resulting in a reduction of moment for beam ends and also column sections. Hence in the latter case the steel requirements also become less. Also in the case of loading condition II there is no difference in optimal solution due to different choice of hinge pattern for the three cases.

6. Conclusions

1. The linear programming formulation of optimal design based on compatibility, limited ductility, equilibrium and serviceability criteria as the

governing constraints, presents a rational unified method for the limit design of reinforced concrete frames.

2. The formulation lends itself to computer programming and such an Optimal Design Program presented here offers the following advantages:

a) Any Code specification for the design of concrete sections, which could be approximated to piece-wise linear relationships may be incorporated in the program.

b) Different choice of plastic hinge pattern to satisfy specific design requirements can be analysed by suitable use of the Plastic Hinge Indicator.

c) Running of the program is economical as judged from the six design examples and promises its wider use in practice. Typical frames of the type described in this paper occupied 25–145 seconds of computing time in the University of London CDC 6600.

3. Practical design examples presented here show significant savings in steel reinforcement which encourages the use of the optimal design method proposed in this paper.

Notation

A_j	Area of reinforcement for section j
c	Total number of critical sections
EI	Flexural rigidity of the member
$\boldsymbol{G}$	Flexibility matrix for the frame $g_{ij} = \int \frac{h_i h_j}{EI} ds$
h_j	Bending moment distribution due to unit action at a basic hinge j
h_{qj}	Value of bending moment at q due to unit action at a basic hinge j
k_{jq}	Angular discontinuity at a basic hinge j due to unit rotation at q ($k_{jq} = h_{qj}$)
l_j	Equivalent length of reinforcement for section j
L_j	Lower bound value of x_j for section $j = \frac{\lambda_*{}^j}{\lambda_u}$
m^j	Moment at a section j
m_0^q	Moment at a section q in the reduced structure due to loads
$\boldsymbol{m}_b$	Column vector $\{m^1, m^2, \ldots, m^\alpha\}$
$\boldsymbol{m}_0$	Column vector $\{m_0^{\alpha+1}, m_0^{\alpha+2}, \ldots, m_0^c\}$
M^j	Moment at section j under working load
M_d^j	Design moment at section j under ultimate load
M_y^j	Yield moment for section j
M_u^j	Factored elastic moment at section j
v_{i0}	Angular discontinuity at a basic hinge section i due to loads $= \int \frac{h_i m_0}{EI} ds$
$\boldsymbol{v}_0$	Column vector $\{v_{10}, v_{20}, \ldots, v^\alpha{}_0\}$

v_j	Volume of reinforcement for section j
V	Total volume of reinforcement for the frame
x_j	Yield safety parameter for section $j = \frac{\lambda^j}{\lambda_u}$
λ^j	Yield moment factor for j section $j = \frac{M_y{}^j}{M^j}$
$\lambda_D, \lambda_L, \lambda_C$	Partial load factors for dead, live and combined loads
λ_u	Ultimate load factor
α	Indeterminacy number for a frame
θ_b^i	Inelastic rotation at a basic hinge i
θ_n^q	Inelastic rotation at a non-basic hinge q
θ_p^r	Permissible rotation at a hinge r
σ_{a1}	Design strength of steel
σ_b	Design strength of concrete

Acknowledgements

The authors are grateful to Professor A. L. L. Baker, Professor of Concrete Structures and Technology, Imperial College, London, for his interest in and encouragement of the work described in this paper. They are also grateful to the Computing Centres of Imperial College and of the University of London.

Appendix I

The cost coefficients (c_j), structural coefficients (a_{ij}) and the stipulations, b_i, of the L.P. problem expressed by Eqs. (28) to (30) are given below:

$$Z = V - \sum_{j=1}^{c} D_1^j + D_2^j L_j,$$

$$c_j = D_2^j U_j \qquad (j = 1, 2, \ldots, c).$$

For $i = 1, 2, \ldots, \alpha$

$$\begin{aligned}
a_{ij} &= g_{ij} M_u^j U_j && (j = 1, 2, \ldots, \alpha),\ j \neq i, \\
a_{ii} &= g_{ii} M_u^i U_i + \delta_i B_2^i |M_u^i| U_i, \\
a_{ij} &= k_{ij} \delta_j B_2^j |M_u^j| U_j && (j = \alpha + 1, \ldots, c), \\
a_{i(c+i)} &= -\delta_i, \\
a_{i(c+j)} &= -k_{ij} \delta_j && (j = \alpha + 1, \ldots, c), \\
b_i &= -v_{i0} - \sum_{j=1}^{\alpha} g_{ij} M_u^j L_j - \delta_i B_1^i \\
&\quad - \delta_i B_2^i |M_u^i| L_i - \sum_{j=\alpha+1}^{c} k_{ij} \delta_j \{B_1^j + B_2^j |M_u^j| L_j\}.
\end{aligned}$$

For $i = \alpha+1, \alpha+2, \ldots, c$

$$\begin{aligned} a_{ij} &= h_{ij} M_u^j U_j \qquad (j = 1, 2, \ldots, \alpha),\ j \neq i, \\ a_{ii} &= h_{ii} M_u^i U_i - M_u^i U_i, \\ b_i &= M_u^i L_i - m_0^i - \sum_{j=1}^{\alpha} h_{ij} M_u^j L_j. \end{aligned}$$

For $i = c+1, c+2, \ldots, 2c$

$$\begin{aligned} a_{i(i-c)} &= -B_2^{(i-c)} |M_u^{(i-c)}| U_{(i-c)}, \\ a_{ii} &= 1, \\ a_{i(i+c)} &= 1, \\ b_i &= B_1^{(i-c)} + B_2^{(i-c)} |M_u^{(i-c)}| L_{(i-c)}. \end{aligned}$$

The coefficients not defined by the above equations would be zero.

References

1. A. L. L. Baker: The Ultimate-Load Theory Applied to the Design of Reinforced and Prestressed Concrete Frames. Concrete Publications Ltd., London, 1956.
2. G. Macchi: A proposal of analysis on the theory of imposed rotations. Bulletin d'Information CEB, No. 21, Jan. 1960, p. 121–158.
3. J. Munro: Ph. D. Thesis. University of London, 1963.
4. A. L. L. Baker and A. M. N. Amarakone: Inelastic hyperstatic frames analysis. Proceedings, International symposium on Fexural Mechanics of Reinforced Concrete, Miami, Florida, 1964, ACI Special Publication, SP-12, p. 85–142.
5. H. A. Sawyer, Jr.: Design of concrete frames for two failure stages. Proceedings, International Symposium on Flexural Mechanics of Reinforced Concrete, Miami, Florida, 1964, ACI Special Publication, SP-12, p. 405–431.
6. J. Munro: The elastic and limit analysis of planar skeletal structures. Civil Engineering and Public Works Review, V. 60, No. 706, May 1965, p. 671–677.
7. M. Z. Cohn: Limit design solutions for concrete structures, Proceedings ASCE, Journal of the Structural Division, V. 93, No. STI, Feb. 1967, p. 37–58.
8. M. Z. Cohn: Limit design of reinforced concrete frames. Proceedings ASCE, Journal of the Structural Division, V. 94, No. ST 10, Oct. 1968, p. 2467–2483.
9. M. Z. Cohn and D. E. Grierson: Optimal design of reinforced concrete beams and frames. Final Publication, 8th IABSE Congress, 1968, p. 215–226.
10. D. E. Grierson: Optimal design of reinforced concrete frames. Ph. D. Thesis, University of Waterloo, Ontario, Canada, 1968.
11. D. E. Grierson and M. Z. Cohn: A general formulation of the optimal frame problem. Transactions ASME, Journal of Applied Mechanics, June 1970, p. 356–360.
12. J. Munro, C. S. Krishnamoorthy and C. W. Yu: Optimal design of reinforced concrete frames. The Structural Engineer. V. 50, No. 7, July 1972, p. 259–264.
13. A. L. L. Baker: Limit-State Design of Reinforced Concrete. Cement and Concrete Association, London, 1970.
14. J. Munro: The stress analysis and topology of skeletal structures. Concrete Structures and technology Research Report, CSTR 14, Imperial College, London, March, 1965.

15. G. B. DANTZIG: Linear Programming and Extensions. Princeton University Press, Princeton 1963.
16. G. HADLEY: Linear Programming. Addison-Wesley Publishing Co., Reading, Mass., 1962.
17. P. B. MORICE: Linear Structural Analysis. Thames and Hudson, London, 1959.
18. CEB – Recommendations for an International Code of Practice for Reinforced Concrete. Cement and Concrete Association, London, 1965.
19. C. S. KRISHNAMOORTHY: Optimum limit design of reinforced concrete frames. Ph. D. Thesis. University of London, 1972.
20. U. NAHHAS and C. W. YU: The elastic-plastic design of reinforced concrete sway frames against instability. Proceedings, Institution of Civil Engineers, London. V. 53, Part 2, June 1972, p. 41–56.
21. J. FOULKES: The minimum weight design of structural frames. Proceedings Royal Society, London, Series A, V. 223, 1954, p. 482.
22. J. MUNRO and D. L. SMITH: Linear programming duality in plastic analysis and synthesis. International Symposium on Computer-Aided Structural Design, University of Warwick 10–14th July, 1972.

Summary

A linear programming model for the optimal design of reinforced concrete frames is presented. This formulation has constraints associated with compatibility, limited ductility, equilibrium and serviceability criteria as the governing constraints, and may or may not place restrictions on the location of plastic hinges. A computer programme that formulates and solves the optimal design problem is described. Practical design examples of six multi-storey reinforced concrete frames have been worked out. The economic merits of these examples are discussed for different cases of hinge patterns.

It is concluded that the proposed linear programming formulation presents an unified approach to the limit design of reinforced concrete frames and the method offers savings in the requirement of steel reinforcement.

Résumé

On présente un modèle de programmation linéaire pour le projet optimal de charpentes en béton armé. Cette formulation a des contraintes associées aux critères de la compatibilité, de la ductilité limitée, des équilibres et de l'utilité. Elle peut donner lieu à des restrictions quant au placement d'articulations flexibles. On décrit un programme d'ordinateur formulant et résolvant le problème optimal du projet.

Des exemples pratiques de six charpentes à plusieurs étages ont été élaborés. Les avantages économiques de ces exemples sont discutés pour différents cas d'articulation. On en tire la conclusion que la formulation proposée du programme linéaire présente une approche unifiée au projet limite de charpentes en béton armé et la méthode offre des économies dans la quantité en acier d'armature.

Zusammenfassung

Es wird ein Modell zur linearen Programmierung für den optimalen Entwurf von Tragwerken aus Stahlbeton vorgelegt. Diese Formulierung bedingt eine Zwangsläufigkeit, die an die Kriterien von Verträglichkeit, Dehnbarkeitsgrenzen, Gleichgewicht und Nützlichkeit gebunden ist. Sie kann Anlass zur Beschränkung bei der Verwendung nachgiebiger Gelenke geben. Es wird ein Computerprogramm zur Formulierung und Lösung zum optimalen Entwurfsproblem beschrieben.

Es wurden praktische Beispiele von 6 mehrstöckigen Tragwerken ausgearbeitet. Die wirtschaftlichen Vorteile werden an diesen Beispielen für verschiedene Fälle von Gelenken diskutiert. Daraus wird geschlossen, dass die vorgeschlagene lineare Programmierungs-Formulierung eine vereinheitlichte Näherung zum Grenzentwurf für Tragwerke aus Stahlbeton bietet und Ersparnisse an Armierung liefert.

Leere Seite
Blank page
Page vide