Statistical concepts in aseismic design

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DISCUSSION PRÉPARÉE / VORBEREITETE DISKUSSION / PREPARED DISCUSSION

Statistical Concepts in Aseismic Design

Concepts statistiques dans les analyses paraséismiques

Statistische Methoden für den Entwurf bei Erdbeben

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INTRODUCTION

In aseismic design, a dynamic analysis is usually required to evaluate the performance of important structures subjected to earthquakes that are likely to occur at a given site. The response of a lightly damped system depends strongly on the history of recorded motions; but since the time histories of future motions corresponding to a given intensity are unpredictable, present seismic design is based on earthquake environments prescribed in the form of a smooth (maximum) response spectrum. The available rules for estimating the responses of multi-degree-of-freedom elastic systems from a given response spectrum, however, are based on heuristic arguments and limited comparisons with results obtained through integration of recorded motions for certain responses [1]*. Moreover, these rules do not give a direct procedure for taking the dispersion of the maximum response into account.

A stochastic approach to aseismic design, in principle, could provide a consistent and systematic means for designing against a set (or ensemble) of motions of prescribed intensity. With this approach, all responses of interest can be considered and explicit consideration can be given to the observed dispersions. Recognition of this fact has attracted much attention to the development of stochastic earthquake models [2], and to the formulation of approximate methods for calculating the distribution of maximum response under random earthquake-type motions [3, 4, 5, 6, 7, 8, 9].

For the stochastic approach to be of practical value, however, it is necessary to clarify the earthquake models that should be used and the technique of maximum response evaluation that is adequate for certain categories of structures. For linear structures considered herein, the results obtained thus far are quite encouraging and indicate the feasibility for practical implementation of certain random vibration results to aseismic design.

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*Bracketed numerals refer to the corresponding references cited.

DISTRIBUTION OF MAXIMUM RESPONSE

Poisson Assumption for Up-Crossings

For structures subjected to earthquake-type ground motions, the exceedance of a high response level can be approximately described as a nonhomogeneous Poisson process with an exceedance rate equal to the rate of the up-crossings. This approximation yields the survival probability as

$$P_s(t) = P\left[\left|X(\tau)\right| < b;\ o \leq \tau \leq t \right] = e^{-2\int_o^t \nu_b\ (\xi)\ d\ \xi} \tag{1}$$

with

$$\nu_b\ (\xi) = \int_o^\infty \dot{x}\ f_{X,\dot{X}}\ (b,\ \dot{x};\ \xi)\ d\ \dot{x} \tag{2}$$

in which $f_{X,\dot{X}}$ is the joint density of the random response of interest X and its derivative $\dot{X}$ at time t= ξ. The probability of exceeding the response level X = $\pm$b at least once within (0,t] is then obtained from

$$P_e(t) = 1 - P_s\ (t) = 1 - e^{-2\int_o^t \nu_b\ (\xi)\ d\ \xi} \tag{3}$$

If the response is a stationary process throughout (0,t], ν_b (ξ) becomes a constant $\bar{\nu}_b$; ignoring a small probability of premature failure, Eq. (1) becomes, in this case,

$$\bar{P}_s\ (t) = e^{-2\ \bar{\nu}_b\ t} \tag{4}$$

Eq. (4), which will be referred to as the homogeneous Poisson approximation, has been applied to the response of single-degree-of-freedom systems by a number of authors [8, 9].

For long durations and high response levels, Eq. (4) is asymptotically correct [10]; otherwise, up-crossings are correlated and the homogeneous Poisson assumption would not strictly be valid. Also, for small values of the integral, Eq. (3) gives results very close to the upper bounds in [5, 7]; however, the homogeneous Poisson assumption may not always provide an upper bound. This is particularly true for small values of t.

To examine the applicability of the Poisson assumption in practical situations of interest, the values of $P_e(t)$ for the displacement response of a linear single-degree-of-freedom system subjected to a base motion described as a Gaussian white noise excitation are compared in Fig. 1 for a system with a 2-second period and for two values of damping, β = 0.02 and 0.08. For this type of excitation the response and its derivative form a Markov vector and the numerical scheme described in [4] was used to obtain accurate results which are then compared to those obtained from the homogeneous and

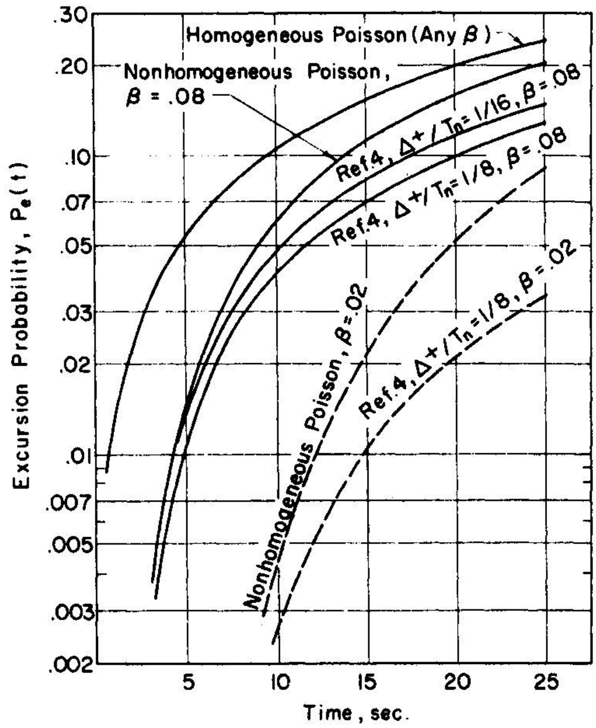

FIG. 1 COMPARISON OF CALCULATED PROBABILITIES OF EXCURSION – T_n = 2 sec., b = 3σ_o, WHITE NOISE EXCITATION

nonhomogeneous Poisson assumptions. The response level is $b = 3\sigma_o$, in which σ_o is the rms value of stationary response, i.e.

$$\sigma_o^2 = \frac{\pi S_o}{2\beta\omega_n^3}, \quad \omega_n T_n = 2\pi \tag{5}$$

It is seen that for durations of 25 seconds or so the nonhomogeneous Poisson assumption improves the results over that of the homogeneous Poisson assumption. This is particularly true for the lower damping value. Zero initial conditions were assumed; hence, the response of a 2-second system with 2% damping is quite nonstationary for the durations considered, which is the reason for the need for a nonhomogeneous Poisson procedure. For higher damping values and systems with shorter periods, the difference between the homogeneous and nonhomogeneous processes becomes less significant for durations of the order of 25 seconds.

Another measure of the reasonableness of the above Poisson process approximation can be seen in terms of the response levels corresponding to a specified probability of exceedance P_e in a given duration. For a 2-second system with β = 2% and P_e (25) = 10%, the required response levels are respectively, $3.3\,\sigma_o$, $3\,\sigma_o$, and $2.6\,\sigma_o$ for the homogeneous and nonhomogeneous Poisson approximations, and the accurate Markov scheme of calculation.

Probabilistic Response Spectra

On the basis of Eq. (1) the median pseudo velocities, $V = \omega_n b$ corresponding to P_e (25) = 0.5, computed for a Gaussian excitation with spectral density

$$G_{\ddot{Y}_1}(\Omega) = S_{o1} \frac{1 + 4\beta_{f_1}^2 \left(\frac{\Omega}{\omega_{f_1}}\right)^2}{\left[1 - \left(\frac{\Omega}{\omega_{f_1}}\right)^2\right]^2 + 4\beta_{f_1}^2 \left(\frac{\Omega}{\omega_{f_1}}\right)^2}, \quad \begin{array}{l} -\infty < \Omega < \infty \\ \beta_{f_1} = 0.642 \\ \omega_{f_1} = 15.5 \text{ sec}^{-1} \end{array} \tag{6}$$

are compared in Fig. 2 with the average pseudo velocities obtained by Housner [11]. The value of S_{o1} used to compute these results is 0.0052 ft^2/sec^3. In [11, 12], which used simulation techniques to generate member functions from Eq. (6), it was found that S_{o1} = 0.00614 ft^2/sec^3 provides a good fit with Housner's spectra. It should be remembered that when using simulation studies, the average spectra calculated do not correspond to a specific value of probability of exceedence; rather, the spectra obtained from a simulation approach are simply the averages of the observed maximum responses at each damping and frequency. Furthermore, in [12] some sensitivity of S_{o1}, required for good fit, to the time interval used for generation of artificial earthquakes was reported. In view of these observations the value of S_{o1} = 0.0052 ft^2/sec^3 which gives a good fit between median pseudo velocity spectra and the Housner spectra is

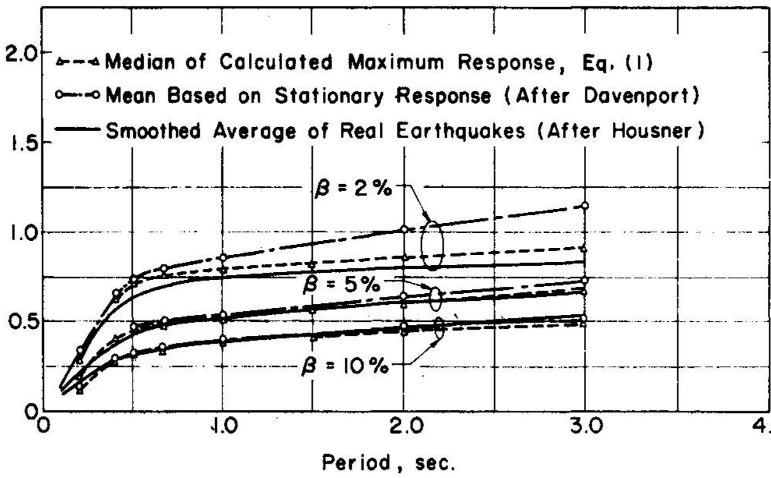

FIG. 2 SMOOTH VELOCITY SPECTRA

considered to be a good agreement between the Poisson assumption and simulation studies. Of course, by adopting the Poisson process approximation, the tedious process of simulation is avoided.

In Fig. 2 are also shown the mean response values computed from an expression due to Davenport [8], which assumes a homogeneous Poisson process. A duration of 25 seconds was used throughout.

Thus, for purposes of earthquake engineering design of linear systems, the significant maximum response statistics can be adequately determined on the basis of the nonhomogeneous Poisson process described above for a reasonably wide range of frequencies. If the excitation, system period, and damping are such that the nonstationarities in the excitation and response can both be ignored, the exceedance process may also be approximated by a homogeneous Poisson process as has been suggested previously [8, 9].

DESCRIPTION OF GROUND MOTIONS

Influence of Nonstationarity

In Fig. 4 are presented the first-passage time probability density, $f_T(t)$, and reliability function $P_s(t) = 1 - \int_0^t f_T(\tau)\, d\tau$, corresponding to the shot noise and truncated white noise inputs shown in Fig. 3. These results were determined with the computational method of [4] for a system with a 2-second period and $\beta = 0.02$; $b = 3\sigma_0$. The comparison shown in Fig. 4 is of interest because the shot noise excitation has been shown to represent quite well the nonstationarity observed in strong-motion records such as El Centro, Taft, and Olympia [14]. Moreover, it is known that long-period systems are more sensitive to input nonstationarity [12] and that for such systems the earthquake excitations can be treated as an uncorrelated process since the effective correlation time of earthquake motions is approximately 0.1 sec.

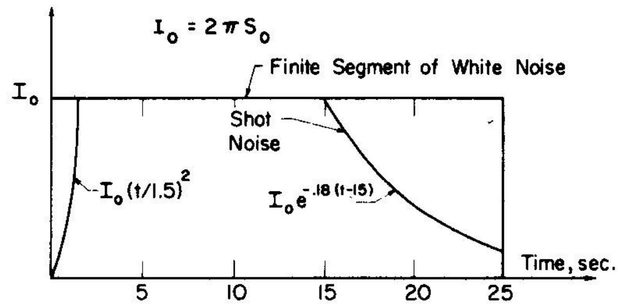

FIG. 3 INTENSITY FUNCTIONS FOR UNCORRELATED EXCITATIONS CONSIDERED

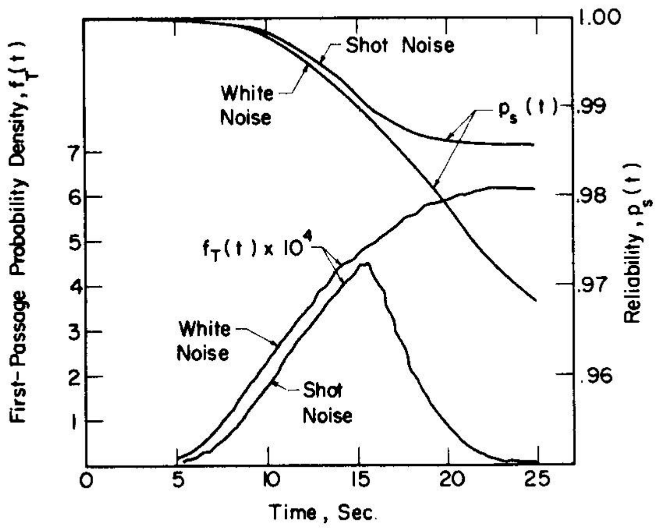

FIG. 4 FIRST-PASSAGE TIME PROBABILITY DENSITY AND RELIABILITY FUNCTIONS -- T_n = 2 sec., $\beta = 0.02$, $b = 3\sigma_0$, Excitations in Fig. 3

The comparison of the probability densities in Fig. 4 shows that the influence of input nonstationarity is controlled by the tail portion of the shot noise; therefore, for long duration responses the computed reliabilities may appreciably be in error. However, for durations of 25 sec. or so which apply to earthquakes of the type considered, the difference in the system reliabilities does not appear to be significant. This is further substantiated by the results of Table 1, which gives the response levels corresponding to an exceedance probability of $P_e(25) = 5\%$ in a 25-second duration.

TABLE 1: RESPONSE LEVELS FOR 5% PROBABILITY OF EXCEEDENCE

Elastic systems with 2% damping; duration = 25 sec.

Period, Sec.	Response Values*	
	Shot Noise	White Noise
1	3.00	3.25
2	2.70	2.85
3	2.32	2.60

* Measured in terms of σ_o, Eq. (5)

The insensitivity of the maximum response of linear damped systems to nonstationarity in the earthquakes of the type considered indicates that in specifying the earthquake environment for purposes of probabilistic design of such systems, primary attention should be given to the specification of the input spectral density.

It should be emphasized that the above conclusion is restricted to damped linear systems. Some exploratory results for elasto-plastic systems show that the effect of nonstationarity tends to increase with increasing values of the ductility factor [13].

Definition of Intensity

For dynamic seismic analysis, the earthquake environment is normally defined in terms of a smooth response spectrum; for firm ground conditions, an example would be the Housner's spectra [11]. Simulation studies [14, 15] have also shown that smooth average pseudo velocity spectra may be obtained from the spectral density of Eq. (6) or any of the following:

$$G_{\ddot{Y}_2}(\Omega) = S_{02}\,\frac{1}{\left[1-\left(\frac{\Omega}{\omega_2}\right)^2\right]^2 + 4\beta_2^2\left(\frac{\Omega}{\omega_2}\right)^2}\,, \quad -\infty < \Omega < \infty, \quad \beta_2 = 0.5, \quad \omega_2 = 3.14\ \text{sec}^{-1} \tag{7}$$

and

$$G_{\ddot{Y}_3}(\Omega) = S_{03}, \qquad |\Omega| < 62.8\ \text{sec}^{-1} \tag{8}$$

Fig. 5 shows these spectral densities with S_{oi} determined so that the density amplitudes are the same for periods ranging from 2 to 4 sec. Clearly, the shapes of these densities are not the same. However, the median pseudo velocity spectra computed for the above densities using Eq. (1), and a low damping (β = .02), are fairly close to each other as shown in Fig. 6a. This is in agreement with the findings of simulation studies and indicates that once a smooth response spectrum is selected for design, any of several proposed spectral densities may be used with Eq. (1) and the spectral density amplitude may be obtained from the specified response spectrum values.

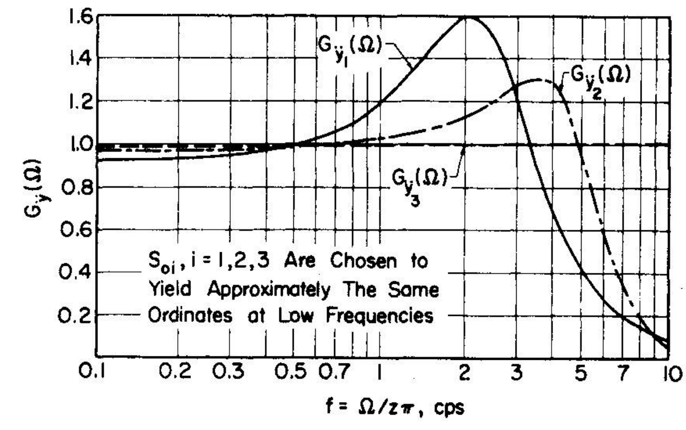

FIG. 5 NORMALIZED POWER SPECTRAL DENSITIES

Fig. 6b shows the pseudo velocity spectra obtained from the spectral densities of Eqs. (6), (7), and (8) in which the constants S_{oi} are selected so as to yield the same rms ground acceleration. These pseudo velocity spectra differ from each other.

Therefore, for purposes of consistency in practical aseismic design, the environment should be defined on the basis of a smooth design response spectrum; and from this, a corresponding spectral density can be obtained for use in the stochastic approach.

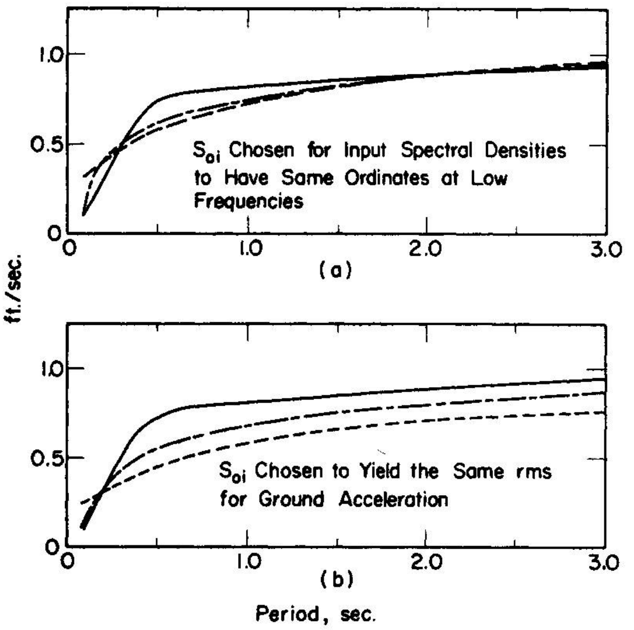

FIG. 6 MEDIAN PSEUDO VELOCITY SPECTRA CORRESPONDING TO TWO NORMALIZATIONS OF POWER SPECTRAL DENSITIES– β = 2 %

PRACTICAL IMPLICATIONS

The stochastic approach described above is intended to improve practical design against earthquake forces; the implications are especially significant with reference to the design of multi-degree-of-freedom systems as illustrated below. An important aspect of earthquake motions that seems to be neglected in present designs, or at least is not explicitly considered, is the fact that the response to actual earthquakes of the same intensity has a wide dispersion as will be illustrated in Table 2.

An Illustrative Application

For the purpose of illustrating the practical implications alluded to above, the system shown in Fig. 7 is considered and the relative story distortions are studied. The ratio k/m is selected to give fundamental mode period of 1sec., and 5% damping is used in all the modes. All the 5 modes are considered in the results reported in Table 2, which provides a comparison for three sets of calculations.

TABLE 2: RESULTS FOR SYSTEM IN FIG. 7

Response x 100. ft. (1)	Normalized Records			Random Vib.		Response Spectrum	
	Average (2)	Range (3)	Second Highest (4)	P_e=.5 (5)	P_e= 0.10 (6)	SRSS (7)	ABS (8)
U_1	353	243 to 587	442	380	434	310	380
(U_2-U_1)	321	216 - 548	402	341	389	282	319
(U_3-U_2)	283	190 - 476	333	290	330	236	292
(U_4-U_3)	221	158 - 351	248	224	251	179	246
(U_5-U_4)	133	109 - 187	152	130	145	102	159

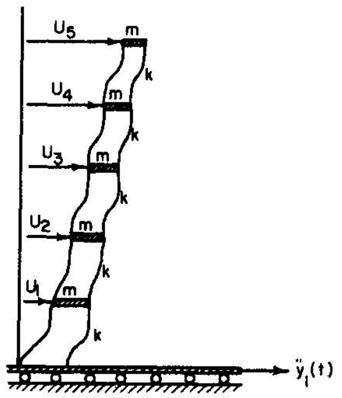

FIG. 7 SYSTEM CONSIDERED

The power spectral density of Eq. (6) is used with S_{01} = 0.0052 ft^2/sec^3. On the basis of Fig. 2, stationary response assumption is acceptable for the period and damping values considered. Assuming Gaussian response, $\bar{\nu}_b$ of Eq. (4) is

$$\bar{\nu}_b = \frac{1}{2\pi}\frac{\sigma_{\dot{X}}}{\sigma_X} e^{-\frac{1}{2}\left(\frac{b}{\sigma_X}\right)^2} \tag{9}$$

in which σ_X and $\sigma_{\dot{X}}$ are the standard deviations of the response X and its derivative $\dot{X}$, respectively. For example, if X is taken to be the second story deformation, then

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*It has been numerically verified that the coupling effect of modes can be ignored in this structure because of separation in modal frequencies.

$$\sigma_X^2 \simeq \sum_{k=1}^{5} \gamma_k^2 \left[\Phi_k(2) - \Phi_k(1)\right]^2 \int_{-\infty}^{\infty} G_{\ddot{Y}_1}(\Omega)\, H_k^2(\Omega)\, d\Omega \tag{10}$$

in which γ_k is the participation factor of mode k, Φ_k (i) is the amplitude of point i in mode k, and H_k (Ω) is the modal frequency response function. The integral in Eq. (10) is adequately approximated by the following:

$$\int_{-\infty}^{\infty} G_{\ddot{Y}_1}(\Omega)\, H_k^2(\Omega)\, d\Omega = \begin{cases} \dfrac{\pi\, G_{\ddot{Y}_1}(\omega_k)}{2\,\beta\,\omega_k^3} \; ; & \omega_k \leq 2\,\omega_{f_1} \\[2ex] \dfrac{\pi\, G_{\ddot{Y}_1}(\omega_k)}{2\,\beta\,\omega_k^3} + \dfrac{\omega_{f_1}\,(1 + 4\,\beta_{f_1}^2)}{2\,\beta_{f_1}\,\omega_k^4}\,\pi\, S_{01} ; & \omega_k > 2\,\omega_{f_1} \end{cases} \tag{11}$$

Similar simplifications are possible for $\sigma_{\dot{X}}$.

Results of such calculations are tabulated in columns 5 and 6 of Table 2, representing the median and the 90 percentile response values.

The results for the normalized records, in Table 2, were obtained through a step-by-step integration of the equations of motion for the two horizontal components of the following records: El Centro (1934, 1940), Taft (1952), and Olympia (1949). Before processing, however, the records were normalized, following Housner [11], to have the same area under the undamped spectrum curve (from T = 0.1 sec. to T = 2.5 sec.); the responses thus normalized were averaged and fitted to the Housner's spectrum with zero damping. This gave the appropriate factors by which each record was multiplied and then used to obtain eight different responses for the multi-degree-of-freedom system. The average, the observed range of each response, and the second highest among the eight values are given in columns 2, 3, and 4 in Table 2. For this structure the highest values of all responses came from the same earthquake, NS El Centro 1934, but different records gave second highest values listed for the five response quantities summarized in Table 2.

The results listed under "response spectrum," columns 7 and 8 of Table 2, were obtained using the median response spectrum of Fig. 2 with β = 5%. The absolute sum (ABS) and the square root of sum of the squares (SRSS) of the peak modal responses thus obtained are given in the columns indicated.

Discussion

For the example structure, almost all of the response U_1 (measure of base shear) is due to the first mode whereas the higher modes increase in importance for the response U_5-U_4. With this in view, the agreement between the U_1 responses in columns 2 and 5 merely indicates that a reasonable normalization of the records has been achieved since it is intended that the collection of records and earthquake model used produce roughly the same average effect on a single-degree-of-freedom system. On the other hand, the agreement between the (U_5 - U_4) values in columns 2 and 5 verifies the applicability of Eq. (1) to multi-degree-of-freedom systems.

Clearly there is a wide range in the responses of the structure to the normalized records; the proposed stochastic procedure with proper selection of P_e provides a consistent means for taking this range into account.

The values in columns 7 and 8 should be compared with those in column 5 because these are all responses to the same average base motion, described by the median response spectrum in Fig. 2. Note that the values of column 7 consistently underestimate those of column 5 whereas the values of column 8 are not consistent for all the responses in the structure.

CONCLUSIONS

The main conclusions of the paper may be summarized as follows:

1. The Poisson approximation for up-crossings, Eq. (1), provides a flexible means for obtaining maximum response statistics under random earthquake-type motion; for single-degree-of-freedom systems, the differences with theory are not large enough to be significant, Fig. 1, and the results thus obtained are in agreement with those from simulation studies, Fig. 2. For the multi-degree-of-freedom system considered, Eq. (1) appears to produce results in fair agreement with those obtained from a normalized set of recorded accelerograms.

2. Responses to a set of normalized recorded motions having the same average response spectrum, which are commonly pooled together, vary in a wide range, Table 2. Unless quantitative means become available to separate differences among these records, it is necessary to take this variability into account. A random vibration approach provides a consistent manner for implementing this goal in practice.

3. For damped linear systems, the influence of nonstationarity of the input motions analogous to El Centro, Taft, and Olympia records can be ignored, Table 1. Starting from an average smooth response spectrum it is then possible to arrive at an appropriate spectral density to be used in a stochastic procedure for design.

ACKNOWLEDGMENT

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SUMMARY

Responses to a set of normalized recorded motions vary in a wide range. It is shown that for linear systems, approximate but adequate methods are available for systematically applying a stochastic approach to aseismic design. For certain commonly used accelerograms nonstationarity of the motions can be ignored when considering linear systems; in these cases the spectral density to be used should be selected after a design intensity is defined by a smooth response spectrum.

RESUME

Le comportement d'un système soumis à un ensemble normalisé de mouvements enregistrés varie énormément. Il est démontré que, pour les systèmes linéaires, il existe des méthodes approximatives mais adéquates pour appliquer systématiquement l'approche stochastique dans les analyses paraséismiques. La non-stationarité des mouvements peut être négligée pour certains accélérogrammes largement utilisés et appliqués aux systèmes linéaires. Dans ces circonstances, la densité spectrale utilisée doit être déterminée à l'aide de l'intensité définie par une réponse spectrale lisse.

ZUSAMMENFASSUNG

Das Verhalten eines Systems gegenüber einer Reihe von beobachteten und normalisierten Bewegungen variiert über einen weiten Bereich. Es wird gezeigt, dass für lineare Systeme ausreichende Näherungsmethoden vorhanden sind, um systematisch stochastische Methoden auf den Entwurf von Konstruktionen gegen Erdbebenbeanspruchung anzuwenden. Für bestimmte, häufig verwendete Beschleunigung-Zeit-Beziehungen kann für lineare Systeme die Veränderung des Ruhepunktes der Bewegungen vernachlässigt werden. In diesen Fällen sollten die zu verwendenden Spektraldichten (PSD) gewählt werden, nachdem die Intensität durch ein ausgeglichenes Verhaltenspektrum definiert ist.

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