Analysis of box girder bridges of arbitrary shape

Analysis of Box Girder Bridges of Arbitrary Shape

Analyse de ponts en poutres-caisson de forme arbitraire

Analyse von Kastenträgerbrücken beliebiger Form

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Introduction

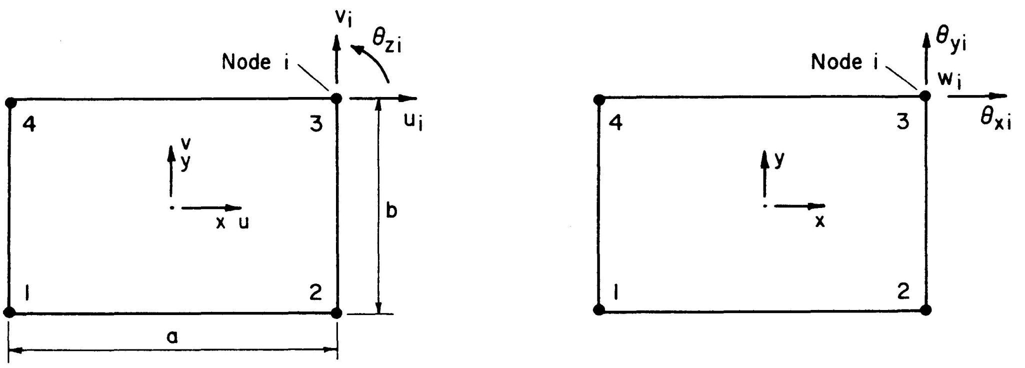
Box girder bridges are increasingly constructed for modern highways and many of them are complex in plan geometry. Several authors have analyzed some particular types of such bridges. In recent years the finite element method has been proved to be the most general and could be applied in practice at a reasonable computer cost for the analysis of skew and curved bridges of variable or constant cross section. The present paper is a review of the recent developments made by several authors and a presentation of the state of the art.

A mention should be made here of other methods of analysis not reviewed in this paper which have advantages in some cases but lack the generality of the finite element method. These are: folded plate method [14], [15], finite segment method [15] and finite strip method [4] for rectilinear right-angle bridges, finite difference method [10] for skew straight bridges, and finite strip method [3, 12] for the analysis of circular bridges with radial support.

In the finite element method a box girder bridge is idealized as an assemblage of thin plate elements as in shell structures. The sufficient degrees of freedom per node are three translations u_i, v_i, w_i in three orthogonal directions x, y and z and three rotations θ_{xi}, θ_{yi} and θ_{zi}. Almost all authors have used plate bending triangular [2] or rectangular [23] element with the degree of freedom w_i, θ_{xi} and θ_{yi} which proved to be adequate for bridge analysis. However, new in-plane elements had to be developed specially for analysis of box girder bridges and, therefore, the in-plane elements are discussed in this paper and not the plate bending elements which are well documented in papers and books [23, 24, 25]. All the developments are outlined in chronological order as follows:

Rectilinear Bridges by Scordelis [15]

In this method box girder bridges are analyzed by using rectangular in-plane element developed originally by ABU GHAZALEH [1] and rectangular plate bending element [22] with w_i, θ_{xi} and θ_{yi} as degrees of freedom at each



(a) Abu Ghazaleh's in-plane element [1] (b) Rectangular plate bending element [22]

Fig. 1. Abu Ghazaleh's in-plane element with corresponding plate bending element.

node (see Fig. 1). The in-plane element has the following degrees of freedom at each node

$$\{\delta\}_i = \begin{Bmatrix} u \\ v \\ \theta_z \end{Bmatrix}_i, \tag{1}$$

where
$$\theta_{zi} = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right). \tag{2}$$

The displacement functions used are

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \sum_{i=1}^{4} \begin{bmatrix} f_1 & 0 & f_2 \\ 0 & f_1 & f_3 \end{bmatrix} \begin{Bmatrix} u \\ v \\ \phi \end{Bmatrix}_i, \tag{3}$$

where *)
$$\begin{aligned} f_{1i} &= \tfrac{1}{4}(1+\xi\,\xi_i)(1+\eta\,\eta_i), \\ f_{2i} &= \frac{b}{8}\eta_i(1-\eta^2)(2+\xi\,\xi_i-\xi^2) f_{1i}, \\ f_{3i} &= -\frac{a}{8}\xi_i(1-\xi^2)(2+\eta\,\eta_i-\eta^2) f_{1i} \end{aligned} \tag{4}$$

*) The coordinates ξ and η are dimensionless values varying between –1 and 1 in the same manner as for the elements in Fig. 6.

and $$\phi_i = \theta_{zi} - \sum_{j=1}^{4} \left\{ \frac{\xi_j v_j (1+\eta_i \eta_j)}{4a} - \frac{\eta_j u_j (1+\xi_i \xi_j)}{4b} \right\}.$$

In this method the elements are rectangular and therefore it is applicable to rectilinear bridges only.

Multi-Cell Rectilinear Bridges by Sawko and Cope [13]

Multi-cell rectilinear box girder bridges are analyzed by representing the cells by in-plane elements alone. The transverse flexural behaviour is approximately simulated by using equivalent diaphragms (see Fig. 2). In this way the generalized six degrees of freedom at each node are reduced to three translations alone.

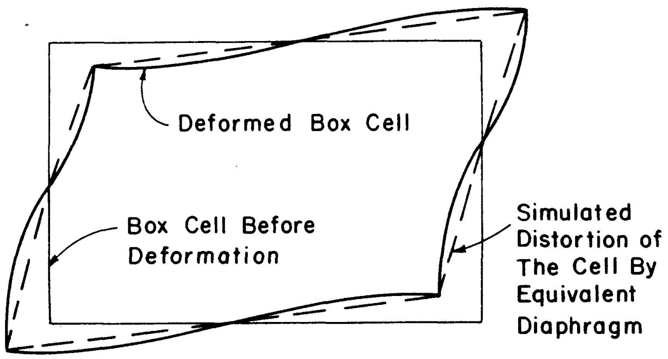


Fig. 2. Equivalent diaphragm to represent flexural distortion of a box-cell.

In this method, at all nodes compatibility of rotations are not satisfied. The method assumes one-way (transverse direction) bending. However, it can yield fast solution for bridges with narrow cells.

Skew and Curved Bridges by Sisodiya et al. [17, 19] *with Existing Elements*

The authors used existing parallelogram and triangular in-plane and bending elements (Fig. 3) to analyze skew straight or curved box girder bridges. The in-plane degrees of freedom are u_i and v_i. The displacement functions [23] for parallelogram element are

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \sum_{i=1}^{4} \begin{bmatrix} f_1 & 0 \\ 0 & f_1 \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix}_i, \tag{5}$$

where $$f_{1i} = \tfrac{1}{4}(1+\xi\,\xi_i)(1+\eta\,\eta_i) \tag{6}$$

and for triangular element [20] the functions are

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} 1 & x & y & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x & y \end{bmatrix} \{A\}, \tag{7}$$

where $\{A\}$ are the constant coefficients.

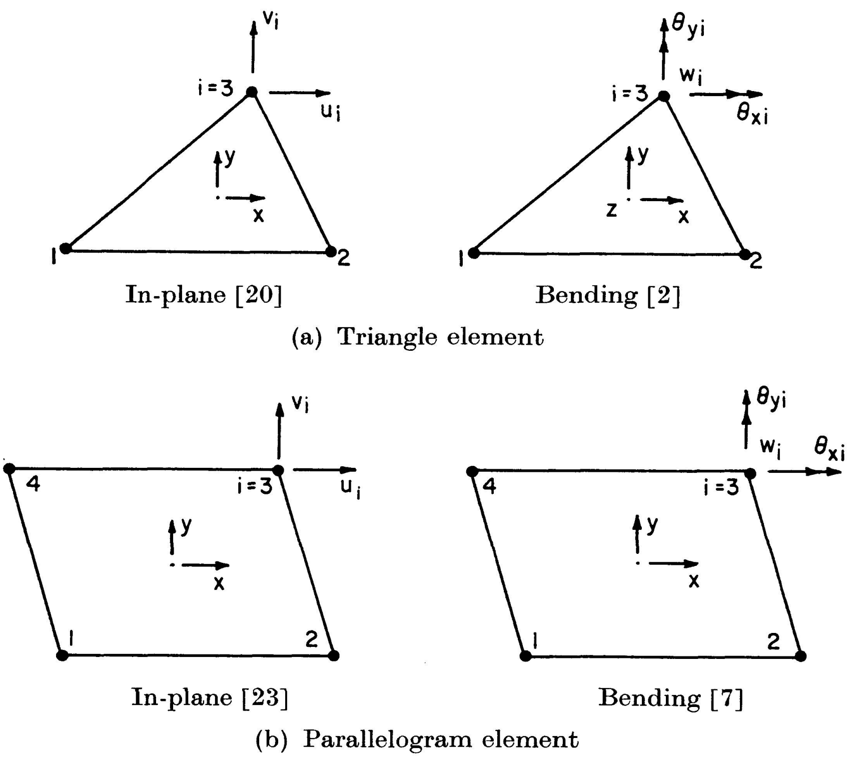


Fig. 3. Triangular and parallelogram elements.

The stiffness coefficients corresponding to in-plane rotation, θ_z, are assumed arbitrarily (with equilibrium maintained) [19].

A reasonable accuracy can be obtained with the above elements only if a large number of elements is used. The linear displacement function, Eq. (5), for quadrilateral element, even though it is widely accepted, proved [17, 19] to be excessively stiff.

William and Scordelis' Computer Program for Cellular Structures of Arbitrary Plan Geometry [21]

William and Scordelis developed finite element program to analyze box girder bridges of constant depth and arbitrary plan geometry. The analysis uses the following elements.

Elements for Top and Bottom Slabs: The slabs are idealized as quadrilaterals or triangles. The in-plane quadrilateral has translational degrees of freedom, u_i and v_i, at four corners and at the middle of the element. The element has also shear strain, ϵ_{xy}, as a nodal parameter at the middle of the element (Fig. 4a). Strains ϵ_x and ϵ_y are derived from the assumed displacement function for u and v, while ϵ_{xy} is assumed constant of same magnitude as ϵ_{xy} at middle of element. The strain energy is associated with these strains in the element.

The plate bending element used is that derived originally by CLOUGH and FELLIPA [5]. The element has 19 degrees of freedom of which internal seven

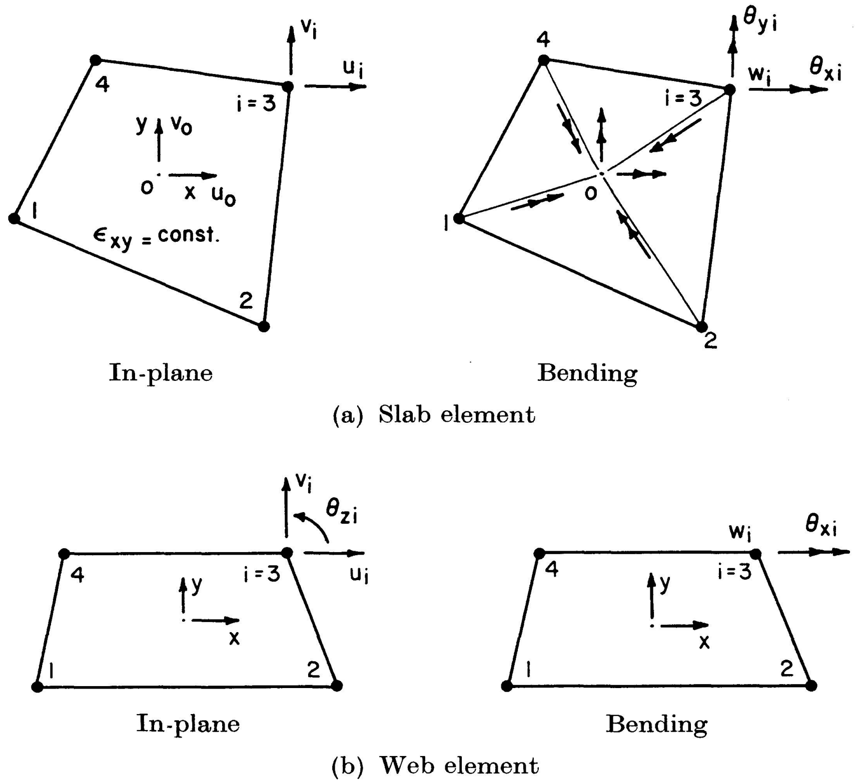


Fig. 4. William and Scordelis' elements for cellular structures [21].

degrees of freedom are eliminated in terms of three degrees of freedom at each node.

The quadrilateral in-plane and bending element degenerates into constant strain triangle and triangular plate bending element LCCT-9 [5], if first and fourth nodes of the quadrilateral coincide.

Elements for the Webs: Quadrilateral elements of different type than the elements for top and bottom slab are used for the webs.

The in-plane elements of constant depth are derived for the degrees of freedon u_i, v_i and $\left(\frac{\partial v}{\partial x}\right)_i$; while one-way plate bending element is derived for degrees of freedom w_i and $\left(\frac{\partial w}{\partial x}\right)_i$ (see Fig. 4b).

The stiffnesses of the above web and slab elements, when assembled, yield the stiffness matrix of a bridge for five degrees of freedom. The sixth degree of freedom corresponding to in-plane rotation is thus omitted.

Analysis of Multi-Cellular Structures by Crisfield [6]

Crisfield developed computer program for the analysis of multi-cell, rectilinear or skew box girder bridges. His analysis assumes symmetry about middle horizontal plane of the bridge and uses the following elements.

Slab Elements: Parallelogram [8] and triangular [9] in-plane elements with u_i and v_i as nodal parameters at corner and midside nodes are used (see Fig. 5a). The corresponding bending element is that of DAWE [7] (Fig. 3b).

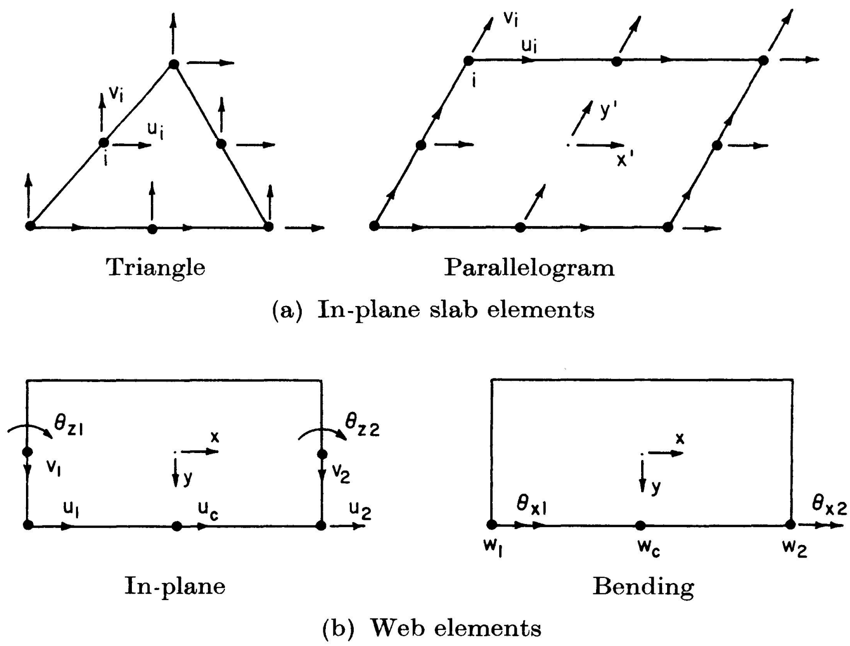


Fig. 5. Elements used by Crisfield [6].

Web Elements: Crisfield developed rectangular in-plane web elements with seven degrees of freedom as shown in Fig. 5b. The displacement functions used assume no vertical straining of the elements and symmetry about middle plan of the element. The corresponding bending element is shown in Fig. 5b.

The stiffnesses of the above elements are given in Ref. [6]. When the slab and web element stiffness is assembled, only one of top or bottom slab stiffness is added twice by assuming the symmetry. Thus the resulting stiffness matrix is for nodes along one slab (top or bottom) of the multi-cell box girder bridge. Thus there are five degrees of freedom at corner nodes and two at midside nodes of the slab elements.

Crisfield makes a further approximation by neglecting plate bending degrees of freedom. The transverse flexural effect of slabs is simulated by equivalent transverse diaphragms [13]. In this way five degrees of freedom are reduced to three translational degrees of freedom at the corner nodes.

New In-Plane Elements for Analysis of Box Girder Bridges of General Geometry by Sisodiya et al. [16, 18]

The following three in-plane elements are derived and are combined with either a parallelogram plate bending element of Ref. [7] or a quadrilateral plate bending element obtained from two triangle elements [2]. The three in-plane elements satisfy compatibility and constant strain criterions.

Parallelogram Element PLC 3 [16]: This element is derived for the nodal parameters (see Fig. 6a)

$$\{\delta\}_i = \begin{Bmatrix} u \\ v \\ \theta_z \end{Bmatrix}_i , \tag{8}$$

where

$$\theta_{zi} = \left(\frac{\partial v}{\partial x}\right)_i . \tag{9}$$

The displacement functions are

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \sum_{i=1}^{4} \begin{bmatrix} f_1 & f_4 & f_5 \\ 0 & f_2 & f_3 \end{bmatrix} \{\delta\}_i , \tag{10}$$

where

$$\begin{aligned} f_{1i} &= \tfrac{1}{4}(1+\xi\,\xi_i)(1+\eta\,\eta_i), \\ f_{2i} &= \tfrac{1}{2}(2+\xi\,\xi_i-\xi^2)\,f_{1i}, \\ f_{3i} &= -\frac{a}{4}\xi_i(1-\xi^2)\,f_{1i}, \\ f_{4i} &= 0, \\ f_{5i} &= 0. \end{aligned} \tag{11}$$

The stiffness matrix is generated by numerical integration using Gaussian quadrature formulae [11].

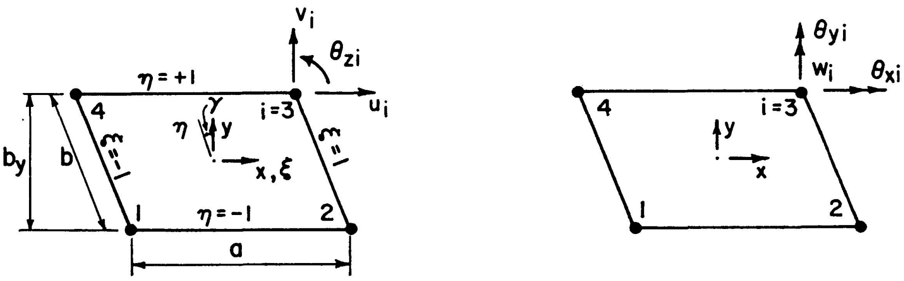
This element was derived almost at the same time as the in-plane element used for the webs by WILLIAM and SCORDELIS [21]. It seems that the two elements are similar (the explicit displacement functions of the latter are not included in Ref. [21]). Element PLC 3 is capable of analyzing skew straight box girder bridges only.

Parallelogram Element PQC 3 [18]: The above element PLC 3 when tested as parallelogram proved to be more stiff than as a rectangle (see Refs. [16] and [18]). Element PQC 3 has the shape functions same as in Eqs. (10) and (11) but with f_{4i} and f_{5i} changed as follows:

$$\begin{aligned} f_{4i} &= \frac{3\,b_y}{8\,a}\xi_i\,\eta_i(1-\xi^2)(1-\eta^2), \\ f_{5i} &= -\frac{b_y}{16}\eta_i(1-\eta^2)(1-2\,\xi\,\xi_i-3\,\xi^2). \end{aligned} \tag{12}$$

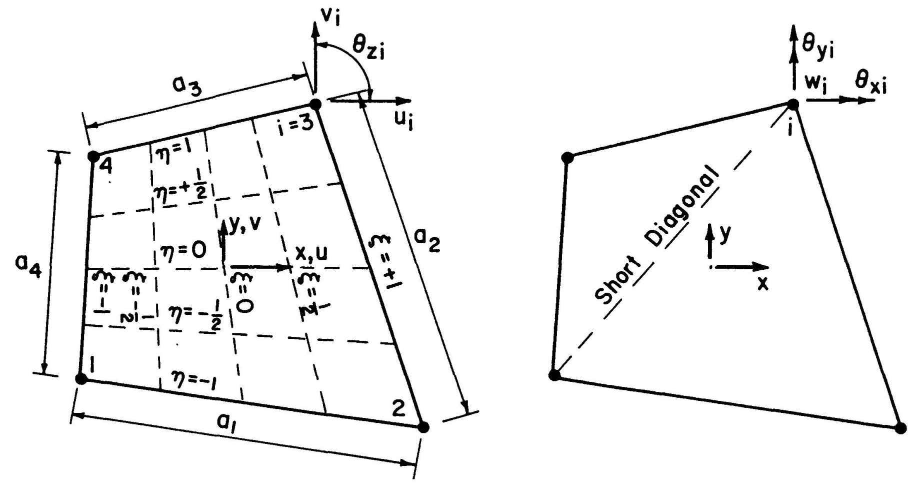
The stiffness matrix is derived using numerical integration in the same way as for element PLC 3. When the elements are rectangles, PLC 3 and PQC 3 give identical results; but as parallelograms PQC 3 demonstrates superior accuracy [18].

For the use of PLC 3 or PQC 3 in the analysis of box girder bridges, the required shell element is derived by combining each of these with Dawe's plate bending element [7] (Fig. 6a).



In-plane PLC 3 [16] or PQC 3 [16] Bending [7]

(a) Parallelogram element



In-plane FLC 3 [18] Plate Bending

(b) Quadrilateral Blement

Fig. 6. Elements for complex box girder bridges.

Quadrilateral Element QLC 3 [18]: None of the parallelogram elements PLC 3 and PQC 3 are capable of analyzing box girder bridges of variable depth or width. For such an analysis a quadrilateral element is developed. The shape functions similar to those in Eqs. (10) and (11) or with modification as in Eq. (12) cannot be used as they do not satisfy the constant strain criterion for a general quadrilateral (Fig. 6b).

The displacement functions are assumed as polynomials

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} 1 & \xi & \eta\,\eta & \xi\,\eta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \xi & \eta & \xi^2 & \xi\,\eta & \xi^3 & \xi^2\,\eta & \xi^3\,\eta \end{bmatrix} \{A\}. \tag{13}$$

The constant coefficients $\{A\}$ are eliminated in terms of the nodal parameters

$$\{\delta\}_i = \begin{Bmatrix} u \\ v \\ \theta_z \end{Bmatrix}_i, \tag{14}$$

where
$$\theta_{zi} = \left(\frac{\partial v}{\partial x_\xi}\right)_i \tag{15}$$

and x_ξ is the distance along lines of equal η from line $\xi = 0$; at a general point B, Fig. 6b.

$$x_\xi = \tfrac{1}{4}\{a_1(1-\eta) + a_3(1+\eta)\}. \tag{16}$$

The elimination of the coefficients $\{A\}$ lead to the displacements u and v expressed in the form of Eqs. (10) and (11) with the functions f_{1i}, f_{2i} and f_{3i} as follows:

$$\begin{aligned} f_{1i} &= \tfrac{1}{4}(1+\xi\,\xi_i)(1+\eta\,\eta_i), \\ f_{2i} &= \frac{1}{16}(1+\xi\,\xi_i)\{2(1+\eta\,\eta_i)(2+\xi\,\xi_i-\xi^2) - \alpha_i\,\xi_i\,\eta_i\,\eta\,(1-\xi^2)\}, \\ f_{3i} &= -\tfrac{1}{4}\,\xi_i\,\beta_i(1-\xi^2)\,f_{1i}. \end{aligned} \tag{17}$$

The coefficients α_i and β_i used in Eq. (17) are

$$\begin{aligned} \alpha_1 &= \alpha_4 = \frac{a_2 - a_4}{a_4}, \\ \alpha_2 &= \alpha_3 = \frac{a_2 - a_4}{a_2}, \\ \beta_1 &= \frac{3\,a_1 a_4 + a_1 a_2 + a_3 a_4 - a_2 a_4}{4\,a_4}, \\ \beta_2 &= \frac{3\,a_1 a_2 + a_2 a_3 + a_1 a_4 - a_3 a_4}{4\,a_2}, \\ \beta_3 &= \frac{3\,a_3 a_2 + a_3 a_4 + a_2 a_1 - a_4 a_1}{4\,a_2}, \\ \beta_4 &= \frac{3\,a_3 a_4 + a_4 a_2 + a_3 a_2 - a_1 a_2}{4\,a_4}. \end{aligned} \tag{18}$$

where a_i ($i = 1$ to 4) are the lengths of the sides of a quadrilateral.

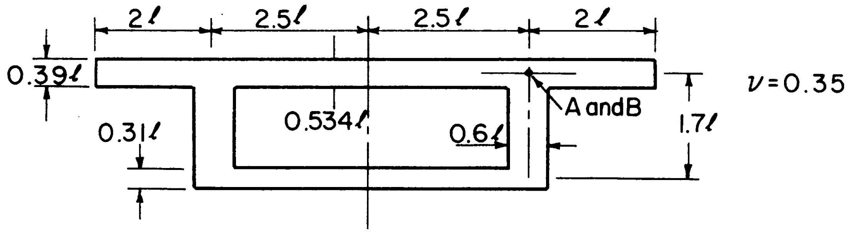
The stiffness matrix is again obtained by numerical integration using Gaussian quadrature formulae. It should be mentioned that the above shape functions in Eq. (18) become same as those in Eq. (11) when the general quadrilateral assumes the shape of a parallelogram.

A quadrilateral shell element is derived by combining QLC 3 with a plate bending quadrilateral element obtained by joining two triangles [2, 22] along the short diagonal of the quadrilateral (Fig. 6b).

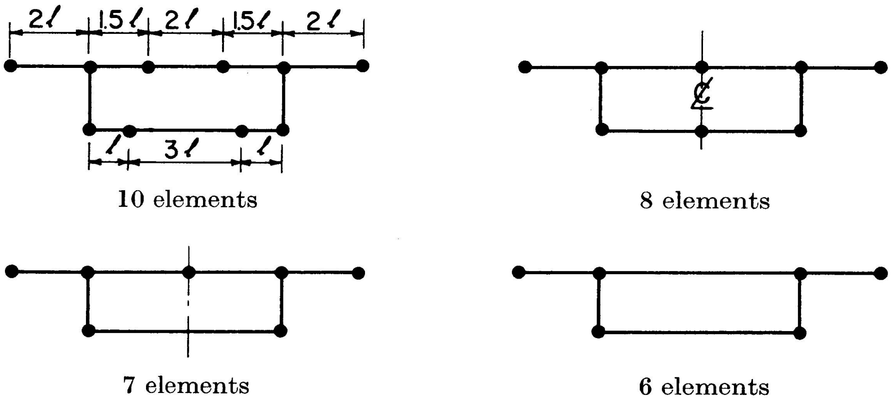
Example Analyses and Discussion of Accuracy

Two examples are included to show the accuracy of the elements of the previous section.

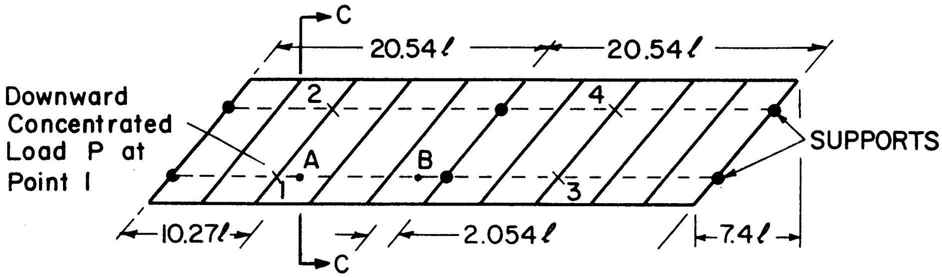
Skew Box Girder Bridge: This example in Fig. 7 is selected to compare elements PQC 3 and QLC 3 [18]. The effect of decreasing the number of elements along the cross section of the bridge is examined. The number of segments along the spans is 10 for all the cases, while the number of elements in the cross section is respectively 10, 8, 7 and 6 as shown in Fig. 7b. It should be stated here that the number of the nodes along the cross section governed the band width in this analysis. Therefore, computer time decreased to less than half by altering the number of nodes from 10 to 6 across the cross section. Some



(a) Cross-section C-C



(b) Finite element idealizatinons in cross-section



(c) Plan and the finite element division in the direction of span

Fig. 7. Straight skew box girder bridge with the constant thickness slabs between the webs.

Table 1. Analysis of the Bridge Model in Fig. 7

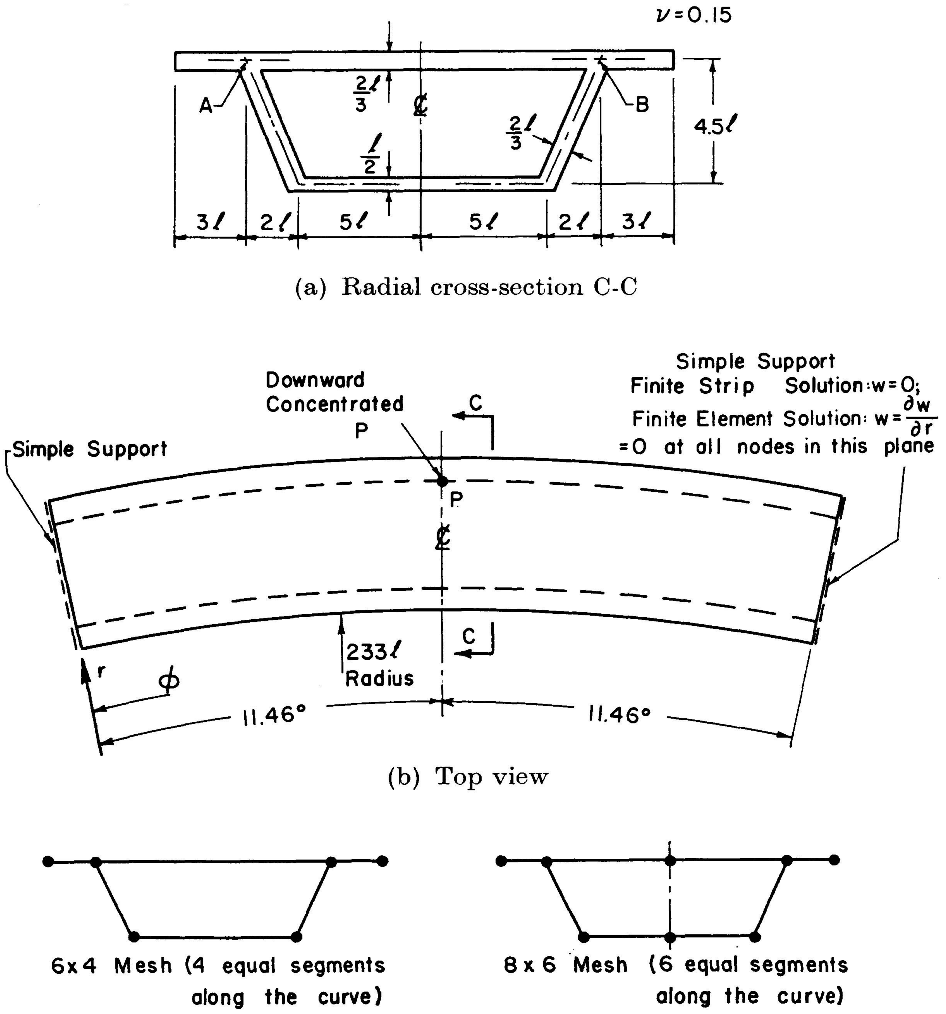
Number of Elements in Cross Section	Type of In-plane Stiffness Elements	Vertical Displacements in Terms of P/El (positive upwards)				Strain Along the Span at Top of Web in Terms of P/El^2		Reactions in Terms of P					
		1*)	2	3	4	A*)	B	5*)	6	7	8	9	10
10	PLC 3	–53.71	–11.43	6.341	4.816	–0.6482	0.2728	0.3642	0.1949	0.6700	–0.1885	–0.0341	–0.0065
	PQC 3	–53.25	–11.25	6.194	4.653	–0.6454	0.2761	0.3610	0.1981	0.6760	–0.1950	–0.0347	–0.0054
8	PLC 3	–52.07	–11.23	6.488	4.676	–0.6332	0.2637	0.3662	0.1924	0.6697	–0.1871	–0.0361	–0.0053
	PQC 3	–53.03	–11.45	6.367	4.583	–0.6279	0.2727	0.3605	0.1982	0.6749	–0.1928	–0.0370	–0.0043
7	PLC 3	–49.00	–11.05	6.724	4.282	–0.6397	0.2629	0.3758	0.1816	0.6634	–0.1783	–0.0393	–0.0033
	PQC 3	–50.62	–11.37	6.334	3.942	–0.6304	0.2631	0.3647	0.1933	0.6747	–0.1908	–0.0403	–0.0016
6	PLC 3	–47.07	–11.43	6.853	3.926	–0.5810	0.2255	0.3800	0.1764	0.6632	–0.1762	–0.0433	–0.0002
	PQC 3	–48.62	–11.72	6.356	3.374	–0.5772	0.2456	0.3673	0.1902	0.6759	–0.1908	–0.0446	–0.0019

*) The location of the points is shown in Fig. 7.

of the results are compared in Table 1 and it can be seen that element PQC 3 gives slightly better accuracy than element QLC 3. However, the latter is more general.

A Curved Box Girder Bridge: Finally, the curved box girder bridge example in Fig. 8 is analyzed. This example is chosen because its plan geometry necessitates the use of quadrilateral element and there is an existing "Finite Strip" solution [3] for comparison. Two types of element divisions are used for the present analysis Fig. 8c: 6×4 mesh and 8×6 mesh. In the 6×4 mesh the cross section is divided into 6 elements and the span of the bridge into 4 equal segments, while in the 8×6 mesh the cross section is divided into 8 elements and the span into 6 equal segments.

The finite strip solution assumes a simple support at each end of each strip. That is, a diaphragm which is infinitely stiff in its own plane but perfectly flexible out of the plane, is assumed to exist at each end cross section of the



(a) Radial cross-section C-C

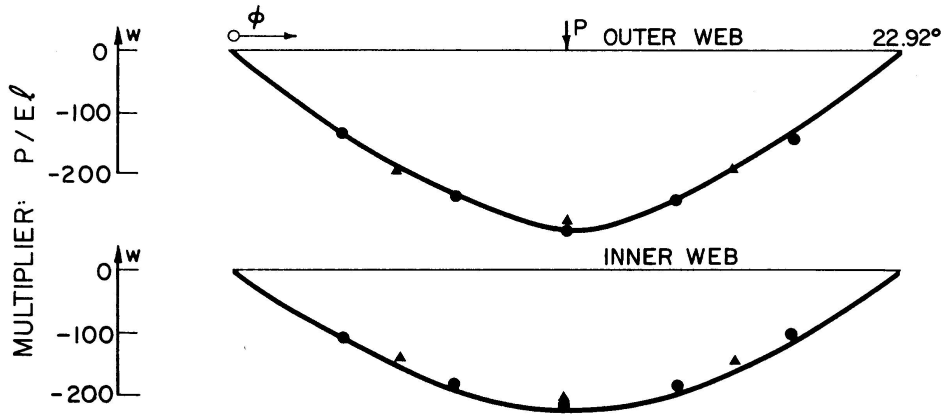
(b) Top view

(c) Finite element idealization

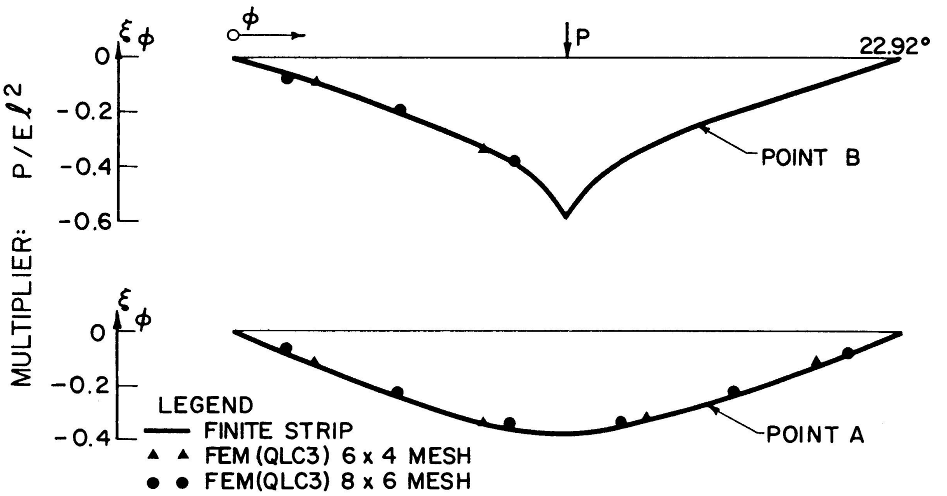
Fig. 8. Curved box girder bridge.

bridge. To take into account such a boundary condition for the finite element analysis, the displacements w and the rotations $\frac{\partial w}{\partial r}$ are zero at all nodes in the end cross section of the bridge.

The vertical displacements along the webs are plotted in Fig. 9a, while the horizontal strains along the curve at points A and B of the web, Fig. 8a, are plotted in Fig. 9b. Even with the coarse mesh (6×4) the results are of reasonable accuracy. It should be mentioned that the execution time taken by 6×4 mesh is only 2 minutes and that by 8×6 mesh is 4 minutes by the 360/50 IBM computer.



(a) Vertical deflections of the webs



(b) Strain aloug curves at points A and A (Fig. 8b)

Fig. 9 Deflections and strains in the curved box girder bridge of Fig. 8.

Conclusions

From the review of the finite element methods for analysis of box girder bridges, the following conclusions can be made.

1. The lower-order quadrilateral and triangle elements with degrees of

freedom u_i and v_i are not economical in the analysis of box girder bridges as a large number of these elements should be used for accuracy, thus requiring long computer time.

2. SAWKO and COPE [13] and CRISFIELD [6] have successfully been able to obtain solution for multi-cell box girder bridges with small computer time by eliminating some degrees of freedom. However, the approximations involved are not valid for large box cells.

3. WILLIAM and SCORDELIS [21] and SISODIYA *et al.* [16, 18] developed higher-order in-plane elements to analyze box girder bridges Williams and Scordelis used 5 degrees of freedom per node; while Sisodiya *et al.* retained all 6 degrees of freedom per node. Although the former decreases some computer time, some generality in computer programs is lost.

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Summary

The finite element method has been proved to be the most general and has been applied in practice at a reasonable computer cost for the analysis of skew and curved bridges of variable or constant cross section. This paper is a review of some recent developments made by various authors and a presentation of the state of the art.

Résumé

La méthode des éléments finis s'est révélée comme étant la plus répandue et elle a été employée en pratique à des frais d'ordinateur raisonnables pour l'analyse de ponts biais et courbes de section variable ou constante. L'article donne une revue de quelques développements récents entrepris par différents auteurs et une présentation de l'état actuel.

Zusammenfassung

Die Methode der endlichen Elemente hat sich als die meistverbreitete erwiesen und wurde in der Praxis zu vernünftigen Computerkosten für die Analyse schiefer und gekrümmter Brücken mit variablem oder konstantem Querschnitt verwendet. Der Artikel bietet eine Übersicht über einige neue Entwicklungsarbeiten verschiedener Autoren und über den gegenwärtigen Stand.