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Objekttyp: **Article**

Zeitschrift: **IABSE publications = Mémoires AIPC = IVBH Abhandlungen**

Band (Jahr): **34 (1974)**

PDF erstellt am: **23.05.2024**

Persistenter Link: https://doi.org/10.5169/seals-26286

Orthotropic Plates with Eccentric Stiffeners

Plaques orthotropes avec raidissements excentriques

Orthotrope Platten mit exzentrischen Aussteifungen

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Introduction

The application of classical flexural theory for elastic thin plate of homogeneous orthotropic material to the problem of orthogonally stiffened plates was first suggested by HUBER [1]. In the analysis, a reinforced concrete slab with different reinforcement in two orthogonal directions is represented by an equivalent orthotropic plate having the same stiffness characteristics. There are two phases to the analysis of stiffened plates by treating the equivalent orthotropic plates. The first phase is the determination of the equivalent stiffnesses and hence the governing differential equations in terms of the transverse displacement. The second is the solution of the governing equations. This latter problem for the so-called Huber equation has been dealt with in the literature for many combinations of boundary and loading conditions and will not be considered further in the following discussions.

The orthotropic plate theory based on Huber equation has since then been applied by many investigators to the analysis of grid systems and orthogonally stiffened plates. GUYON [2] was the first to apply the theory to the analysis of grid systems in which he considered only the bending rigidities. MASSONNET [3] extended the analysis of Guyon by introducing the effect of St. Venant torsion. The eccentrically stiffened slab commonly used in the construction of bridge decks was analysed by BARES and MASSONNET [4]. In their analysis the bending rigidities were determined by considering the full interaction between the slab and the stiffening grid system. However, this interaction was not taken into consideration in the evaluation of the torsional rigidity, which was taken as the simple sum of the St. Venant torsional rigidities of the slab and the grid system. As will be shown later, this is true only if the in-plane shear rigidity of the stiffening grid system is negligibly small.

In deriving the bending rigidities and hence the moment curvature relations, Bares and Massonnet assumed that the Poisson's ratio vanishes. Consequently the location of the neutral surface of the bending stresses becomes a cross-sectional constant and the complexity arising as a result of the unknown location of the neutral surface was thereby avoided. TIMOSHENKO and WOINOWSKY-KRIEGER [5] derived the moment curvature relations by considering a biaxial state of stresses for the slab and the analysis was limited to plates stiffened symmetrically. Therefore the complexity due to the unknown location of the neutral surface was similarly avoided. GIENCKE [6] considered the same influence of Poisson's ratio not only for the slab but also for the stiffening grid system. The unknown location of the neutral surface was considered in his derivation of the bending rigidities. Noting that, in the grid system, a biaxial state of stress exists only at the intersections, CUSENS, ZEIDAN and PAMA [7] modified the expressions of Giencke by considering the coupling influence through Poisson's ratio only at these intersections for the grid system.

Since no consideration is given to the interaction between the slab and the stiffening grid system in the determination of the torsional rigidity, the existing orthotropic plate theory of stiffened plates based on Huber equation is subject to an error, excepting the case where the in-plane shear rigidity of the grid system is negligibly small in comparison with that of the plate. The existence of this error can be readily demonstrated by considering an hypothetical stiffened plate consisting of two identical plates, perfectly bonded at the interface, one representing the slab and the other the stiffening grid system. While the bending rigidities suggested in the literature, when applied to the hypothetical plate, coincide with that for the single plate which is twice as thick, the torsional rigidity of the former, which is the sum of the torsional rigidities of each plate, amounts to only one fourth of that of the latter.

A theory which considers the extensibility of the middle plane of the plate which introduced additional shear stresses was first formulated by PFLUGER [8] for the treatment of buckling problems of stiffened plates. This theory was later applied to orthotropic plate problems by TRENKS [9], MASSONNET [10] and CLIFTON, CHANG and AU [11]. The governing differential equations are expressed in terms of the in-plane as well as transverse displacement components of the middle plane of the plate. The theory compensates to a certain extent the discrepancy in the torsional rigidities discussed above through the additional shear strain at the middle plane. However, the extent of this compensation depends on the geometry of the structure and the loading and boundary conditions. For example, consider the same hypothetical stiffened plate mentioned previously under the action of self-equilibriating anticlastic corner forces which subject the plate to a pure torsional stress field. It can be easily shown [12] that vanishing in-plane displacements and anticlastic transverse displacement satisfy all the governing equations and boundary conditions. The transverse displacements, which are inversely proportional to St. Venant torsional rigidity computed for the two perfectly bonded identical plates according to the equations given by MASSONNET and CLIFTON, CHANG and AU, the latter for plates with open stiffeners, are respectively four and eight times that for the single plate which is twice as thick.

In the following, orthotropic plates with orthogonally placed eccentric open stiffeners are studied, taking into account the full interaction between the plate and the stiffening system. The equivalent bending and torsional stiffnesses to be used in Huber equation are determined for plates with stiffeners placed only on one side of the slab. The resulting torsional rigidity is much larger than those presented in the literature. The corresponding result for the hypothetical stiffened plate consisting of two perfectly bonded identical plates coincides with that for the single plate which is twice as thick.

Stress Resultant-Displacement Relations

Torsion

Consider the typical element of a plate which is monolithic with two orthogonal systems of closely spaced stiffeners shown in Fig. 1. The dimensions of the element in x and y directions, a and b respectively, are assumed to be very small in comparison with the widths of the entire orthotropic plate. In order to analyze the behavior of the element under the action of twisting moment, it is first separated into two individual parts, the plate and the

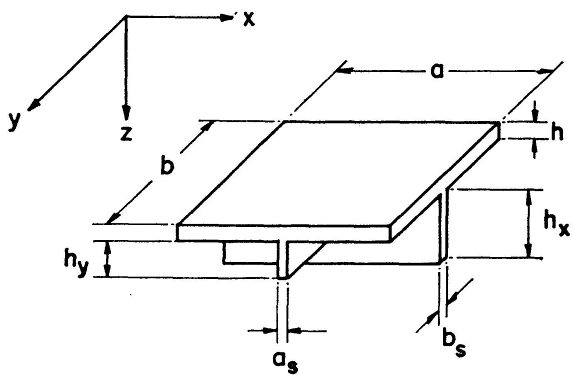

Fig. 1. Typical Element of Eccentrically Stiffened Plates.

stiffening grid system. The two parts, under suitable loading conditions, are then combined to form a monolithic structure by observing the continuity condition at the interface.

Allowing in-plane deformation of the middle surface, the small displacement theory of thin plates under Navier-Bernoulli hypothesis implies the shear strain distribution

$$\gamma_p = \left(\frac{\partial u_p}{\partial y} + \frac{\partial v_p}{\partial x}\right) - 2z\frac{\partial^2 w}{\partial x\,\partial y}, \tag{1}$$

where γ = shear strain; x, y, z = cartesian coordinates with the xy-plane located on the undeformed middle surface of the plate shown in Fig. 1; u, v, w = displacements in x, y and z directions respectively; and p = subscript denoting the plate. The shear strain at the bottom of the plate is given by

$$(\gamma_p)_{z=h/2} = \left(\frac{\partial u_p}{\partial y} + \frac{\partial v_p}{\partial x}\right) - h\frac{\partial^2 w}{\partial x\,\partial y}, \tag{2}$$

where h = thickness of the plate.

Under the action of torsional moment, the shear strain in the stiffener of narrow rectangular cross section is considerable only on planes parallel its middle plane [13]; hence for relatively deep open stiffeners as shown in Fig. 1, the shear strain at the top surface of the stiffening grid system is assumed to vanish. Hence the shear strain distribution on horizontal sections of the stiffener is constant regardless of the depth z, excepting the vicinity of the bottom edge which can be approximately corrected and the shear strain of the grid system on any horizontal section is given by

$$\gamma_s = \frac{\partial u_s}{\partial y} + \frac{\partial v_s}{\partial x}, \tag{3}$$

where u_s, v_s = displacements u and v of the centroidal axes of the stiffeners in y and x directions respectively.

Equating Eqs. (2) and (3), the continuity condition at the interface is expressed by

$$\frac{\partial u_p}{\partial y} + \frac{\partial v_p}{\partial x} - h\frac{\partial^2 w}{\partial x\,\partial y} = \frac{\partial u_s}{\partial y} + \frac{\partial v_s}{\partial x}. \tag{4}$$

This continuity condition is satisfied by the presence of suitable indeterminate forces which cause in-plane deformation of the middle surfaces of the plate and the grid system. Rearranging Eq. (4) and noting that $\partial^2 w/\partial x\,\partial y$ is regarded as constant in each element, it is obvious that $(u_p - u_s)$ and $(v_p - v_s)$ are linear functions of y and x respectively. Thus Eq. (4) can be satisfied only when the indeterminate forces subject both the plate and the grid system to pure shear fields. The indeterminate forces acting on the plate, therefore, are uniformly distributed shear stresses while those acting on the grid system take the form

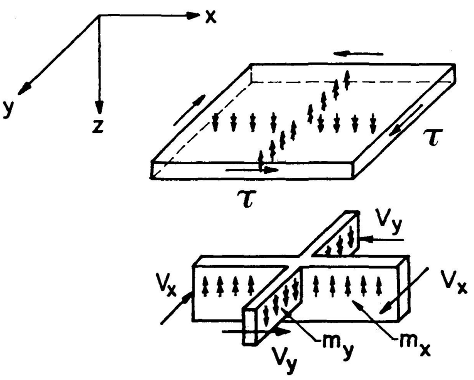

Fig. 2. Indeterminate Distributed Moments and Shears.

of edge shears and self-equilibrating uniformly distributed moments acting along the centroidal axis of the stiffeners as shown in Fig. 2. The presence of these uniformly distributed moments is necessary in order for the grid members to remain straight, i.e. to be free from bending moments. The latter is realized if

$$V_x - m_x = 0, \qquad V_y - m_y = 0, \tag{5}$$

where m_x, m_y = uniformly distributed moments per unit length about z-axis acting on stiffeners in x and y directions respectively; and V_x, V_y = shear forces acting on the stiffeners in x and y directions respectively. Since only twisting moments are applied to the monolithic structure, the equilibrium of forces in x and y directions require respectively that

$$V_x + \tau h b = 0, \qquad V_y + \tau h a = 0, \tag{6}$$

which in turn leads to

$$V_x a - V_y b = 0. \tag{7}$$

Eqs. (5) and (7) imply that the uniformly distributed moments m_x and m_y are in self equilibrium.

The stress-strain relation for an elastic material is given by

$$\tau = G\gamma, \tag{8}$$

where G = shear modulus. Since the grid elements are subjected to a pure shear field, the shear deformation leads to the force displacement relations

$$\begin{aligned} u_s &= \frac{\kappa V_y}{G A_y a} y + u_{s0}, \\ v_s &= \frac{\kappa V_x}{G A_x b} x + v_{s0}, \end{aligned} \tag{9}$$

where A_x, A_y = cross sectional areas of individual stiffeners in x- and y-directions per unit width along y and x axes, respectively; κ = a numerical factor depending on cross sectional shape, being 1.2 for rectangular section [14]; and u_{s0}, v_{s0} = displacements u_s and v_s at $y=0$ and $x=0$, respectively. For the plate, the stress-displacement relation is

$$\frac{\partial u_p}{\partial y} + \frac{\partial v_p}{\partial x} = \frac{\tau}{G}. \tag{10}$$

Solving Eqs. (4), (6), (9) and (10) yields the indeterminate shears in term of the transverse displacement w,

$$\begin{aligned}
\tau &= \frac{G}{\frac{1}{h} + \kappa\left(\frac{1}{A_x} + \frac{1}{A_y}\right)} \frac{\partial^2 w}{\partial x\, \partial y},\\
V_x &= \frac{-G b h}{\frac{1}{h} + \kappa\left(\frac{1}{A_x} + \frac{1}{A_y}\right)} \frac{\partial^2 w}{\partial x\, \partial y},\\
V_y &= \frac{-G a h}{\frac{1}{h} + \kappa\left(\frac{1}{A_x} + \frac{1}{A_y}\right)} \frac{\partial^2 w}{\partial x\, \partial y}.
\end{aligned} \tag{11}$$

The total twisting moment acting on the element shown in Fig. 1 is contributed by the plate, the stiffener and the indeterminate distributed forces acting at the middle surface of the plate and at the mid-depth of each stiffener. In view of Eqs. (1), (8) and (11) and St. Venant torsion of the stiffeners [13], the twisting moment-displacement relations are given by

$$\begin{aligned}
M_{xy} &= -D_{xy} \frac{\partial^2 w}{\partial x\, \partial y},\\
M_{yx} &= -D_{yx} \frac{\partial^2 w}{\partial x\, \partial y},
\end{aligned} \tag{12}$$

where

$$\begin{aligned}
D_{xy} &= G\left[\frac{h^3}{6} + \frac{J_x}{b} + \frac{h\left(\frac{h_x + h}{2}\right)}{\frac{1}{h} + \kappa\left(\frac{1}{A_x} + \frac{1}{A_y}\right)}\right],\\
D_{yx} &= G\left[\frac{h^3}{6} + \frac{J_y}{a} + \frac{h\left(\frac{h_y + h}{2}\right)}{\frac{1}{h} + \kappa\left(\frac{1}{A_x} + \frac{1}{A_y}\right)}\right],\\
J_x &= K b_s^3 h_x, \qquad J_y = K a_s^3 h_y,
\end{aligned} \tag{13}$$

in which M_{xy}, M_{yx} = torsional moments about x and y axes per unit width in y- and x-directions respectively, positive as shown in Fig. 3; h_x, b_s = depth

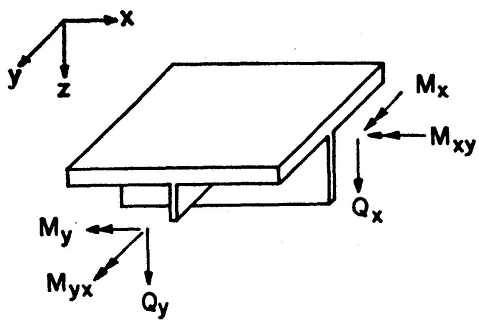

Fig. 3. Positive Directions of Stress Resultants.

and width of stiffeners in x-direction; h_y, a_s = depth and width of stiffener in y-direction; J_x, J_y = St. Venant torsional constants of individual stiffeners in x- and y-directions respectively; and K is the coefficient of the torsional constant of a rectangular section [13]. In order to approximate the continuity between the slab and the stiffener, the coefficient for a section twice the depth of the stiffeners below the slab may be used.

It should be noted that no indeterminate torsional moment develops for plates siffened in only one direction. Hence for this type of structures the third term in the bracket of Eq. (13) vanishes. This can be easily seen in Eq. (7) by equating the shear force corresponding to the non-existing stiffener equal to zero or by considering the limiting case of either A_x or A_y approaching zero in Eq. (13).

It is of interest to note that, in view of Eq. (3), no indeterminate torsional moments develop for a hypothetical structure consisting of two identical grid systems which are perfectly bonded together. Hence the torsional rigidity is given by the sum of the rigidities of each system. On the other hand, for a hypothetical structure consisting of two plates with thicknesses h_1 and h_2 bonded together, the torsional rigidity is

$$D_{xy} = G\left[\frac{h_1^3}{6} + \frac{h_2^3}{6} + h_1 h_2 \left(\frac{h_1 + h_2}{2}\right)\right]. \tag{14}$$

The torsional rigidities of these two hypothetical structures are limiting cases of Eq. (13).

Bending

Consider the element shown in Fig. 1 under the action of bending moments. The stress-strain relations for the plate are

$$\begin{aligned} \sigma_x &= \frac{E}{1-\nu^2}(\epsilon_x + \nu\,\epsilon_y), \\ \sigma_y &= \frac{E}{1-\nu^2}(\epsilon_y + \nu\,\epsilon_x), \end{aligned} \tag{15}$$

in which E = modulus of elasticity; σ_x, σ_y = normal stresses in x- and y-directions respectively; ϵ_x, ϵ_y = normal strains in x- and y-directions respectively;

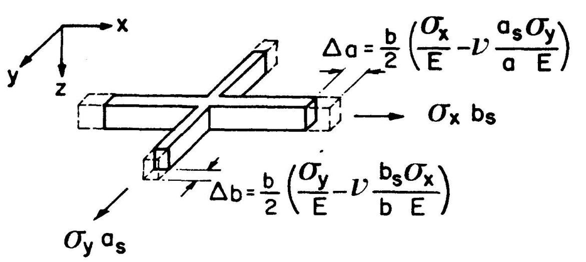

Fig. 4. Forces and Displacements of Stiffeners.

and ν = Poisson's ratio. The state of stress in the stiffeners are assumed to be biaxial at the joint between two intersecting stiffeners, and uniaxial elsewhere. Fig. 4 shows a horizontal slice of the typical element of the stiffeners of unit depth. If the stiffeners are subjected to axial forces resulting in uniformly distributed stresses σ_x and σ_y, the average strains in x- and y-directions are given by

$$\epsilon_x = \frac{\sigma_x}{E} - \nu\alpha\frac{\sigma_y}{E},$$
$$\epsilon_y = \frac{\sigma_y}{E} - \nu\beta\frac{\sigma_x}{E}, \tag{16}$$

where $\alpha = a_s/a$ and $\beta = b_s/b$. In the following analysis, it is arbitrarily assumed that $h_x \geqq h_y$. In view of Eq. (16), the stress-strain relations for the stiffeners are, for $h/2 \leqq z \leqq h/2 + h_y$,

$$\sigma_x = \frac{E}{1-\nu^2\alpha\beta}(\epsilon_x + \nu\alpha\epsilon_y),$$
$$\sigma_y = \frac{E}{1-\nu^2\alpha\beta}(\epsilon_y + \nu\beta\epsilon_x) \tag{17}$$

and, for $\frac{h}{2} + h_y \leqq z \leqq \frac{h}{2} + h_x$,

$$\sigma_x = E\epsilon_x. \tag{18}$$

Adopting the usual assumptions of small displacement and no shear deformation during bending, the normal strains are related to w by

$$\epsilon_x = \epsilon_{x0} - z\frac{\partial^2 w}{\partial x^2},$$
$$\epsilon_y = \epsilon_{y0} - z\frac{\partial^2 w}{\partial y^2}, \tag{19}$$

where $\epsilon_{x0}, \epsilon_{y0} = \epsilon_x$ and ϵ_y at $z = 0$ respectively.

The equilibrium of normal forces at the edges of the element requires that

$$\int \sigma_x dA = 0, \qquad \int \sigma_y dA = 0. \tag{20}$$

In view of Eqs. (15) through (19), solving Eq. (20) for ϵ_{x0} and ϵ_{y0} yields

$$\epsilon_{x0} = e_x\frac{\partial^2 w}{\partial x^2} - \nu e_{xy}\frac{\partial^2 w}{\partial y^2},$$
$$\epsilon_{y0} = e_y\frac{\partial^2 w}{\partial y^2} - \nu e_{yx}\frac{\partial^2 w}{\partial x^2}, \tag{21}$$

where

$$e_x = \frac{1}{A_e}[t_y s_x - \nu^2\beta(t_p + \alpha t_{x1})s_y],$$
$$e_{xy} = \frac{1}{A_e}[t_p s_y + \alpha(t_{x1}s_y - t_y s_{x1})], \tag{22}$$

$$
\begin{aligned}
e_y &= \frac{1}{A_e}\left[t_x s_y - \nu^2 \alpha (t_p + \beta t_{y1}) s_{x1}\right], \\
e_{yx} &= \frac{1}{A_e}\left[t_p s_x + \beta (t_{y1} s_x - t_x s_y)\right], \\
A_e &= t_x t_y - \nu^2 (t_p + \alpha t_{x1})(t_p + \beta t_{y1}), \\
t_p &= \frac{h}{1-\nu^2}, \\
t_{x1} &= \frac{h_y}{h_x} \frac{A_x}{1-\nu^2 \alpha \beta}, \\
t_{x2} &= \frac{h_x - h_y}{h_x} \frac{A_x}{1-\nu^2 \alpha \beta}, \\
t_{y1} &= \frac{A_y}{1-\nu^2 \alpha \beta}, \\
t_x &= t_p + t_{x1} + t_{x2}, \\
t_y &= t_p + t_{y1}, \\
s_{x1} &= t_{x1} \frac{h + h_y}{2}, \\
s_{x2} &= t_{x2} \frac{h + h_y + h_x}{2}, \\
s_x &= s_{x1} + s_{x2}, \\
s_y &= t_{y1} \frac{h + h_y}{2}.
\end{aligned}
\tag{22}
$$

It should be noted that ϵ_{x0} and ϵ_{y0} are functions of both $\partial^2 w/\partial x^2$ and $\partial^2 w/\partial y^2$ and, in view of Eqs. (19) and (21), the location of the neutral surface does not remain fixed but depends on displacement w, hence on the loading and boundary conditions. Since no axial force is present, the bending moments per unit width can be defined about any reference axis along z coordinate. Selecting this reference axis at $z=0$, the bending moments are then given by

$$
M_x = \frac{1}{b}\int \sigma_x z\, dA, \qquad M_y = \frac{1}{a}\int \sigma_y z\, dA. \tag{23}
$$

In view of Eqs. (15) through (19) and (21), Eq. (23) yield the moment-curvature relations

$$
\begin{aligned}
M_x &= -\left(D_x \frac{\partial^2 w}{\partial x^2} + D_1 \frac{\partial^2 w}{\partial y^2}\right), \\
M_y &= -\left(D_2 \frac{\partial^2 w}{\partial x^2} + D_y \frac{\partial^2 w}{\partial y^2}\right),
\end{aligned}
\tag{24}
$$

where

$$
\begin{aligned}
D_x &= E\left(I_x - s_x e_x + \nu^2 \alpha s_{x1} e_{yx}\right), \\
D_y &= E\left(I_y - s_y e_y + \nu^2 \beta s_y e_{xy}\right), \\
D_1 &= \nu E\left(I_p + \alpha I_{x1} + s_x e_{xy} - \alpha s_{x1} e_y\right), \\
D_2 &= \nu E\left(I_p + \beta I_{y1} + s_y e_{yx} - \beta s_y e_x\right)
\end{aligned}
\tag{25}
$$

and

$$
\begin{aligned}
I_p &= \frac{h^3}{12(1-\nu^2)}, \\
I_{x1} &= \frac{b_s h_y}{b(1-\nu^2\alpha\beta)}\left[\frac{h_y^2}{12}+\left(\frac{h+h_y}{2}\right)^2\right], \\
I_{x2} &= \frac{b_s(h_x-h_y)}{b(1-\nu^2\alpha\beta)}\left[\frac{(h_x-h_y)^2}{12}+\left(\frac{h+h_y+h_x}{2}\right)^2\right], \\
I_x &= I_p+I_{x1}+I_{x2}, \\
I_{y1} &= \frac{a_s h_y}{a(1-\nu^2\alpha\beta)}\left[\frac{h_y^2}{12}+\left(\frac{h+h_y}{2}\right)^2\right], \\
I_y &= I_p+I_{y1}.
\end{aligned}
\tag{26}
$$

When the width of the stiffeners are very small compared with the spacing of the stiffeners, the influence of biaxial state of stress at the intersection of the stiffeners may be neglected. The coefficients in the moment-curvature relations are then obtained by substituting $\alpha=\beta=0$ in Eqs. (22), (25) and (26).

Discussions and Conclusions

The equilibrium of forces in z-direction is prescribed by [5]

$$\frac{\partial^2 M_x}{\partial x^2}+\frac{\partial^2 M_{xy}}{\partial x\,\partial y}+\frac{\partial^2 M_{yx}}{\partial x\,\partial y}+\frac{\partial^2 M_y}{\partial y^2} = -q, \tag{27}$$

where q = intensity of distributed load. Substituting Eqs. (12) and (24) into Eq. (27) yields the governing differential equation for orthotropic plates with eccentric stiffeners,

$$D_x\frac{\partial^4 w}{\partial x^4}+2H\frac{\partial^4 w}{\partial x^2\,\partial y^2}+D_y\frac{\partial^4 w}{\partial y^4} = q, \tag{28}$$

where

$$2H = D_{xy}+D_{yx}+D_1+D_2. \tag{29}$$

The boundary conditions are the same as those given in References [3], [4], [5] for Huber equation and are omitted here for the sake of brevity. The bending moments and twisting moments are given by Eqs. (24) and (12) respectively, and the shearing stress resultants are determined, as usual, from the equilibrium conditions

$$
\begin{aligned}
Q_x &= \frac{\partial M_x}{\partial x}+\frac{\partial M_{yx}}{\partial y}, \\
Q_y &= \frac{\partial M_y}{\partial y}+\frac{\partial M_{xy}}{\partial x},
\end{aligned}
\tag{30}
$$

where Q_x and Q_y are the shearing forces per unit width in y- and x-directions respectively as shown in Fig. 3.

As pointed out earlier, the uncertainty in using Huber equation to analyse orthotropic plates with eccentric stiffeners arises mainly in the evaluation of D_{xy} and D_{yx}. A comparison of the proposed expressions for $(D_{xy}+D_{yx})$ and $2H$ with those suggested by other investigators [3], [4], [5], [6]. [7] may be of interest. For this purpose, three eccentrically stiffened slabs are chosen as examples. The spacing and depth of the stiffeners are varied while the thickness of the slab and the width of the stiffeners are kept constant. The values of $(D_{xy}+D_{yx})$ and $2H$ for $a=10h$ are plotted in terms of the torsional rigidity

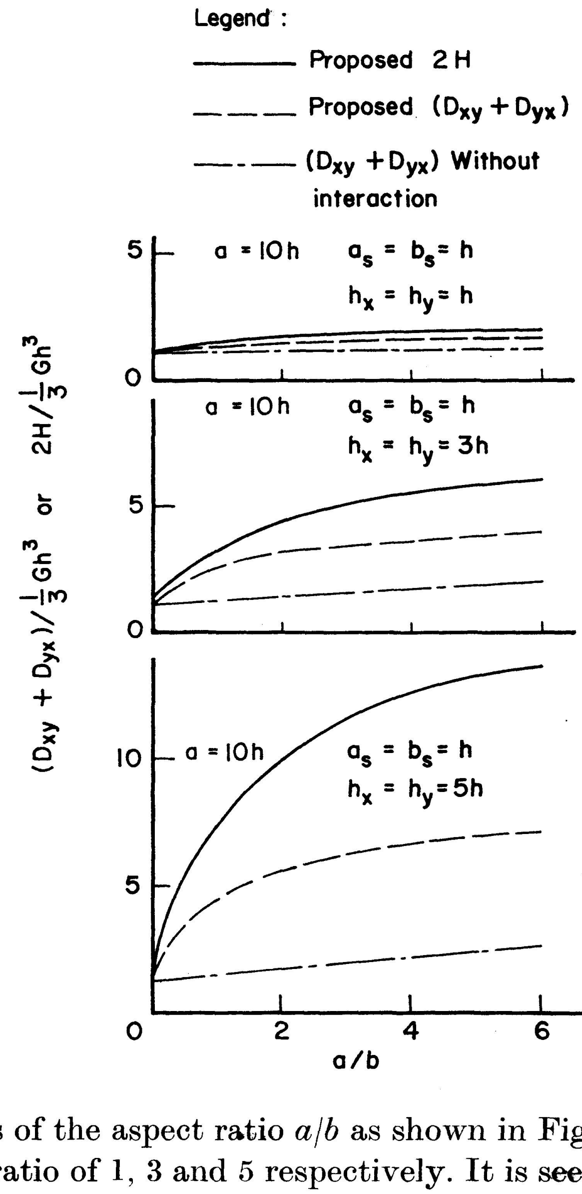

Fig. 5. Torsional Rigidity Plotted Against Arguments of a/b.

of the top slab against arguments of the aspect ratio a/b as shown in Fig. 5 for stiffener depth to slab thickness ratio of 1, 3 and 5 respectively. It is seen that the values of $(D_{xy}+D_{yx})$ obtained by summing the St. Venant terms of the stiffeners and slab only are far below the values computed by means of the proposed theory. The difference increases with increasing stiffener depth.

In Eq. (28), the term $2H$ includes the bending terms D_1 and D_2. To examine the contribution of the latter, the values of $2H$ are also plotted in Fig. 5 which shows that in some instances the contribution of D_1 and D_2 is greater than the

St. Venant terms. It should be mentioned however that the contributions of $(D_{xy}+D_{yx})$ and (D_1+D_2) are quite different depending on the loading and boundary conditions. For anticlastic loading, for example, D_1 and D_2 play no role whatsoever. The load distribution characteristics of orthotropic plates are influenced as much by D_{xy} and D_{yx}, if not more so in same instances, as by D_1 and D_2.

A theoretical analysis has been presented for orthotropic plates with eccentric stiffeners resulting in Huber type governing equation. The extensibility of the middle surface of the plate as well as the monolithic action of the plate and the stiffeners are taken into account in evaluating the elastic rigidities of the structures. It has been reported [9], [10], [15], [16] that Huber equation yields only approximate solution for the problem due to the fact that the influence of the stretching of the middle surface of the slab was neglected in evaluating the elastic rigidities and that the problem may be more rigorously described by three simultaneous differential equations of equilibrium of forces in three orthogonal directions. The solution of the latter requires very laborious calculations. The Huber type equation with the proposed rigidities derived in this study overcomes this difficulty and the proposed torsional rigidities which incorporate the interaction between the plate and the stiffeners remove, at least in part, the uncertainty [10] with regard to the torsional behavior of orthotropic plates with eccentric stiffeners.

References

1. HUBER, M. T.: Die Theorie der kreuzweise bewehrten Eisenbeton-Platte, nebst Anwendungen auf mehrere bautechnisch wichtige Aufgaben über rechteckige Platten. Bauingenieur, Vol. 4, 1923, p. 354–360, 392–395.
2. GUYON, Y.: Calcul des ponts larges à poutres multiples solidarisées par des entretoises. Annales des Ponts et Chaussées, Vol. 24, No. 5, Sept.-Oct. 1946, p. 553–612.
3. MASSONNET, C.: Méthode de calcul des ponts à poutres multiples tenant compte de leur résistance à la torsion. Publications, International Association for Bridge and Structural Engineering, Vol. 10, 1950, p. 147–182.
4. BARES, R. and MASSONNET, C.: Analysis of Beam Grids and Orthotropic Plates. Crossby Lockwood and Sons, London, 1968.
5. TIMOSHENKO, S. and WOINOWSKY-KRIEGER, S.: Theory of Plates and Shells. 2nd ed., Mc Graw-Hill Book Co., New York, 1959.
6. GIENCKE, E.: Die Grundgleichungen für die orthotrope Platte mit exzentrischen Streifen. Der Stahlbau, Vol. 24, No. 6, 1955, p. 128.
7. CUSENS, A. R., ZEIDAN, M. A. and PAMA, R. P.: Elastic Rigidities of Ribbed Plates. Building Science, Vol. 7, 1972, p. 23–32.
8. PFLÜGER, A.: Zum Beulproblem der anisotropen Rechteck-Platte. Ingenieur Archiv, Vol. 19, 1947, p. 111–120.
9. TRENKS, K.: Beitrag zur Berechnung orthogonal anisotroper Rechteck-Platten. Der Bauingenieur, Vol. 29, No. 10, 1954, p. 372–377.
10. MASSONNET, C.: Plaques et coques cylindriques orthotropes à nervures dissymétriques.

Publications, International Association for Bridge and Structural Engineering, Vol. 19, 1957, p. 201–230.
11. CLIFTON, R. J., CHANG, J. C. L., and AU, T.: Analysis of Orthotropic Plate Bridges. Journal of Structural Division, Proc. of ASCE, Vol. 89, No. ST 5, Oct. 1963, p. 133–171.
12. DEY, B.: Flexural Properties of Reinforced Concrete Ribbed Slabs. M. Eng. Thesis, Asian Institute of Technology, Bangkok, Thailand, 1972.
13. TIMOSHENKO, S. P. and GOODIER, J. N.: Theory of Elasticity. 3rd ed., McGaw-Hill Book Co., New York, 1970.
14. TIMOSHENKO, S. P.: Theory of Elastic Stability. 2nd ed., McGraw-Hill Book Co., New York, 1961.
15. American Institute of Steel Construction: Orthotropic Steel Plate Deck Bridges. American Institute of Steel Construction, New York, 1963.
16. TROITSKY, M. S.: Orthotropic Bridges Theory and Design. The James F. Lincoln Arc Welding Foundation, Ohio, 1967.

Summary

An analysis of orthotropic plates with eccentric stiffeners which leads to Huber type governing equation is presented. The extensibility of the middle surface of the plate as well as the interaction between the plate and the stiffeners in torsion are taken into account in evaluating the elastic rigidities of the structures. The proposed torsional rigidities are much larger than those currently accepted for use in design practice.

Résumé

La contribution traite une analyse de plaques orthotropes avec raidissements excentriques ce qui mène à l'équation selon M. T. Huber. L'extensibilité de la surface moyenne de la plaque ainsi que l'interaction entre la plaque et les raidissements sous torsion sont prises en considération en évaluant les rigidités élastiques des structures. Les rigidités torsionnelles proposées sont beaucoup plus élevées que celles prévues en général dans les projets.

Zusammenfassung

Der Beitrag behandelt eine Berechnung orthotroper Platten mit exzentrischen Versteifungen, welche zur Gleichung nach M. T. Huber führt. Die Ausdehnbarkeit der mittleren Oberfläche der Platte sowie die Wechselwirkung zwischen der Platte und den Versteifungen unter Torsion werden durch die elastischen Steifigkeiten der Konstruktion abgeschätzt. Die vorgeschlagenen Torsionssteifigkeiten sind weit höher als die im Entwurf für die Praxis im allgemeinen angenommenen.