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II

DISCUSSION PRÉPARÉE / VORBEREITETE DISKUSSION / PREPARED DISCUSSION

Safety in Large Panel Construction

La sécurité dans la construction par grands panneaux

Sicherheit in der Großtafelbauweise

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Modern Engineering and the Safety Concept.

It has long been recognised by the engineering profession that absolute safety against all possible conditions and hazards can never be achieved. The problem is one of reducing risk, rarely, if ever, its total elimination. Indeed, one fundamental responsibility of the engineer is to achieve acceptable safety at acceptable cost.

Safety is related to both the risk and structural consequence of particular events relevant to the satisfactory behaviour of the structure. In general, past experience has shown that this combination has been adequately dealt with since few serious failures have occurred. To a large extent this has been fortuitous since the older forms of construction had an inherent strength which could cope with conditions not allowed for in design.

Modern developments in design, analysis, building material and techniques have resulted in the refinement of our structures to suit more precisely the loading and environmental conditions assumed in design. The accuracy and adequacy of these design assumptions have therefore assumed much greater importance since a precisely designed structure may be sensitive to a greater or different loading condition and the reserve strength previously available may be absent. At the same time the size of buildings has increased considerably, particularly with regard to height.

The statistical risk of any particular event occurring has probably changed but little: the structural consequences, however, may have changed radically. It is clearly no longer sufficient to assume that a structure designed for normal conditions will react satisfactorily for the abnormal or accidental condition. If we are to design our structures with both precision and safety we must make a conscious assessment of all conditions and hazards that might arise, however remote a possibility they may represent.

This does not mean that we must design against all hazards. It simply means that we should consider the combination of risk and consequence of the hazard so that appropriate action, if any, can be determined to achieve an acceptable and uniform standard of safety.

Definition of a Required Standard of Safety.

In defining a required standard of safety, two main aspects need to be considered: cost and risk to life.

Cost

Given the statistical risk of a particular cause of failure and how this risk may be varied with added or reduced cost, the cost consequence of the failure, the prevailing rate of interest and the proposed building life, it is possible to arrive at a design which represents minimum overall cost. Providing the relevant data are available this could be applied to any important building or structure. It could also be beneficially applied to less important or parts of structures.

Risk to Life.

This aspect is more difficult since emotional and political issues are raised, particularly in relation to housing, since people understandably expect to be 100% safe in their own homes. However, some comparison can be made with those risks which already exist as part of our modern way of life.

For example the risk of a person being killed on the roads is approximately 0. 7%: the risk for a person flying 10 hours each year over a period of 70 years is also appreximately 0. 7%, while the risk for a person doing, say, 300 railway journeys for each of his 70 years life is about 0. 2%.

It is not for engineers to decide what risk to life is acceptable as a basis for structural design. This is a matter for the politicians and other representatives of the community at large. The engineer, however, can and should advise on the cost and other implications associated with any desired standard of safety. Above all, the engineer should ensure that any given expenditure is used to greatest advantage.

Design Against Progressive Collapse.

Progressive collapse is defined as collapse originating and spreading from an area of local failure. Such collapse may be above, below or to the sides of the area of initial damage.

There are three ways of designing against progressive collapse: -

(a) eliminate the hazards which may lead to local failure, or reduce the risk to an acceptable value.

(b) design so that the hazard, if it occurs, does not cause any local failure.

(c) allow the local failure to take place, but design the structure so that progressive collapse does not occur.

Methods (b) and (c) above involve a quantitative assessment of the hazard, part of which is to be allowed for in design. Anything in excess of this must then represent an acceptable risk.

Possible sources of hazards in Buildings.

The first step is to consider, in terms of both the statistical risk of their occurrence and their structural consequence, the possible hazards which might lead to local failure. Many such hazards exist. In the first instance, all should be considered, however remote a possibility they may represent, as follows:-

(i) Explosions - internal and external.
(ii) Fire.
(iii) Faulty design, materials or workmanship.
(iv) Differential settlement or local foundation failure.
(v) Wind.
(vi) External impact.
(vii) Local overload.

There may be other hazards depending upon the location of the building and its intended use. For example, in some areas of the world even sabotage may need to be considered and, at the very least, the saboteur's job should not be made too easy.

In this paper, only internal explosions will be considered in depth to illustrate the intended design philosophy. Similar reasoning could be readily applied to other hazards.

Internal Explosions

The explosion risk itself falls into three parts:-

(i) the risk of any explosion occurring.
(ii) the intensity of pressure which may be reached and the period over which it will act.
(iii) the area upon which the explosion pressure will be effective.

Taking all domestic explosions into account, a total of 1889 occurred in the United Kingdom during the period 1957-1966, and very approximately, the risk of an explosion occurring from any source including domestic town gas is 12 per million dwellings in any one year. This risk is less in flats where some sources of explosion do not exist.

With regard to the intensity of pressure reached in these explosions, very little information indeed is available but some guidance can be obtained from the extent of damage which occurred. For example it is known that the damage resulting from 50% of these explosions was confined to windows or doors and of the remainder only 40% caused cracking or movement of the walls, floors or ceiling joists. Only in very few cases indeed did severe explosion damage extend into the neighbouring dwellings.

Bearing in mind that most of the dwellings involved must have been simple brick terraced housing with timber floors, the equivalent static pressure (for brick walls but not necessarily for other types of construction) would appear to be, conservatively, as follows:-

0 - 1 p. s. i.		-	50%
1 - $2\frac{1}{2}$ p. s. i.		-	30%
$2\frac{1}{2}$ - 5 p. s. i.		-	15%
5 - 25 p. s. i.	$<$	-	5%

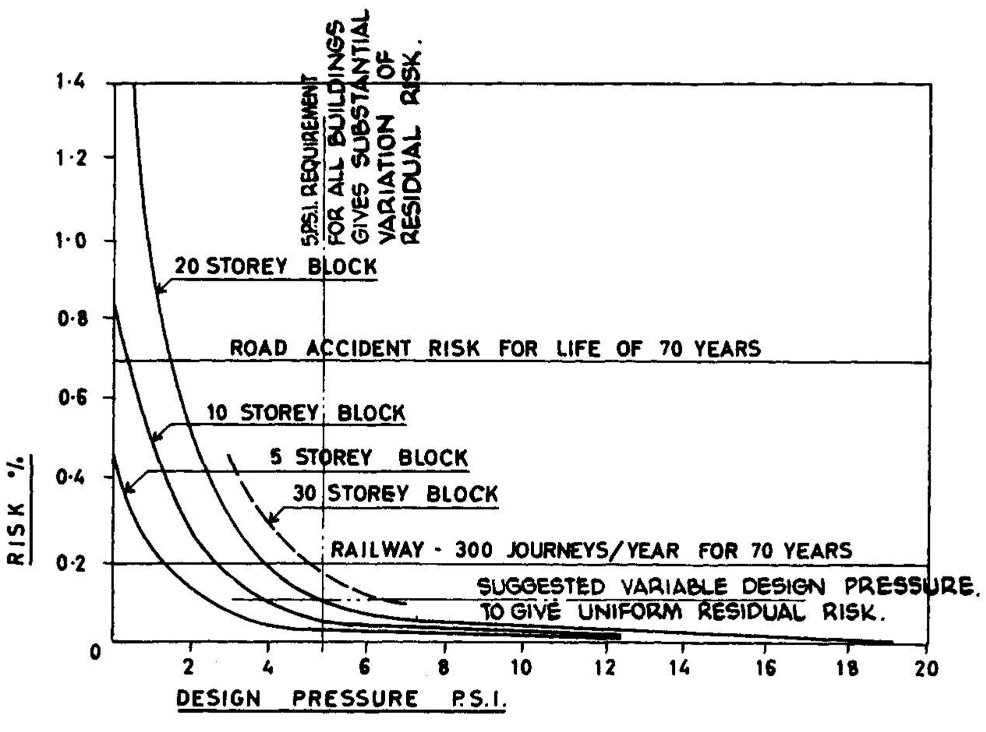
These figures are only the roughest of guides and are included here to illustrate principle only. An extreme pressure of 25 p. s. i. for town gas seems a reasonable extrapolation from information and test results related to propane explosions allowing for the venting likely to occur in domestic dwellings. A variation in explosion pressure is most likely since the probability is remote that ignition would occur precisely at the moment of worst concentration and volume of explosive mixture. Obviously these figures would need to be checked by research which should include the more careful recording and assessment, by a structural engineer, of the damage actually incurred during a number of domestic explosions.

The period of the explosion has an important bearing on the reaction of any structural component resisting the pressure, since the loading is of very short duration. The inertia of the structural member and the deflection it can sustain before failure will have an important influence on its resistance. For example a long span prestressed beam would be heavy and would also deflect a considerable distance before it failed. The time required to produce this movement may be much greater than the period of the explosion, particularly if venting can occur. In this case, a comparison between the explosion period and the period of vibration in the elastic range only would be, in the authors' opinion, erroneous and misleading. On the other hand, some structural elements can suffer only very small movement before failure and the effective pressure would then be near the peak. Load bearing brickwork would be in this category and therefore the pressure frequency referred to above probably represents an even more conservative assumption for most other types of structural elements of equivalent mass.

The area over which the explosion pressure acts is another variable about which little is known. Considering domestic dwellings supplied with town gas, the extreme case would be an explosion occurring in the whole dwelling. At the other end of the scale, the explosion would be confined to the room containing the gas appliance. In the absence of any suitable information, an arbitrary assumption regarding this has to be made taking into account that all explosions must involve at least one room and that very few, if any, involve a complete dwelling. A gradation from 1 in 1 to say 1 in 10 may be reasonable to allow for the proportion of the dwelling affected by the explosion.

Using the above reasoning and assumed pressure frequency figures, it would be possible to relate a chosen design pressure with the remaining risk of an explosion occurring giving a greater pressure. The design pressure is the basis for determining the extent of local failure for bridging purposes, or for designing to prevent local failure and if it is exceeded progressive collapse may occur.

If, in the event of progressive collapse one life is lost for each storey that collapses, it is possible to estimate the remaining risk to the inhabitants for any given design pressure. It will be seen from the accompanying diagram, that there is a very substantial reduction of risk as the design pressure is increased. If all the above assumptions are correct, and they will need to be proved by tests or other evidence, then for a 20 storey block designed to resist a pressure of 5 p. s. i. or designed to bridge over the damage resulting from a 5 p. s. i. pressure, the risk is reduced to something less, and probably much less, than 0.1%. If the risk is to be maintained at a constant figure, so that people are equally safe wherever they live, then the design pressure should be varied with the height of the building. For example, in a 5 storey building, the design pressure could be reduced to $2^1/2$ p. s. i. while in a 30 storey building the pressure should be increased to 7 p. s. i. to maintain the same level of risk.



VARIATION OF RESIDUAL RISK WITH ADOPTED DESIGN PRESSURE

BASED ON :-

1. BUILDINGS DESIGNED TO PREVENT OR TO BRIDGE OVER THE LOCAL DAMAGE RESULTING FROM THE DESIGN PRESSURE.

2. ASSUMED EXPLOSION PEAK PRESSURE FREQUENCY OF :-

0 - 1 P.S.I. = 50% : 1 - 2½ P.S.I. = 30% : 2½ - 5 P.S.I. = 15% : 5 - 25 P.S.I. = 5%

THIS DISTRIBUTION IS PROBABLY CONSERVATIVE BUT REQUIRES VERIFICATION.

THIS DIAGRAM IS TO ILLUSTRATE PRINCIPLE ONLY AND PARTICULARLY THAT, IF, THE RESIDUAL RISK IS TO BE KEPT AT A CONSTANT LEVEL THE DESIGN PRESSURE SHOULD VARY WITH THE NUMBER OF STOREYS.

Having, on the above basis, decided the design pressure, we must also decide the area over which it acts. The same diagram can be used for determining the pressure/area relationship to maintain a constant level of acceptable risk. For the purposes of illustration, let us assume that the probability of the explosion occurring in a combination of rooms in a four roomed flat, is as follows:-

1 room affected	100%	)	rooms would be defined as
2 roomsaffected	70%	)	bounded by substantial walls
3 rooms affected	30%	)	or floors, having a certain
4 rooms affected	10%	)	minimum mass.

If we consider a 20 storey block, a constant level of risk would be obtained if pressures are adopted as follows:-

1 room affected	5 p. s. i.
2 rooms affected	4 p. s. i.
3 rooms affected	2. 5 p. s. i.
4 rooms affected	1 p. s. i.

All the above relates to domestic dwellings containing town gas. The incidence of explosions in other types of building, the resulting pressures and their structural effects will all vary with the type of building, itsuse, and the size of rooms or spaces in which the explosion might occur. Other influencing factors will be the venting which might occur through the light and weak elements bounding the space and whether or not forced ventilation is provided. With adequate research and other investigations, it should be possible to allow for all these factors so that explosion ratings could be provided for use in design, as they are for fire. Such ratings should be based upon a statistical assessment of both the risk and consequence, with the objective of achieving a uniform and acceptable level of safety.

Reverting to the three ways of reducing the risk of progressive collapse to an acceptable value, as described earlier, (a) could be dealt with by consideration of venting, ventilation, or the removal of some of the sources of explosion, or a combination of the three, so that the hazard itself becomes an acceptable risk. Methods (b) and (c) could be dealt with by choosing an appropriate design pressure as already discussed.

Other Hazards.

In principle these could all be dealt with statistically using a design philosophy similar to that described above. Of greatest importance is the assessment of the sensitivity of any particular structure or part of the structure to the particular hazard being considered. This needs to be done not only for the accidental conditions but also for what would be considered as a normal loading condition.

In some cases, consideration of the hazard will involve a bridging ability, or alternative path, for the loss of a single structural element. In other cases, a combination of such elements may have to be allowed for.

Application of the Philosophy to Large Panel Structures.

Large panel structures are sensitive to the explosion hazard because the vertical load bearing elements present large areas on which the explosion pressure may act. On the other hand large panel structures can be designed and built to give massive overall strength so that overall stability is retained in spite of even severe local damage.

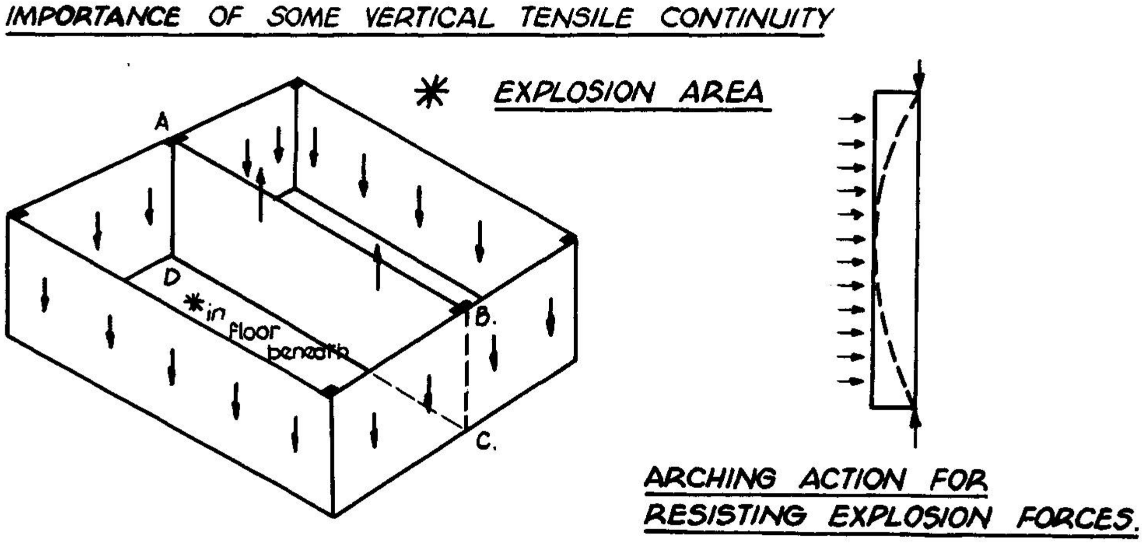
Before the application of the philosophy of design described, it is preferable to adopt a plan form which will realise the potential strength of this form of construction so that the structure is not sensitive to the loss of an individual structural member or a combination of such members.

Having chosen a suitable plan, the described design philosophy can be applied to determine which elements or combination of elements are damaged by the particular hazard.

Three particular points are worth noting:-

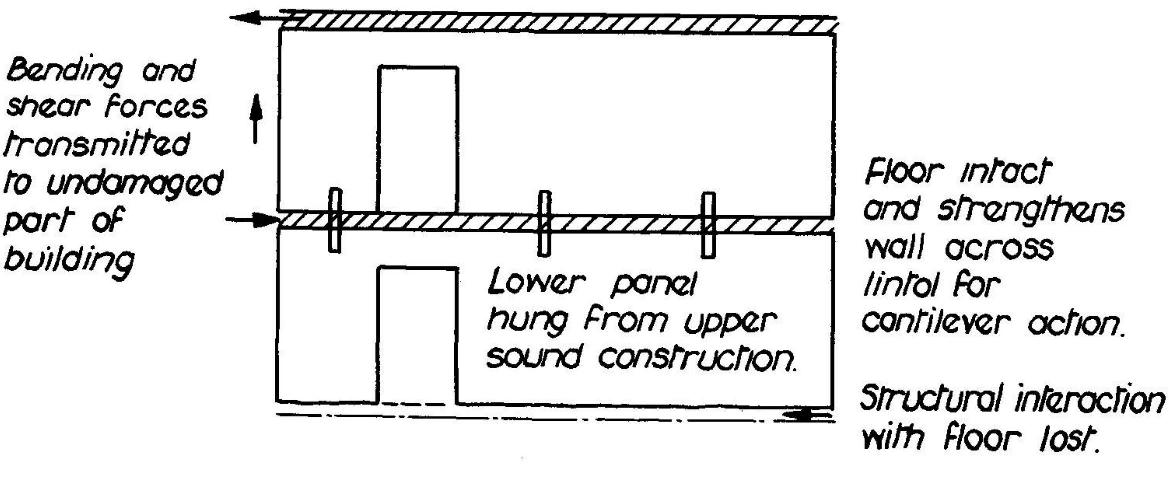
1. Local Damage.

The resistance of the wall elements is increased by a vertical arching action, which can be considerable if the load of a large portion of the structure can be gathered over the wall subjected to the pressure.

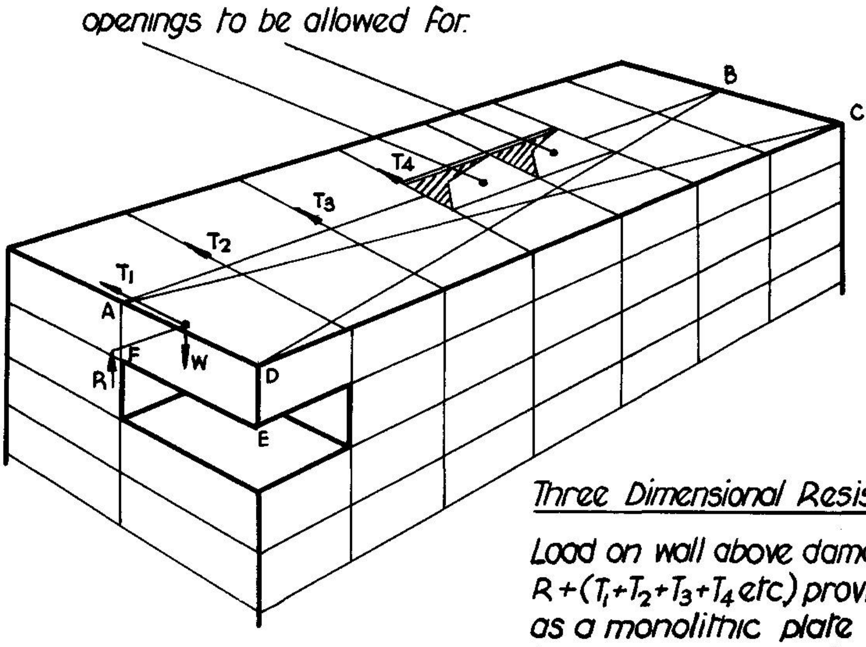


2. Bridging Action.

If the floors and walls are properly interconnected then beams of at least one storey in height can be obtained. Where openings exist, interaction between the wall and the floors at top and bottom is required. Cantilever or beam action can be developed by these composite structures. Since the floor at the level of the explosion may be damaged, it may be necessary to make provision for each wall to hang from the structure above.



It is also very helpful in assessing the building strength to take into account the three dimensional characteristics of the structure:-



Three Dimensional Resistance to Collapse.

Load on wall above damage can be resisted by $R+(T_1+T_2+T_3+T_4 \text{ etc})$ providing ABCD can act as a monolithic plate and is adequately jointed to wall ADEF. Openings in floor must be allowed for and all joints checked for required continuity.

3. Prevention of Progressive Collapse Downwards.

If local failure is permitted as in method (c) and progressive collapse downwards is to be prevented, the building must be able to withstand the impact loads from debris and other disturbances arising from the explosion area. Of primary importance is, first, the prevention of shear or bearing failure due to impact load, so that a maximum amount of the kinetic energy of the falling parts is absorbed in bending, and second, the structural interaction of components to limit the number of falling parts.

A building designed and constructed on the basis already described would almost certainly cater for any of the other hazards. Many buildings would require little or no special action. Others may require very special attention and extra cost to achieve the required level of safety. Nonetheless, in our opinion, an assessment of the hazards and their structural consequence should be made.

SUMMARY

The paper presents a design philosophy based on the assessment of the hazards and their consequential effects on the behaviour of structures. Internal explosions in buildings are taken as an example to illustrate the principles which can also be applied to other exceptional loads.

As a particular case, progressive collapse in large panel construction is treated in terms of the philosophy.

RESUME

Une philosophie de conception basée sur les probabilités de charges exceptionnelles et de leurs effets sur le comportement des structures est présentée. Le problème des explosions à l'intérieur des bâtiments est pris comme exemple pour illustrer les principes de base.

L'article traite en particulier, le cas de l'effondrement progressif dans les structures à grands panneaux préfabriqués.

ZUSAMMENFASSUNG

Dieser Beitrag zeigt ein Entwurfsverfahren unter Einschätzung des Zufalles und dessen Folgewirkung auf das Verhalten der Bauten. Um das Verfahren zu veranschaulichen, wurde als Beispiel eine innere Explosion angenommen; es können aber auch andere Ausnahmelasten berücksichtigt werden.

Als ein besonderer Fall wurde der fortschreitende Einsturz von Grosstafelbauten behandelt.

II

Probability Considerations in Design and Formulation of Safety Factors

Considérations des probabilités dans la conception des projets et dans la formulation des facteurs de sécurité

Wahrscheinlichkeitsbetrachtungen beim Entwurf und bei der Ableitung von Sicherheitsbeiwerten

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The analysis of structural safety requires a two-sided activity. On one side is the description of the loading environment and the analysis of load effects; on the other side, we have the description of material properties and the prediction of structural capacity. These may be referred to, respectively, as "stress analysis" and "strength analysis". Results of these analyses then form the basis for design. It is in the consideration of safety and serviceability that the results of stress and strength analyses become meaningful.

Except for the simplest cases, however, the analysis of the loading and its associated load effects, and the analysis of structural capacity necessarily entails a number of factors whose influences on the accuracy of the design calculations are difficult, if not impossible, to assess. Such factors as the unknown inaccuracies arising from the idealization of the loading function and structural system, the assumptions underlying all analyses and failure prediction formulas, and the unknown variances of construction and fabrication, are indeed difficult to evaluate. These difficulties are compounded by the fact that loads and material properties are generally statistical variables; moreover, available data are invariably limited such that estimates of the required statistical parameters are approximate at best. Thus, even if statistical information can be modeled with probability concepts, the difficulties associated with the unknown uncertainties cited above and the general lack of data to properly evaluate the necessary parameters, still remain. That is, the use of statistical and probability models cannot circumvent the above difficulties. These uncertainties can only be treated subjectively through the exercise of engineering judgments, which may be in the form of multiplicative or additive factors. Alternatively, such judgments may be expressed in the form of judgmental probabilities; this serves to express the unknown uncertainties in terms of subjective probabilities, which are, however, unfamiliar and thus confusing in general to engineers at this time. Nevertheless, in appropriate situations, such judgmental probabilities may be a suitable alternative to the conventional form of expressing engineering judgment.

$$P(R < R_p) = p$$

$$P(S > S_q) = q$$

The basic requirement for safety against a specified limit state is then expressed in terms of the characteristic values as,

$$R_p = \gamma\ S_q \tag{1}$$

where R_p is the required structural capacity, and γ is the overall safety factor. In more general terms, γ is composed of several components γ_m, γ_s, γ_c, called the "partial safety factors" [1]. The safety factor $\gamma > 1.0$ (or its constituent partial safety factors) is necessary to take account of the unknown uncertainties and other considerations, as well as the influence of statistical variabilities (e.g., for steel $\gamma_m = 1.15$ whereas for concrete $\gamma_m = 1.50$ are the recommended values of CEB on the grounds that concrete has a wider statistical dispersion of strengths than steel).

It might be observed that using the probability-based nominal values R_p and S_q, the major influences of statistical variabilities have already been accounted for through Eq. (1); on this basis, the calculated design resistance will increase with the degree of statistical dispersion even if the same value of γ were used. The use of larger values for γ in situations where large dispersions are expected must, therefore, be to take care of the eventualities of encountering $R < R_p$ and/or $S > S_q$. These eventualities can and ought to be treated in the context of probability; i.e., the influence of statistical variabilities on γ can be evaluated objectively.

Classical Reliability Theory

Much has been written on the classical reliability theory, beginning with the early papers of Freudenthal [4], Pugsley [5], and Prot and Levi [6]. However, it should be emphasized that relative to structural safety, the classical reliability theory is predicated on the tacit assumption that the statistical distributions of the loading and structural resistance are known precisely, and that there are no other imponderables and uncertainties in the analysis of structural safety. In the premise of the classical theory, structural safety becomes solely a problem of determining the risk associated with the statistical variabilities of the load and strength. The safety of a structure is then measured by the "probability of survival" or reliability, and conversely the "probability of failure" is the calculated risk against an unsatisfactory performance or collapse. That is, if the random load (or load effect) is S, and the structural resistance is R, then assuming no other effects, failure can be defined as the occurrence of the event $(R < S)$; accordingly, in general terms, its probability is

$$p_f = P(R < S) = \int_0^\infty F_R(s) f_S(s)\ ds \tag{2}$$

where F_R and f_S are, respectively, the distribution and density functions of R and S. This can be calculated simply if R and S are both normal random variables; i.e., Other considerations in design must include the importance and projected use of a structure, and the possible consequences in case of damage or collapse. Also, when treating combined loadings, consideration must be given to the reduced likelihood of encountering two or more extreme loads at the same time.

MODERN BASES OF STRUCTURAL SAFETY

The basic concepts underlying two modern approaches to structural safety are reviewed briefly below: these are namely, the limit-state approach [1]* which is the basis of the European Concrete Committee recommendations for safety [2], and the classical reliability theory [3]. The classical reliability theory offers the correct rationale for the treatment of statistical variables in structural safety consideration, whereas the limit-state format offers the necessary flexibility to account for unknown uncertainties and the simplicity required for conventional design implementation. These features can be combined in a consistent and logical manner to yield a formulation which retains a basic simplicity necessary for practical implementation. This review is presented, therefore, to identify the technical advantages and shortcomings of these methods, for the purpose of showing that capitalizing on the best features of each of these two methods, a third method emerges which is tantamount conceptually to a generalization of the reliability theory incorporating the basic format and intent of the limit-state approach.

Limit-State Approach

Loads and structural material properties are often statistical variables, such that there is no single load nor structural capacity that can be used in design without some risk of encountering some unfavorable state of performance, including collapse, because the no-risk load would be excessively too high whereas the no-risk capacity may require an absurdly massive structure. For purposes of design, it is therefore sensible to specify nominal values of loads and structural capacities on the basis of finite probability levels. In this regard, the consideration of safety would dictate that the nominal value for resistance must be on the low side, whereas the corresponding value of the load must be on the high side of the respective ranges of possible values. This observation naturally leads to the conclusion that the most appropriate nominal values are the "characteristic strength" and "characteristic load" as defined in the limit-state approach.

In general, the characteristic resistance and characteristic load are R_p and S_q which, for normal variates, are

$$R_p = \overline{R}(1 - k_p\delta_R)$$

$$S_q = \overline{S}(1 + k_q\delta_S)$$

where: $\overline{R}$ and $\overline{S}$ are the mean resistance and mean load (or load effect),

δ_R and δ_S are the coefficients of variation of R and S,

k_p is the number of standard deviation σ_R between R_p and $\overline{R}$,

k_q is the number of standard deviation σ_S between S_q and $\overline{S}$.

In more general terms, R_p and S_q are values corresponding to specified probability levels, and can be defined as follows:

* Number in brackets corresponds to reference cited.

$$p_f = \frac{1}{\sqrt{2\pi}} \int_{\frac{\bar{R}-\bar{S}}{\sqrt{\sigma_R^2 + \sigma_S^2}}}^{\infty} e^{-\frac{1}{2}x^2}\, dx \tag{2}$$

which is easily evaluated using tables of normal probabilities. The probability of survival, or reliability, then is simply

$$p_s = 1 - p_f$$

For specified probability distributions, the probability of failure p_f is related to the safety factor γ, as defined in Eq. (1). For example, if R and S are normal variates, it is clear from Eq. (2) that,

$$\frac{\bar{R} - \bar{S}}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \Phi^{-1}(1 - p_f) \equiv k_{pf}$$

where $\Phi^{-1}(1 - p_f)$ is the value of the standard normal function at a cumulative probability of $(1 - p_f)$. From this equation, we obtain,

$$R_p = \frac{1-k_{pf}^2\delta_S^2}{1-k_{pf}\sqrt{\delta_R^2 + \delta_S^2 - k_{pf}^2\delta_R^2\delta_S^2}} \left(\frac{1 - k_p\delta_R}{1 + k_q\delta_S}\right) S_q$$

and the safety factor, therefore, is

$$\gamma = \frac{1-k_{pf}^2\delta_S^2}{1-k_{pf}\sqrt{\delta_R^2 + \delta_S^2 - k_{pf}^2\delta_R^2\delta_S^2}} \left(\frac{1 - k_p\delta_R}{1 + k_q\delta_S}\right)$$

where k_{pf} is the number of standard deviation $\sqrt{\delta_R^2 + \delta_S^2}$ that $(\bar{R} - \bar{S})$ is above zero, such that the failure probability is equal to p_f.

Clearly, therefore, k_{p_f} is a function of p_f; for example from tables of normal probabilities, the values of k_{p_f} for specific values of p_f are given in the second column of Table 1, from which we obtain the safety factors given in the third and fourth columns of the same table.

TABLE I: VALUES OF γ WITH $p = 0.10$, $q = 0.01$

p_f	k_{pf}	γ for $\delta_R=.15$, $\delta_S=.20$	γ for $\delta_R=.20$, $\delta_S=.20$
10^{-3}	3.090	1.206	1.467
10^{-4}	3.719	1.466	2.152
10^{-5}	4.265	1.788	3.656
10^{-6}	4.753	2.215	10.509

The safety factors formulated using other distribution functions for R and S can be similarly evaluated; as shown in Fig. 1, the safety factor γ varies widely for a given p_f depending on the assumed distribution function.

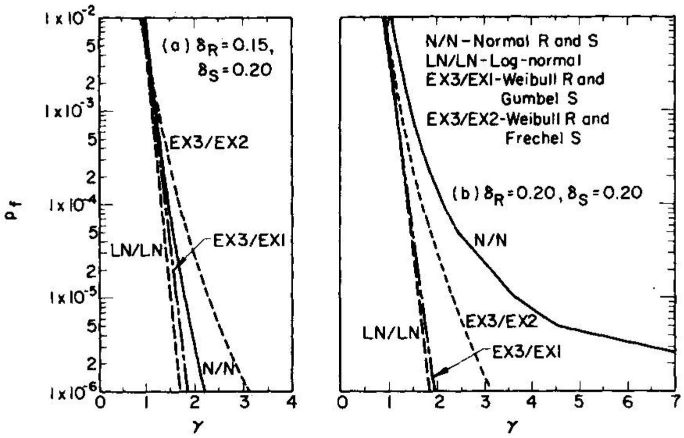

FIG. 1 SENSITIVITY OF γ TO DISTRIBUTION FUNCTIONS (CLASSICAL RELIABILITY BASIS)

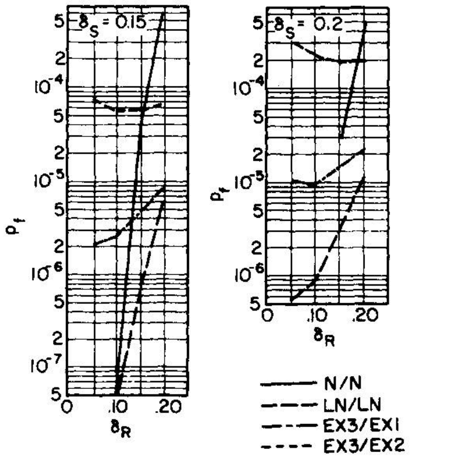

FIG. 2 INFLUENCE OF DISTRIBUTION FUNCTIONS ON PROBABILITIES OF FAILURE

There are other formidable difficulties and limitations associated with the classical reliability theory relative to its practical design implementation; practical design situations are invariably shrouded with many uncertainties and unknowns, not all of which are necessarily statistical or probabilistic. These difficulties have been emphasized by Freudenthal [7], which we quote as follows:

1. "the existence of non-random phenomena affecting structural safety which cannot be included in a probabilistic approach,"
2. "the impossibility of observing the relevant random phenomena within the ranges that are significant for safety analysis, and the resulting necessity of extrapolation far beyond the range of actual observation,"
3. "the assessment and justification of a numerical value for the 'acceptable risk' of failure, and"
4. "the codification of the results of the rather complex probabilistic safety analysis in a simple enough form to be usable in actual design."

It should be emphasized that the first three difficulties quoted above are especially significant because in the range of failure probabilities that may be considered acceptable (10^{-4} to 10^{-6} or less) the calculated probabilities of failure are extremely sensitive to the underlying distribution functions of R and S, as illustrated in Fig. 2. These distributions, however, are most difficult to ascertain because of the general lack of data. As expected, this sensitivity is reflected also on the design obtained from a specified failure probability [8], as well as on the safety factor, as shown in Figs. 1 and 3.

In summary, the classical reliability concept is an idealized theory based on assumptions and requirements that are not tenable in practice. Nevertheless, it is a sound and necessary formalism for any rational analysis of structural safety.

EXTENDED RELIABILITY CONCEPT

Basic Principles

From the above review, we recognize that it is desirable to have a method that can overcome the shortcomings but that would retain the rationality of the reliability concept, and possesses the practical flexibility of the limit-state approach. Such a method also should not be too sensitive to the distribution functions of the statistical variables but should reflect the influences of the major statistical variabilities through certain key quantities such as the means and variances (or coefficients of variation) without necessarily knowing the precise underlying distributions. A method developed on the basis of the "extended reliability concept" [8] comes close to fulfilling all of these requirements.

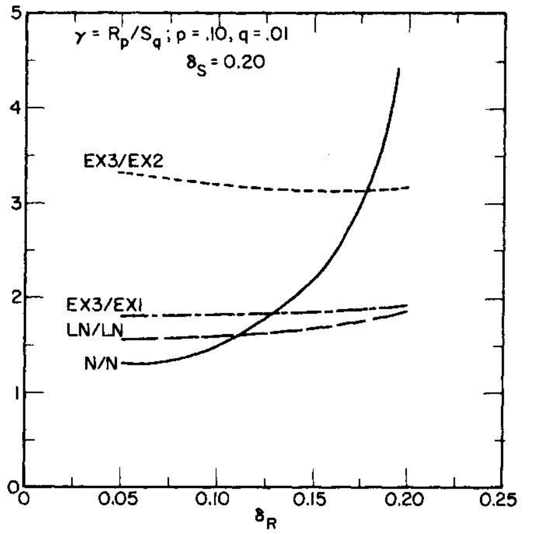

FIG. 3 VARIATION OF γ WITH δ_R BASED ON CLASSICAL RELIABILITY ; $p_f = 10^{-6}$

Following the basic format of the limit-state approach, the unknown uncertainties are covered by a nominal requirement in terms of characteristic values,

$$R_p = \nu S_q \tag{3}$$

where ν is a "judgment factor," and is necessarily greater than 1.0 to take account of the unknown uncertainties. The factor ν must be determined using engineering judgment in much the same way that the γ-factor is chosen in the limit-state approach. However, in contrast to the factor γ, the factor ν does not include the influence of known statistical variabilities.

Since ν is in reality an ignorance factor, $(R < \nu S)$ must represent a state of unsatisfactory performance or unsafety; therefore, by requiring Eq. (3) alone, the safety of a structure may still be jeopardized if $R < \nu S$, which will occur primarily when $R < R_p$ or $S > S_q$. The logical measure of the occurrence of such eventualities is the probability $P(R < \nu S)$, which can be called the "probability of unsafety", and is clearly a generalization of the classical failure probability. Hence, an additional requirement for structural safety must be,

$$P(R < \nu S) \leq \alpha \tag{4}$$

where α is a small probability necessary to insure that the occurrence of $R < \nu S$ is sufficiently rare.

In other words, Eq. (3) is a nominal requirement for safety; however, with this nominal requirement imposed, the remaining question is: "In view of statistical variabilities, what is the reliability of this nominal design against these latter eventualities?" This reliability is measured by $P(R < \nu S)$, and Eq. (4) accordingly serves to assure a required level of this reliability. Clearly, if R and S are both deterministic, then Eq. (3) is sufficient; whereas, known statistical variabilities should be treated with probabilistic models and Eq. (4) is the appropriate model for this purpose consistent with Eq. (3).

Significantly, it turns out that if the nominal requirement, Eq. (3), is imposed, the risk or probability of unsafety is bounded [8] as follows:*

$$pq < P(R < \nu S) < (p + q - pq) \tag{5}$$

It might be emphasized again that $(R < \nu S)$ will occur primarily when $R < R_p$ or $S > S_q$; but the first part of Eq. (5) says that the probability of such an occurrence is greater than pq. Hence, if Eq. (3) is required, there is no point in specifying the acceptable probability α to be less than pq.

In view of the minimum possible value of the probability of unsafety indicated in Eq. (5), the two requirements for structural safety, i.e. Eqs. (3) and (4), can both be satisfied by the following single requirement:

$$P(R < \nu S) = \alpha; \qquad \text{with } \alpha \nless pq \tag{6}$$

Thus, Eq. (6) is the desired basis for design, and the evaluation of safety factors in design. It can be observed that Eq. (6) is similar to the safety requirement of the classical reliability approach. In fact, the probability of unsafety reduces to the classical failure probability if $\nu = 1.0$. In this case it is significant to observe that Eq. (5), which remains valid, means that if $R_p = S_q$ is nominally required the associated probability of failure is also bounded as $p_f > pq$.

Formulation of Safety Factors

Through Eq. (6), specific design formulas can then be derived for given distribution functions of R and S [8]. For example, if R and S are normal variates, Eq. (6) yields

$$R_p = \nu \frac{1 - k_\alpha^2 \delta_S^2}{1 - k_\alpha \sqrt{\delta_R^2 + \delta_S^2 - k_\alpha^2 \delta_R^2 \delta_S^2}} \left(\frac{1 - k_p \delta_R}{1 + k_q \delta_S}\right) S_q$$

and the requisite safety factor, γ of Eq. (1), therefore is

$$\gamma = \nu \frac{1 - k_\alpha^2 \delta_S^2}{1 - k_\alpha \sqrt{\delta_R^2 + \delta_S^2 - k_\alpha^2 \delta_R^2 \delta_S^2}} \left(\frac{1 - k_p \delta_R}{1 + k_q \delta_S}\right) \tag{7}$$

where $k_\alpha = \Phi^{-1}(1-\alpha)$. It might be emphasized that δ_R is the overall measure of variation of the appropriate resistance, which may consist of the variations of several factors or components; e.g., dispersions in material properties and geometrics of structural members, which may be functions of workmanship quality. For example, in the formulas for bending capacity of an under-reinforced

*Eq. (5) really refers to a conditional probability; i.e., the probability of $(R < \nu S)$ given $R_p = \nu S_q$, or $P(R < \nu S | R_p = \nu S_q)$.

concrete beam, $M_u = f_y A_s jd$, δ_R is the coefficient of variation of M_u, which is a function of the variations in f_y, A_s, and d. Similarly, δ_S may also consist of the variations from several contributory factors or components.

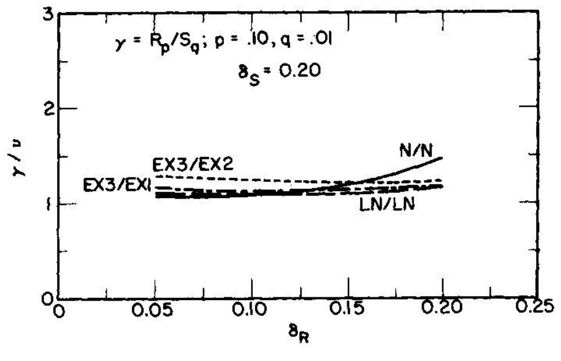

FIG.4 VARIATION OF γ/ν WITH δ_R BASED ON EXTENDED RELIABILITY

Formulas for the safety factor γ corresponding to other probability distributions for R and S can be similarly derived on the basis of Eq. (6). The expressions obtained for these other distribution functions will be different from that of Eq. (7); however, the calculated values of γ for the same ν and coefficients of variation will not differ much. In other words, γ obtained on the basis of Eq. (6) will not be too sensitive to the distribution functions of R and S, as can be seen from the results presented in Fig. 4, which should be contrasted with those of Fig. 3

The formulation typified by Eq. (7) clearly distinguishes the unknown uncertainties from the known statistical variabilities; the unknown uncertainties are handled through a subjective factor ν, whereas observed statistical variabilities are handled by the remaining factor which is a function of the coefficients of variation. This distinction is important. On this basis, it emphasizes that statistical information should not be confused with ignorance and should be handled objectively through appropriate probability models. The vagueness that is unavoidable in the exercise of judgment, which is necessary in the consideration of subjective factors, is however unnecessary when treating information with measured statistical dispersions.

The part of the safety factor necessary to account for unknown uncertainties, i.e. ν, should theoretically remain constant unless the state of ignorance changes; in any event, this part should not change with the measured variability of the observed statistical information. The overall safety factor γ, of course, may change with the degree of statistical dispersion, but this can be done objectively and more consistently with the form suggested by the extended reliability approach.

We observe from Fig. 4 that the variation of the factor γ/ν with δ_R (or δ_S) depends on the distribution functions of R and S. For certain distributions, this factor may even decrease with δ_R (and δ_S) as shown in both Figs. 3 and 4; this is because γ is described in terms of R_p and S_q which are also functions of δ_R and δ_S, respectively. However, it should be emphasized that in spite of this, the resulting designs will always increase with δ_R and δ_S.

For the normal distribution, however, γ/ν is monotonically increasing with δ_R, which is perhaps a desirable property from the standpoint of consistency with conventional thinking. Since the extended reliability approach is somewhat independent of the distribution functions, the normal function therefore may be adopted for general design applications. However, if information or data suggests that other distributions are more appropriate, such distributions can always be used to obtain more precise designs at the expense of more involved computational efforts.

Design Codification

One of the purposes of the proposed extended reliability concept is for the formulation and evaluation of safety factors to be used in a design code.

For this purpose, values of ν must be given for appropriate situations; these values require subjective analysis and may be obtained in much the same way that the partial safety factors are currently obtained, and may similarly be decomposed into several sources of uncertainties; e.g., $\nu = \nu_r \nu_s$, in which ν_r is the judgmental correction necessary to take account of the unknowns in the prediction of resistance; and ν_s is the corresponding factor to include the possible inaccuracies in the analysis of the load and load effects, and the unlikely occurrence of two or more extreme loads at the same time. These factors may each be further broken down into components if necessary to facilitate analysis, as suggested in the limit-state approach [1].

The influences of measured or known statistical variabilities should not be included in the subjective analysis of ν, since these are evaluated through the formula given in Eq. (7).

In its initial implementation, the value of ν may be evaluated on the basis of current designs; i.e., assuming typical values of certain parameters, its value should be such that the same safety factor is obtained as currently used. For example, suppose that based on the recommendations of the CEB, the overall safety factor is $\gamma = \gamma_m \gamma_s \gamma_c = 1.80$, in which the characteristic values are assumed to be based on $p = .05$ and $q = 0.02$, whereas $\delta_R = 0.20$ and $\delta_S = 0.25$; then in order to obtain the same design for this typical case, the judgment factor ν, according to Eq. (7), must be

$$\nu = 1.80 \left(\frac{0.132}{0.400}\right)\left(\frac{1.512}{0.670}\right) = 1.34$$

For subsequent designs of the same or similar types of structures under similar conditions, this value of ν must be held constant, whereas depending on the quality of material and variability of the loadings, the value of the safety factor γ would vary in accordance with Eq. (7).

Other considerations, such as the importance and projected use of a structure, may be taken into account through the specification of the nominal design load S_q; the S_q for an important structure intended for human occupany should correspond to a smaller q than a structure of lesser importance.

SUMMARY AND CONCLUSIONS

Uncertainties in design can be identified to be of two types; namely, unknown uncertainties arising from the lack of perfect knowledge and information, and measured statistical variabilities. The unknown uncertainties can be treated only subjectively through the use of engineering judgments, whereas known statistical information can and should be treated objectively using probability concepts.

The probability-based characteristic load and resistance are suitable nominal design values. In terms of these characteristic values, the unknown uncertainties can be accounted for through a "judgment factor" (or factors) expressed nominally in a conventional format; in these terms, statistical variabilities are also largely accounted for. The remaining concern is then primarily the risk against having a resistance less than the characteristic value or encountering a load greater than the specified characteristic load. However, in view of the nominal requirement, this risk is theoretically limited by a lower bound. Thus, the acceptable risk need not be smaller than the indicated minimum.

The essence of the proposed extended reliability concept can be summarized as follows:

If, $$R_p = \nu S_q \tag{3}$$

then $$P(R < \nu S) > pq \tag{5}$$

for all values of ν, including $\nu = 1.0$. Hence, pq is an acceptable risk when Eq. (3) is nominally required, and the appropriate basis for safe design is,

$$P(R < \nu S) = \alpha \tag{6}$$

with $\alpha \nless pq$. On this basis, reliability-based design procedures (in conventional form) can be developed that are not too sensitive to the assumed distribution functions, thus permitting the adoption of the normal distribution for most practical purposes. However, the approach also allows the use of other distributions if necessary and warranted.

In the context of the above extended reliability concept, a design safety factor derived from Eq. (6) consists of two parts--a subjective part represented by ν, and an objective part for evaluating the influence of statistical information. In this way, the variation of the safety factor with statistical dispersions can be evaluated systematically and objectively.

ACKNOWLEDGMENT

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SUMMARY

Unknown uncertainties in design are formulated in terms of a nominal requirement through a subjective judgment factor. In view of this nominal requirement, the risk against unfavorable performance due to statistical variabilities is theoretically limited. Hence, a minimum acceptable risk is available to permit the formulation of an extended reliability basis for safe design and evalutation of safety factors.

RESUME

Les variables aléatoires inconnues, dans la conception des projets, sont exprimées sous forme d'une exigence nominale grâce à un facteur subjectif de jugement. Considérant ce facteur arbitraire, la détermination du risque d'un comportement insatisfaisant créé par les variations statistiques, est théorétiquement limitée. Ainsi, un risque minimal acceptable est utile afin de permettre la formulation de bases sérieuses pour une conception sûre et pour l'évaluation des coefficients de sécurité.

ZUSAMMENFASSUNG

Mit Hilfe eines subjektiven Beurteilungswertes werden die unbekannten Unsicherheiten beim Entwurf in Gliedern einer Nennanforderung ausgedrückt. Im Hinblick auf diese Nennforderung ist das Risiko gegen unerwünschtes, aus statistischer Streuung hervorgerufenes Verhalten theoretisch begrenzt. Dadurch wird ein kleinstes, annehmbares Risiko nutzbar für die Formulierung eines sicheren Entwurfes sowie der Sicherheitsbeiwerte aufgrund eines erweiterten Zuverlässigkeitsbereiches.

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II

Structural Safety and Catastrophic Events

Sécurité et accidents des constructions

Bauwerksicherheit und -schäden

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1. INTRODUCTION

In modern theories of structural safety it is customary to assign a certain "probability of failure" (P_f) to a structure. This P_f can be derived from the probability distribution of the strength and the probability distribution of the loads. Failure is thought to occur if the loads exceed the strength.

The intention is, to choose P_f so small, that an economic optimum is reached, where the sum of building costs, maintenance and risk (possibly also remainder value after the end of the fixed lifetime) is made as small as possible.

Many authors have studied the possibilities of assigning a certain value to P_f if the variations of loads and strength are known [1]. Practical application is still difficult, because there is not sufficient knowledge of the probability of extreme loads and extreme material properties. Nevertheless it seems probable that the results of this theory are not completely realistic, because in the theory it is assumed that the structure as a whole - although with an unfavorable combination of material properties - must be able to sustain the normal types of loads (like floor loads and wind), without being damaged appreciably, not even when the loads have an exceptional magnitude. Very little attention has been paid to what happens to a structure that has been damaged locally by an overload or materials defect. For complicated structures, comprising many structural elements this is not satisfactory. Furthermore the theory as usually applied does not allow for abnormal types of load, differing considerably from the standard loads given in the building codes (like explosions, collisions and fire) and of abnormal material properties caused by building errors, chemical attack or fire.

In this paper the author will try to point out some factors that in reality have a great influence on the probability of failure of a structure. For this end he will use on one hand simple statistical considerations, and on the other hand data obtained from building failures.

2. ELEMENTARY STATISTICAL CONSIDERATIONS

For a simple structural element the difference between the strength of the critical cross section and the load can be calculated. This is compared with some quantity (like the standard deviation) that represents the scatter in this difference. The probability that failure will occur depends from the type of frequency distribution and from the ratio between difference and standard deviation.

If the strength of a structure is obtained by addition of the strength of a number of cross sections (like e.g. a statically indeterminate beam or a rigid block supported on a great number of piles) the scatter in the strength is smaller than that of the individual cross sections. It is not reasonable therefore to calculate statically indeterminate structures with the same "factor of safety" as statically determinate structures.

On the other hand many structures contain a number of elements that are linked in a series, like the links of a chain, the consecutive elements of the cable of a suspension bridge, or the columns that are situated one above the other in a high building.

It is obvious that the chain is no stronger than the weakest link. If mean $\bar{x}_1$ and standard deviation σ_1 of the strength of a single element are known, the mathematical mean and standard deviation of the weakest of a series of n elements can be calculated approximately [2]. In fig. 1 the necessary parameters r_n and S_n are given. The weakest element of a series of n elements has a mean strength $\bar{x}_n = \bar{x}_1 - r_n\sigma_1$ with a standard deviation $\sigma_n = S_n\sigma_1$.

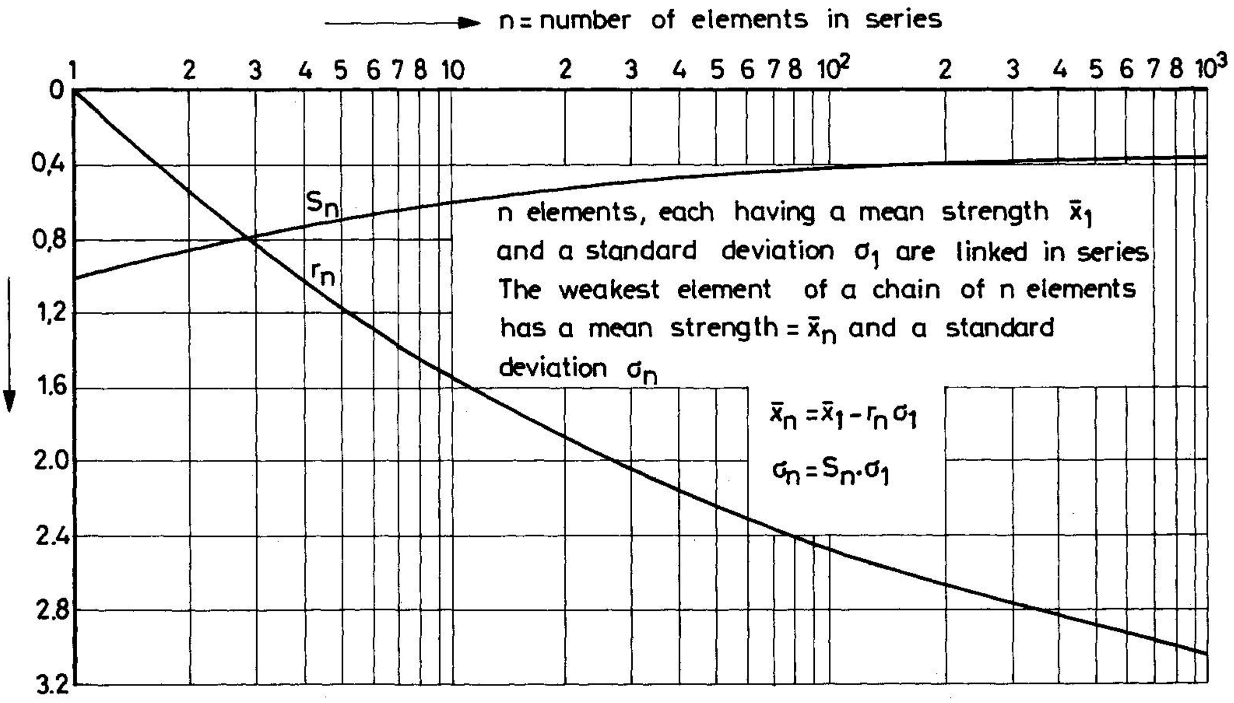

fig.1 The strength of the weakest link of a chain of n elements

The mean strength of a series of elements is therefore smaller than that of a single element. A greater value of the coefficient of safety will be needed in such cases. This is true especially in cases where the scatter in the loads is relatively great. In cases where the loads are known rather accurately the effect is mildered by the fact that the standard deviation σ_n for the series is smaller than σ_1 for the single element.

The same sort of phenomenon occurs in greater structures. It seems appropriate to consider a greater structure as an assembly of a great number of structural elements. Each of these has to fulfill certain specifications in respect to safety.

Let the greater structure consist of n elements. Each element has a (small) probability of failure = p_1. This does not fix the probability of failure of the whole structure. In the worst case each individual element will by collapsing bring about a total destruction of the whole structure. This is the case for example in a completely statically determinate structure. In this extreme case the probability of failure of the whole structure will be approximately $p_n = np_1$. If p_n must have an acceptable low value, $p_1 = \frac{p_n}{n}$ must be extremely small.

A much more normal situation will be, that only a smaller number of elements (n_2) will bring about a complete failure by failing individually, whereas the other $n - n_2$ elements cause only local damage that can be repaired.

In such a case p_n may be set equal to $p_n = n_2p_2$ (p_2 is the probability of failure of the critical elements). For similar reasons as before $p_2 = \frac{p_n}{n_2}$ will have to be considerably lower than p_n.

The designer has to know what elements are critical, so that he can make these elements sufficiently safe. The safety requirements for the other elements that can cause only local damage may be less stringent.

If there are 100 critical elements in a greater structure, and if $p_1 = p_n = 10^{-3}$, then p_2 will have to be $p_2 = 10^{-5}$. For the normal elements this means that the difference between strength and expected loads has to be equal to 3.1 x the standard deviation, for the critical elements this difference becomes 4.2 x the standard deviation.

Calculations make it seem simple to do this. If the loads and the strength both have a standard deviation of 10 %, the mean strength of the normal elements has to be 1.58 x the mean value of the expected loads caused by the most unfavorable load combination, and the strength of the critical elements must be designed with a factor of safety = 1.90 in order to have $p_2 = 10^{-5}$.

In reality this is nonsense. For not too small levels of probability (order of magnitude 10^{-3}) it is completely reasonable to consider loads and strength as quantities that may vary in magnitude, but retain more or less the same character. If however one has to look for smaller probabilities, the probability of occurrence of completely other types of load (like those brought about by explosions, collisions, inundations, earthquakes, etc.) becomes sufficiently great to make it necessary that these too are considered. The same is true for the strength, where far more abnormal situations (fire, chemical attack, etc.) come within the range of possibilities.

This seems to indicate that it is not sufficient to increase the conventional coefficient of security in order to diminish the probability of failure of a certain structural part below a certain - normal - limit. If this is necessary at least some qualitative insight in the causes of structural damage is needed, as well as some idea of the frequency of occurrence in practice.

3. STATISTICAL DATA ON STRUCTURAL DAMAGE

In the daily papers mention is made regularly of occurrencies where structural damage has been involved. Dependable statistical data are not available. The "news-value" is rather independent of the extent of the damage, so that for a structural engineer the selection by the daily papers seems completely haphazard. Due to the fact that in many cases conflicts arise between several parties on questions of who is responsible for the damage etc., it is not easy to publish freely about specific cases where details are known on causes and extent of structural damage.

This makes necessarily the following statistics a rough estimate. Nevertheless the facts are remarkable enough.

For 1967 the following causes for structural damage in the Netherlands can be enumerated (where in all there are about 3.000.000 houses, flats and other buildings):

15000 fires, known at the fire brigade offices (in 1500 of these fires flameover occurred in at least one room).

200 individual cases, where wind loads caused rather severe structural damage (i.e. more severe than fallen chimneys and roof tiles). Among these was a whirlwind which caused considerable damage to many houses, several roofs were torn of apartment buildings etc., a sport hall was blown over etc.).

200 explosions caused structural damage. Part of them occurred outside buildings (ship carrying ammunition, oil refinery, tank transport vehicle), another part occurred in the buildings themselves (gas explosions of natural gas, sewer gas, acetylene cylinder, gasoline, chemical experiments, detonating gas in an industrial accident, etc.).

100 collisions (ship against bridge, truck against bridge, car or tram against building, building crane falling down on building, airplane against guying of television mast, etc.).

50 total or partial collapses under almost normal circumstances, due to materials defects and/or faulty design.

20 total or partial collapses caused by local overloading (among these a complete roof of an industrial building coming down as the result of an extremely high loading by iron dust on a very small part of the roof.

The total damage may be estimated at H fl. 400.000.000,- (this is about 1/2 % of the national income, and 5 % of the total budget of the whole building industry in the Netherlands). Moreover about 100 people were killed in the accidents described. Roughly one half of the damage was caused by fire.

Another important aspect is that in many cases the indirect damage (e.g. caused by the loss of a vital part of an industrial process) or the injuries to people and the loss of goods that were in the damaged building have caused far greater losses than the structural damage in the building itself.

As far as can be seen this year is not exceptional. In 1968 there was somewhat less damage caused by wind. In the beginning of 1969 some 10 collapses due to snow loads occurred, which had not been present in the previous years.

All this happened without earthquakes, civil war, sabotage, flood disasters, hurricanes and other disasters striking a large area entering the picture.

If it is assumed that only in the case flameover occurs fire causes structural damage, in one year more than 2.000 buildings are damaged in one way or another. This means that from the 3.000.000 buildings in the Netherlands some 100.000 (3 %) will be damaged during their lifetime.

In structural calculations the coefficients of security normally adopted would lead to expect a very low probability of failure (order of magnitude 10^{-4} or 10^{-5}) due to "normal" causes. The designer ought to be more conscious of the adverse possibilities of loading by fire, explosions, collisions, etc. This may be expected to have a relatively great influence on the real safety of structures. Only if this is done, advantage can be reached by using refined calculating methods and quality control.

In the next chapter some more details will help to visualize the risks that a structure runs.

4. CAUSES OF BUILDING FAILURES

There can be discerned three main causes for structural damage:

1. fire,
2. brute violence (explosions, collisions, some cases of wind damage, inundations, earthquakes, sabotage, war actions),
3. an unfortunate combination of material, structural and loads.

Most of the somewhat spectacular failures can be found in the first two categories. In many cases a minute accident triggered off a sequence of events, leading to substantial damage and loss of human lives. Mostly a great total damage occurs when a relatively great part of a building is damaged. Sometimes however even a failure of a minor structural part (e.g. a sewer pipe) can cause considerable damage in the industrial sector.

By fire great losses occur if the room where the fire starts has great dimensions, if the contents are very costly or if the fire can spread later on to adjacent rooms or buildings.

The risk that during the lifetime of a building flameover will occur in one of its rooms may be estimated at 2 %. This makes it obviously a sensible thing to take precautions for diminishing the risk of spread of fire to adjacent rooms. Very large individual rooms should be avoided wherever possible.

Brute violence causes some damage to about 1/2 % of all buildings. Most building codes do not take explicit precautions against this sort of calamities - nor do more advanced ideas on structural design current in the technical litterature. There is no reason to believe that this risk is automatically covered by the conventional coefficient of security.

In normal circumstances quite a lot of communication is needed between the several people concerned with the design and the erection of a building. Even in the design phase no one concerned can effectively supervise all the different viewpoints (economic, heating and ventilation, structural, aesthetic, etc.).

The designer has in mind a definite purpose. Even during erection unexpected circumstances may arise. During the long life of a structure changes in use, additions and internal reorganization may alter the circumstances of several structural parts considerably. It is not to be wondered that in some cases by an unhappy coincidence of structural design, execution and loads part of a structure fails.

This again is an argument, which makes it clear, that in a good design the possibility of a local failure ought to be considered.

5. RISK CONSCIOUSNESS

Especially in cases where a failure may endanger the lives of many people (high apartment building) or where great industrial damage may be caused, it is urgent that methods are developed to make the design "fail-safe".

In aeroplane industry this is commonplace, in shipbuilding watertight compartments have since long been completely normal. Why has it taken such a long time, before the need for "risk consciousness" for the structural engineer became apparent?

Obviously one of the main causes is that for small structures there is not much difference between the extra margin of safety that is obtained by a coefficient of security and by some form of risk consciousness. At this moment however a magnification in scale causes more, bigger and more complicated structures to be built than ever before. In such cases the risk of a complete failure induced by a local failure cannot be covered by the use of a coefficient. The type of design is the only factor that can help without exceptionally high costs.

Necessarily some money will be needed to make a structure so that a local failure cannot cause severe damage to a greater part of the structure. The certainty that a local failure will only cause local damage will make it possible however to choose a higher probability of failure (i.e. a smaller coefficient of safety) for the design of the individual structural elements. This may offset the greater part of the extra costs of the main structure.

In all cases one ought to seek for a solution which makes the sum of building costs, exploitation and risk as small as possible. In a greater object the risk becomes more prominent. As an example a total failure of a normal one family house will cause a damage of say H fl. 100.000,- and there is a reasonable chance that no human lives will be lost in such a failure. If however by a similar cause a high apartment building containing 100 flats collapses, the damage is 100 times as great and there is a reasonable probability that some hundred people will be killed. Moreover the odds that some clumsiness of one of the people living in the building causes the initial calamity is equally great as in 100 one family houses.

This makes it clear that the greater and more complicated buildings and structures that are becoming more and more common now must have some capacity of sustaining completely unexpected loads and local failures. Very accurate calculations seem out of place, but it ought to be investigated at least intuitively and with some rough calculations what can happen in exceptional circumstances.

During the last war prof. J.F. Baker used similar considerations for reinforcing the roof trusses of factory halls. He wished to avoid that a small bomb that e.g. blew away one of the columns would cause the roof to come down completely. In order to increase the risk consciousness of the structural engineers it seems useful to include in building codes and similar documents a sentence like "The structure shall be designed in such a way, that local damage cannot induce disproportionately great damage in the structure as a whole or cause disproportionately great effects on the function of the structure". Such a sentence has effect only if building authorities act upon it.

6. SCIENTIFIC EVALUATION OF RISK

By now the behaviour of most structures under deterministic circumstances is known well enough to enable a specialist to calculate the real behaviour under loads in considerable detail. In many cases it will be possible to calculate the behaviour of a given structure under a given sequence of loads. At the end of this sequence the final state can be described by a number of parameters fixing e.g. the deflections, the crack widths, etc. in a number of typical points. The necessary calculations can be made very rapidly using a computer.

There is a method, called "Monte Carlo method" or "simulation". This means that a rather great number of possible structures is chosen (taking into account the known frequency distributions of material properties, dimensions, etc.). In the same way for each of them a certain sequence of loads occurring during the "lifetime" can be chosen. Some of these loads have exceptional magnitudes - like those due to removal of furniture - others have an abnormal character - like fire, which occurs in varying severity in about 2 % of the cases -. With a computer all the typical parameters of the structural behaviour at the end of the load sequence chosen for that structure are calculated.

The situation of some tens of thousands more or less similar structures under more or less similar conditions can be determined in this way. The data can be evaluated statistically in the same manner as experimental data, and give - within the range of our knowledge of loads and material properties - a realistic estimate of the risk that the structure will become unservicable.

It is obvious that for a complicated structure this type of analysis will be difficult, because so many assumptions have to be made on scatter and frequency distributions of loads, material properties and dimensions.

Even for a rather simple structural part however this type of analysis may lead to unexpected results that can serve as a guide for future work. As an example it would be extremely interesting to investigate in this way the behaviour of a simple reinforced concrete slab. The cover, the quantity and quality of the reinforcement bars, their diameter, the concrete quality and the slab thickness may be taken as design parameters. It seems certainly possible, that this may lead to the conclusion that several normal design procedures are unrealistic (like multiplying body weight and external loads with the same load factor, determining the amount of steel of different qualities from the yield moment at normal temperature and determining the cover from tests in pure bending where the crack width is observed).

It is hoped that this type of analysis will lead in future to methods of structural analysis, where as well the scatter in loads and structural properties as the influence of abnormal loading like fire and structural defects are treated in an orderly way.

7. CONCLUSIONS

Abnormal types of load like fire and brute violence occur too frequently to be neglected in structural design. If these are considered in an adequate manner the real safety of structures can be improved materially. This is especially so for greater structures built from a great number of structural elements.

The probability that in such a building one of the elements is loaded far heavier than normal is so great, that it must be explicitly avoided that any such element causes a complete disaster in failing. This can be ensured by providing alternative paths of load if one element fails. Critical elements must be located and special precautions must be taken to insure their safety.

In most cases rough calculations and qualitative insight will suffice. The more refined modern building codes (like e.g. the CEB regulations) build up a coefficient of security from a great number of separate factors. As a kind of check list on all the influences this procedure may be useful. From a statistical point of view multiplication of a number of these factors is nonsense. Moreover the great numerical accuracy achieved in that way leads to the neglecting of more important aspects of safety.

Good statistical data on exceptional loads and on building failures are not available. For the time being a more realistic approach must therefore make use of extremely rough estimates. Some increase of knowledge in this area will lead to much more increase of structural safety and economy than most of the structural research going on in laboratories all over the world now (including my own!).

[1] See e.g. The analysis of structural safety. Final report of the Task Committee on factors of safety ASCE by A.M. Freudenthal, J.M. Garrelts and M. Shinozuka. Journal of the Structural Division Proc. ASCE, Febr. 1966 (page 4682 etc.) and J. Ferry Borges & M. Castanheta "Structural Safety", LNEC Lisbon, 1968.

[2] Van Douwen, Kuipers and Loof "Correcties op gemiddelde waarde en standaardafwijking bij proevenseries met symmetrische proefstukken" (in Dutch).
Report Oe 5, Stevin Laboratory Technical University Delft (May '58).

SUMMARY

In greater structures there is a difference between failure of a structural part and failure of the structure as a whole. A part can fail by overloading and materials defects but also by fire or brute violence. Statistical data show that this happens during the lifetime of 3 % of the buildings in the Netherlands. A good structure has to be "fail-safe" as well as sufficiently strong in the normal situation. Critical elements must be located.

RESUME

Pour les constructions d'une certaine importance, il y a lieu de distinguer entre la défaillance d'un membre et l'écroulement de la structure entière. La rupture d'un membre peut être occasionnée par des surcharges excessives et par des défauts de matériaux, mais aussi par le feu ou la violence. Les statistiques montrent que 3 % des bâtiments en Hollande subissent un dommage pendant leur durée de service. Une structure bien faite ne doit pas s'écrouler, même en cas d'avarie à l'un de ses éléments. Les parties critiques de la structure doivent être localisées.

ZUSAMMENFASSUNG

In grösseren Bauwerken muss man zwischen dem Bruch eines Gliedes und dem Zusammenbruch des Ganzen unterscheiden. Ein Teil kann sowohl durch Ueberbelastung und Materialmängel als auch durch Feuer und rohe Gewalt versagen. Die Statistiken weisen aus, dass in Holland 3 v.H. Gebäuden innerhalb der Lebensdauer Schaden erleiden. Eine zweckmässige Konstruktion muss bruchsicher und im Regelfall hinreichend tragfähig sein. Die kritischen Teile müssen lokalisiert werden.

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II

Probabilistic Evaluation of Safety Factors

Evaluation probabilistique des coefficients de securité

Wahrscheinlichkeitstheoretische Auswertung der Sicherheitsfaktoren

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INTRODUCTION

Over the last two decades, there has been an increased interest in the study of safety of structures from a probabilistic viewpoint. In these studies, two schools of thought can be identified: a classical probability analysis of the problem of safety exemplified in the works of Freudenthal [1], and an engineering approach to design codes, based on probabilistic concepts but aiming to maintain the simplicity of existing codes [2,3,4]. This paper pursues the latter approach.

It is recognized that the probability of failure of a structure is fundamental to a rational measure of the safety in view of the stochastic nature of resistance and load. The present state of knowledge permits ordinarily only an evaluation of the probability of failure of individual components (i.e. members) of a structure. The search for methods to calculate the probability of failure of structural systems remain an active field of research. In the spirit of the codes currently in use, this paper is concerned immediately with the design of individual components.

The load and resistance of a structure are functions of many stochastic variables. These variables are inter-related and their influence on the probability of failure is therefore very complex. Some design codes (e.g. the CEB Recommendations [5], attach partial safety factors on the effect of each specified variable. However, if the aim of a code is to achieve a constant probability of failure, it may not be valid *a priori* to assume that the effect of the stochastic variables can be separated; the partial safety factor would in general be mutually dependent. Therefore it would seem that a probabilistic design objective can only be achieved within a partial safety factor scheme at the expense of prohibitive complications in the expressions for the partial safety factors or, alternatively, by introducing coarse simplifications. Furthermore, the advantage of partial safety factors is partly lost [6] if they are selected arbitrarily.

Yet, the partial safety factor format remains attractive from a practical point of view, and it is worth the effort to examine how well it can be reconciled with the stochastic approach. The work reported in the following shows that it is always possible to derive a set of partial safety factors in such a way that consistency in the probability of failure is achieved with reasonable accuracy.

Following Cornell [2], the resistance R may be regarded as a product of three variables, M representing material strength, F representing fabrication and P representing the influence of professional assumptions, that is, the errors involved in the calculation of the resistance. For example, P includes variation within the limited discrete member sizes available, and accuracy of the formula for resistance used. The load S may be regarded [2] as a product of two variables: total load T and a factor E representing the uncertainty in engineering analysis of the evaluation of the load effect (for example; maximum moment) assuming that the actual loads were given.

Design then consists in the selection of 'characteristic values' of these five variables. The characteristic value of a load variable is the value at a specified number of standard deviations above the mean. This specified number may be called the 'characteristic coefficient' and is related to the probability of exceedance. Characteristic values of strength variables are defined in a corresponding manner, following established notions about strength and loads [5]. The ratio of the characteristic to the mean value of a variable is the corresponding central partial safety factor. Thus, it is seen that this partial safety factor for each variable depends only on its coefficient of variation and its characteristic coefficient.

This formulation permits selection of the coefficients of variation of the above variables, depending on experience and the particular design situation, in order to determine a set of partial safety factors.

DERIVATION OF PARTIAL SAFETY FACTORS

For the random variables, resistance R and load effect S (which may be an applied load, or applied moment, for example), with means $\bar{R}$ and $\bar{S}$, and coefficients of variation V_R and V_S, we may define the central safety factor θ as

$$\theta = \bar{R}/\bar{S} \tag{1}$$

Referring to Fig. 1, failure occurs when the resistance R is less than the applied load S, that is, when the stochastic variable (R - S), the safety margin, is less than zero.

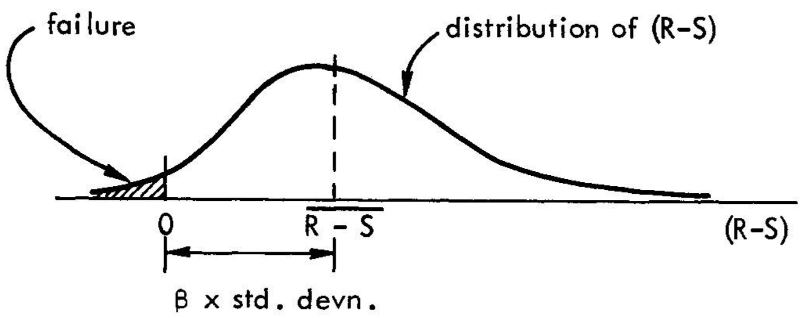

FIG 1
DEFINITION OF
SAFETY INDEX β

A measure of the degree of reliability β, called the 'safety index' is defined as the number of standard deviations of (R - S) between its mean value and zero. With a knowledge of the actual distributions of R and S, one can calculate the probability of failure of an element for any specified β. Thus:

$$\beta = \frac{\overline{R-S}}{\text{std. dev. }(R-S)} = \frac{\bar{R}-\bar{S}}{[(V_R\bar{R})^2 + (V_S\bar{S})^2]^{\frac{1}{2}}} = \frac{\theta - 1}{[\theta^2 V_R^2 + V_S^2]^{\frac{1}{2}}} \tag{2}$$

We now effect a linearization of the square root function, for any x and y, by introducing a function $\alpha = \alpha(x/y)$ defined by the relation:

$$(x^2 + y^2)^{\frac{1}{2}} = (x + y) \cdot \alpha(x/y) . \tag{3}$$

It is easily shown that α always lies between 0.707 and 1. Moreover, if x and y are roughly of the same magnitude, α is practically constant. For example, the assumption that α has a constant value of 0.75 would introduce a maximum error less than 10% for $0.25 < x/y < 4.0$.

By Eq. 2 and Eq. 3, the safety index is:

$$\beta = \frac{\theta - 1}{\alpha(\theta V_R + V_S)} \tag{4}$$

From Eq. 1 and 4, we get for the central safety factor:

$$\theta = \frac{1 + \alpha\beta V_S}{1 - \alpha\beta V_R} = \theta_R \theta_S \tag{5}$$

where $\theta_S \equiv 1 + \alpha\beta V_S$ is the partial safety factor on the load effect and $\theta_R \equiv (1 - \alpha\beta V_R)^{-1}$ is the partial safety factor on the resistance.

Now, θ_R can be separated into partial safety factors on the component variables M, F, and P as follows:

$$\theta_R = (1 - \alpha\beta V_R)^{-1}$$

$$= [1 - \alpha\beta (V_M^2 + V_F^2 + V_P^2)^{\frac{1}{2}}]^{-1} \tag{6}$$

By repeated use of Eq. 3, the partial safety factor on the resistance becomes:

$$\theta_R = [1 - \alpha\alpha_1\beta V_M - \alpha\alpha_1\alpha_2\beta V_F - \alpha\alpha_1\alpha_2\beta V_P]^{-1} \tag{7}$$

Factorizing, and keeping the term containing V_M independent of the other terms, we get:

$$\theta_R = [(1 - \alpha\alpha_1\beta V_M)(1 - \frac{\alpha\alpha_1\alpha_2\beta V_F}{1 - \alpha\alpha_1\beta V_M})(1 - \frac{\alpha\alpha_1\alpha_2\beta V_p}{1 - \alpha\alpha_1\beta V_M - \alpha\alpha_1\alpha_2\beta V_F})]^{-1}$$

$$= [(1 - \alpha\alpha_1\beta V_M)(1 - C_1\alpha\alpha_1\alpha_2\beta V_F)(1 - C_2\alpha\alpha_1\alpha_2\beta V_p)]^{-1}$$

$$= [(1 - K_M\beta V_M)(1 - K_F\beta V_F)(1 - K_p\beta V_p)]^{-1} = \theta_M\theta_F\theta_P, \tag{8}$$

where K_M, K_F, and K_p are functions of V_M, V_F, and V_p.

Each θ_i may be regarded as a partial safety factor on the variable i. It is shown below that the K_i are approximately constants, in the range of practical designs, so that each characteristic coefficient $(K_i\beta)$ varies predominantly with β only.

Similarly, the partial safety factor on the loads may be re-written:

$$\theta_S = (1 + K_T\beta V_T)(1 + K_E\beta V_E) = \theta_T\theta_E, \tag{9}$$

where K_T and K_E are functions of V_T and V_E and, as will be shown below, are practically constants. θ_T and θ_E are the partial safety factors on T and E respectively.

Furthermore, the effects of dead load and live load variations can be separated into individual partial safety factors, θ_D and θ_L respectively, which may be combined into the partial safety factor on total load, θ_T, by proportional addition as in the ACI 318-63 Code. Also, it can be shown that these additional partial safety factors depend only on the coefficients of variation of the loads.

Returning to Eq. 5, using Eq. 8 and 9, we get for the central safety factor

$$\theta = \theta_R \theta_S = \theta_M \theta_F \theta_P \theta_T \theta_E \tag{10}$$

CALIBRATION TO AN EXISTING CODE

The process of selecting appropriate values for the parameters in a code is called calibration [3]. A new code may be calibrated to an existing code so as to produce approximately the same member proportions as produced by current design, and, in the process, to produce approximately the same probability of failure, cost of failure, etc.

A convenient way to calibrate the proposed code format is first to calculate the implied value of β in the existing code by using a realistic set of $\{V\} = \{V_o\}$ of coefficients of variation of the variables M, F, P, T and E, and a calibration value of the central safety factor $\theta = \theta_o$.

With this value of β and for different combinations of the set $\{V\}$, the values of the set $\{K\} = \{K_M, K_F, K_P, K_T, K_E\}$ are calculated. The value of each K is approximately constant in the practical range of the set $\{V\}$ as shown (in the example for K_M) in Fig. 2. Accordingly, the uncertainty in the value of $\{V\}$ assumed in calibration to the existing code has very little influence on the resulting calibration, $\{K\} = \{K_o\}$

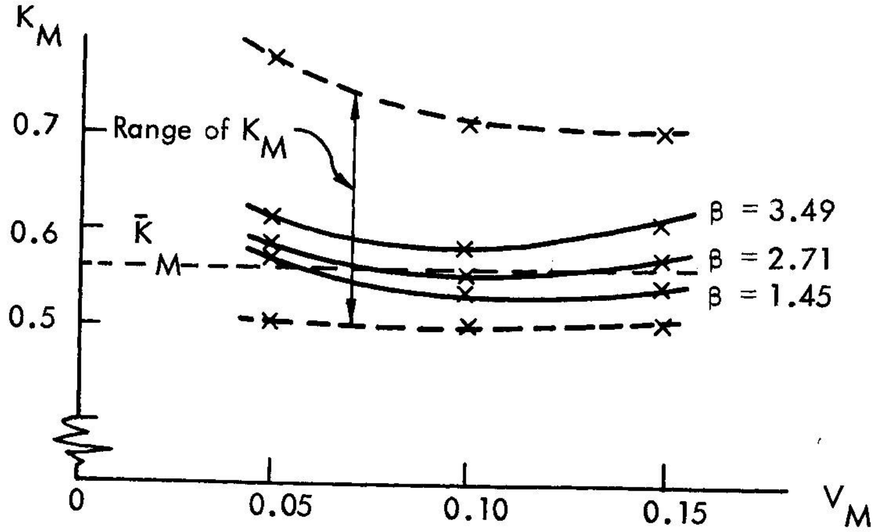

FIG 2.
VARIATION OF K_M WITH V_M.

Fig. 2 shows, furthermore, that the value of K_M as an approximation, can be replaced by a constant. In fact, the other functions in the set $\{K\}$ can similarly be assumed to be constant. Averaged over the domain of combinations of realistic values of the set $\{V\}$, we may put $\{K\} = \{0.56, 0.52, 0.58, 0.56, 0.50\}$. Moreover, we may simplify the results by inverting the expressions for θ_M, θ_F, and θ_P and neglecting terms of second and higher order. Finally we may even choose a global value, optimized over a realistic domain, of $K = 0.60$, say. Accordingly,

$$\theta_i = 1 + K\beta V_i; \quad i = M, F, P, T, E \tag{11}$$

can be used to calculate all partial safety factors for different conditions of materials, inspection etc.

The error in θ according to Eq. 10 arising from using Eq. 11, embodying all these approximations, rather than the correct expressions, Eq. 8 and Eq. 9, was determined using a digital computer over the unweighted practical ranges of the five coefficients of variations. Fig. 3 shows the distribution of this error.

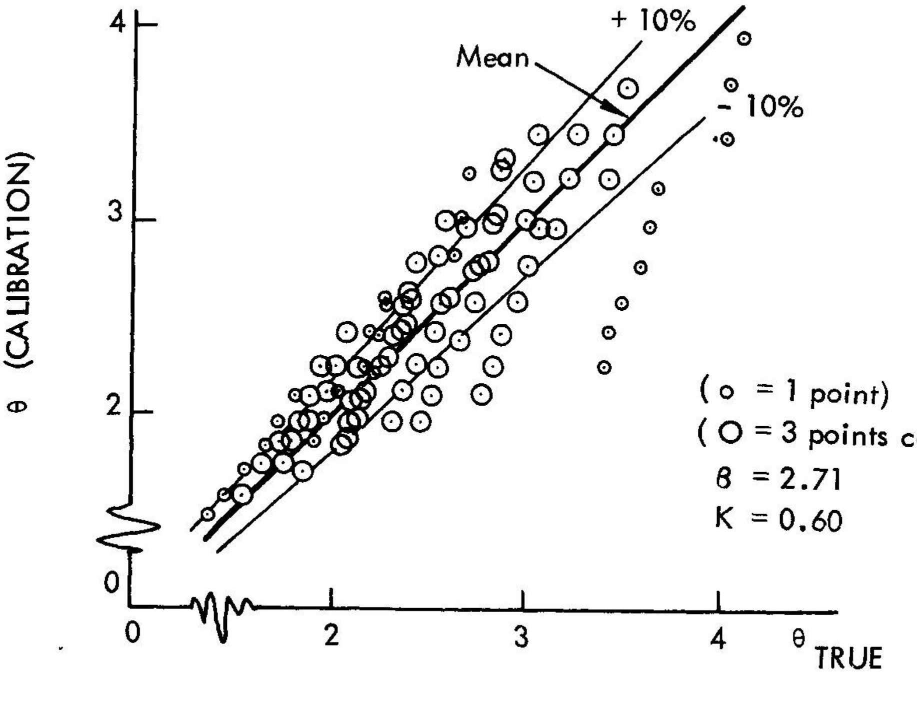

FIG 3
CALIBRATED VS
EXACT CENTRAL
SAFETY FACTOR

DESIGN PROCEDURE

In actual design the value of K as determined by the code authority could be given in the code and the designer might be free to select the set $\{V\}$ according to conditions. If the consequences of failure were particularly severe, a higher value for β would be specified. The central safety factor θ to be used would be calculated from Eq. 10 and 11.

Alternatively, the partial safety factors θ_i might be specified in the code in the manner similar to the C.E.B. Recommendations.

ILLUSTRATION

A partial safety factor code is to be calibrated to an existing code, (assumed to be National Building Code of Canada 1965 [7]).

As calibration point, we here select (somewhat arbitrarily, for the purpose of illustration only);

Office building	:	Nominal live load	= 50 psf.
Supported area	:	20 ft. span at 10 ft. c.c.	= 200 psf.
Dead load (6 in. slab, plus self weight, etc.)			= 80 psf.
Steel beams, simply supported $f_{all} = 0.6\, f_y$			= 21,900 psi.

Here, f_y is the mill test nominal minimum yield strength (for A36 steel). Actual yield strengths are assumed to have a mean of $\bar{f}_y = 36{,}000$ psi with a coefficient of variation for such beams equal to 12%, on the basis of tests [8] assumed to be relevant. The mean office live loading is assumed to be 25 psf.[9] and with a coefficient of variation equal to $c/A^{\frac{1}{2}} = 0.92$ for this particular area[10].

The central safety factor implied is therefore:

$$\theta = \bar{R}/\bar{S} = (80 + 50) \times \frac{20^2}{8} \times \frac{36{,}000}{21{,}900} / (80 + 25) \times \frac{20^2}{8} = 2.0$$

A realistic set of coefficients of variation is taken as:

$V_M = 0.12$, $V_F = 0.05$, (Good Control)

$V_P = 0.05$, (High accuracy), $V_L = 0.92$, $V_D = 0.05$, (Average)

$V_E = 0.10$ (Ordinary analysis).

Combining the loads, $T = L + D$, we get:

$$V_T = [(\bar{L}\ V_L)^2 + (\bar{D}\ V_D)^2]^{\frac{1}{2}} / (\bar{L} + \bar{D}) = [(25 \times 0.92)^2 + (80 \times 0.05)^2]^{\frac{1}{2}} / (25 + 80) = 0.22$$

Using these values, we calculate the coefficients of variation of the resistance and the load, respectively, as:

$$V_R = (V^2_M + V^2_F + V^2_P)^{\frac{1}{2}} = 0.14$$

$$V_S = (V^2_T + V^2_E)^{\frac{1}{2}} = 0.24$$

By equation 2, the safety index β is equal to 2.71. The set $\{K\}$ in Eqs. 8 and 9 is found using this value of β. The result is:

$$\{K\} = \{K_M, K_F, K_p, K_T, K_E\} = \{0.52, 0.44, 0.47, 0.53, 0.41\} \tag{12}$$

The desired partial safety factor code should result in approximately the same safety level as in the existing code at the calibration point. Therefore, we select $\beta = 2.71$ for the new code. The code is to acknowledge the variability in all five variables as shown in Table 1, where the coefficient of variation of each condition is listed. For each of these conditions, the resulting partial safety factors from Eq. 8 and 9 range as shown in the Table. The values shown are the averages of the exact values for the entire domain of combinations of the coefficients of variation given in Table 1.

It can be seen that the proportioning of the safety margin between load and strength is quite different from that of the reference code.

TABLE 1

Partial safety factors derived for a safety index of 2.71.

RESISTANCE	Good Conditions	Average Conditions	Poor Conditions
Coefficient of variation	0.05	0.10	0.15
θ_M	1.09	1.17	1.29
θ_F	1.07	1.15	1.27
θ_P	1.08	1.18	1.33
LOAD	low variability	average variability	high variability
Coefficient of Variation	0.05	0.20	0.40
θ_T	1.08	1.31	1.65
STRUCTURAL ANALYSIS	accurate	average	approximate
Coefficient of Variation	0.05	0.10	0.15
θ_E	1.06	1.12	1.18

DISCUSSION

The performance of the partial safety factor code format suggested here, relative to the first order probabilistic code format can be judged from Fig. 3. Bearing in mind that the total cost of a structure near the optimum range is insensitive to the variations in the safety factor [11], most of the deviations are seen to be of no practical consequence. Moreover, practical limitations in feasible probabilistic codes, as reflected in the presence of the vague parameters V_F, V_P and V_E in Cornell's format [2], invalidate any attempts at increased accuracy at the expense of simplicity.

When the safety index β is reduced, the distribution narrows. For example, for β equal to 1.45 the ratio θ/θ_{true} is always between 0.97 and 1.10. Conversely, when it is attempted to raise the reliability level by increasing the safety index, the ratio θ/θ_{true} may be significantly below unity; but always for unreasonable combinations of the coefficients of variation.

The range of the ratio θ/θ_{true} can be reached considerably at several stages of the derivations, by optimization of the parameters; this is best done by an individual code committee after the operating range of the parameters and the calibration points have been carefully selected.

Figure 3 also reflects the variation in the actual central safety factor typically inherent in partial safety factor code formats. If fewer than five factors are used to represent the variation of design reality, greater error relative to the probabilistic ideal must occur.

It can be shown by partial differentiation of Eq. 2 that an error of 20% in either of the coefficients of variation of resistance or load, produces an error of approximately 10% in the calibrated value for β. Such an error in β would only alter the probability of failure a fraction of an order of magnitude [2]; this should be acceptable.

The value of the safety index β, that is, the ratio of the mean of safety margin to the standard deviation of $(R - S)$, is directly related to the probability of failure of the element. If the distributions for the variables M, F, P, T and E are given, the probability of failure is practically constant for all combinations of $\{V\}$, provided that the shape of the distribution of $(R - S)$ does not change significantly.

It is seen from Table I that in order to achieve a constant safety index under varying control conditions, a variable control safety factor is required; also from this table, it can be inferred -- and verified by calculation -- that the constant central safety factor computed using present deterministic procedures does not assure a constant level of safety.

The partial safety factors separate the effect of each stochastic factor, such that the individual influence of each variable can be directly appreciated as a valuable guide for decisions in design or research planning.

CONCLUSIONS

1. A first order probabilistic design, based on a consideration of the first and second moments of the stochastic variables in design can be made without introducing any new notions beyond that of the partial safety factor. In other words, a partial safety factor code can be derived, which may maintain the accepted concepts of deterministic design and which is also self-consistent in the probabilistic sense; that is, if achieves a sensibly constant probability of failure in all design situations.

2. It is possible effectively to separate the influence of the interdependent stochastic variables on the central safety factor, using a set of partial safety factors. These factors can be calculated by Eqs. 8 and 9. As in some present code formats, each of these partial safety factors is dependent on the coefficient of variation of the corresponding stochastic variable. However, the factors are not arbitrarily selected here and they are directly related to the safety index as defined in Eq. 2. A code committee can evaluate its code parameters and characteristic values from the derivation presented herein.

3. The results justify the common approach in code writing, whereby load criteria and strength criteria are separately prescribed -- often by separate code writing authorities. In contrast to present codes, the central safety factor can be evaluated explicitly even when the statistical data are limited.

ACKNOWLEDGEMENTS

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SUMMARY

A set of partial safety factors are derived from purely probabilistic concepts. In contrast to present codes, one may derive central safety factors for design which maintain a specified level of safety over a domain of the component variables. The analysis considers only the first and second moment of the distributions of the variables, thus not requiring the detail distribution to be specified.

Using these factors, one may evaluate, rationally, the 'characteristic values' and multiplicative, heretofore arbitrary, safety parameters.

RESUME

On dérive un ensemble de coefficients partiels de sécurité à l'aide de concepts probabilistiques. On peut aller plus loin que les normes actuelles et dériver des facteurs centraux de sécurité pour des calculs qui exigent un niveau donné de sécurité sur un domaine des variables. L'analyse ne considère que les premiers et seconds moments des distributions des variables stochastiques; ainsi il n'est pas nécessaire de spécifier la forme exacte de la distribution.

L'utilisation de ces facteurs permet d'évaluer d'une manière rationnelle les valeurs caractéristiques et multiplicatives des coefficients partiels de sécurité, qui étaient jusqu'à maintenant arbitraires.

ZUSAMMENFASSUNG

Ein Satz von Teilsicherheitsfaktoren wird aus der reinen Wahrscheinlichkeitslehre abgeleitet. Heutigen Vorschriften entgegen kann man zentrale Sicherheitsfaktoren für eine vorgeschriebene Sicherheitshöhe über einem Bereich der unabhängigen Zufallsvariablen auswerten. Die Berechnung zieht nur die ersten und zweiten Momente der Zufallsvariablen in Betracht, wobei die Verteilungsart unbekannt sein kann. Mit diesen Faktoren kann man auf einfache Weise die "charakteristischen Werte" und die multiplikativen, bisher beliebigen Sicherheitsbeiwerte schätzen.

II

STRUCTURAL SAFETY AND OPTIMUM PROOF LOAD

Sécurité des constructions et charge d'essais optimale

Bauwerksicherheit und optimale Prüflast

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1. Introduction

In a recent paper[1] dealing primarily with aerospace structures, the author pointed out the importance of proof-load test in conjunction with the optimum structural design based on reliability concept. In fact, Ref. 1 developed an approach to an optimum design (either minimum weight design or minimum expected cost design) introducing the proof load as an additional design parameter and demonstrated the advantage of the use of proof load in terms of weight saving (under constraint of expected cost). From the view point of probabilistic safety analysis, it was also pointed out, the advantage of performing the proof-load test was two fold; it could improve not only the reliability value itself but also the statistical confidence in such a reliability estimate since the proof-load test eliminates structures with strength less than the proof-load. In other words, the structure which passes the proof-load test belongs to a subset, having the strength higher than the proof load, of the original population. The fact that the proof-load test truncates the distribution function of strength at the proof load alleviates the analytical difficulty of verifying the validity of a fitted distribution function at the lower tail portion where data are usually non-existent. Evidently the difficulty still remains in the selection of a distribution function for the load. However, the statistical confidence in the reliability estimation now depends mainly on the accuracy of the load prediction. The question of how to deal with the statistical confidence of the load distribution was also discussed in Ref. 1.

Consider now civil engineering structures such as bridges, transmission towers and buildings. Because of their characteristic construction processes, these structures usually undergo tacit processes of proof-load test during the construction. If a structure does not fail during and upon completion of construction, it implies that all of its structural components and therefore the structure itself have sufficient strength to withstand at least the dead load. This is the information that must be taken into consideration as the lower bound of the strength distribution for the reliability estimation of an existing structure, although the lower bound thus established may in some cases be too small to be of any practical significance. Furthermore, if a structure under construction survives a live load due to severe wind or earthquake acceleration, which are referred to as secondary live load in many design codes but of primary importance for safety consideration of existing structures, the combined action of such a live load and of the dead load (existing at the time of occurrence of the live load) can be interpreted as a proof-load test. The fact that the partially completed structure has survived such a proof-load test should be taken into consideration in the reliability analysis since this fact usually makes it possible to establish a better lower bound of the strength of each of structural components (existing in the partially completed structure).

Although the subject of such implicit processes of proof loading appears to be an interesting item for future study, the present paper places an emphasis on the explicit proof-load test for civil engineering structures to be performed before the structures are placed into service, and examines the conditions under which the explicit proof-load test is economically advantageous.

An important implication of the above argument is that separate considerations are given to the safety of a structure during and after completion of its construction. This seems quite reasonable since the cost of detection possibly by means of proof-load test and the cost of the replacement of that part of the structure which failed because of a member or members with insufficient strength may be absorbed as the construction cost or otherwise, whereas any failue after the structure is placed into service by the client would produce much more serious contractual and socioeconomic problems, possibly involving human lives.

2. Expected Cost and Optimum Proof Load

The present discussion deals with a structure designed under a conventional design code with a specified design load S_d. The structure is supposed to withstand a system of proportional loads with a reference value S which is statistical. This system of loads is hereafter referred to as the load S, and the design load is meant by the same system of loads with a particular reference value S_d. Furthermore, it is assumed that the proof load to be applied is also the same system of loads with a reference value mS_d, in which a positive number m indicates the magnitude of the proof load in terms of the design load. For example, when a bridge is designed for a design uniform load w, the proof load is the uniform load with intensity mw. This assumption is made essentially for simplicity of discussion and does not imply the limitation of the proof load approach presented here. An obvious example in which the proof and the design loads cannot be of the same type is a tower structure designed for wind pressure. In such a case, how to specify a system of (proportional) loads as well as its magnitude that should most effectively (in some sense) be used as a proof load, is not a trivial problem. Evidently, it is possible to proof-test structural components individually before they are assembled (an approach discussed in Ref. 1). This approach, however, appears to be too expensive to be applied to civil engineering structures.

Under these circumstances, it seems reasonable, for the purpose of presenting the essential idea of optimum proof load, to assume the following form of expected cost EC of a structure.

$$EC = q_o C_o + p_f C_f \quad \text{or} \quad EC^* = q_o \gamma + p_f \qquad (1)$$

where $EC^* = EC/C_f$ = the relative expected cost, $\gamma = C_o/C_f$, q_o= the expected number of the (candidate) structures that fail under the proof load before the one that can sustain it is obtained, C_o = the cost of a proof load test including the cost of loss of a (candidate) structure (during the proof load test), p_f = the probability of structural failure (that might occur after the structure is placed into service) and C_f = the cost of structural failure (that might occur after the structure is placed into service) such as cost of the structure, loss of prestige, etc. It is noted that Eq. 1 takes only the costs of failure and of proof-load test into account, although more elaborate forms are obviously possible and may even be desirable depending on the specific problem at hand.

Since the proof load is applied to the (entire) structure, not to its components individually as in Ref. 1, there is a probability p_o that it will produce a failure of the entire structure unless a method is devised to replace the component that exhibits an initiation of failure at a magnitude of proof load less than the prescribed value before the structural failure developes. If the proof load can produce only component failures because of such a device or otherwise, it seems reasonable to consider that the ratio γ is as small as 10^{-4} or even smaller. If, however, the proof load can lead only to structural failures, the ratio does not seem to be so small. In the present discussion, it is assumed that the proof load may produce only structural failures and that the ratio γ ranges from 10^{-4} to 10^{-1}.

The expected number q_o of candidate structures that will fail under the proof load can be shown to be

$$q_o = p_o/(1-p_o) \tag{2}$$

in which the probability p_o (defined previously) is given by

$$p_o = \int_0^{mS_d} f_{R_o}(x)dx = F_{R_o}(mS_d) \tag{3}$$

with $f_{R_o}(\cdot)$ and $F_{R_o}(\cdot)$ being respectively the density and the distribution functions of the resistance R_o of the structure on which the proof-load test has not been performed yet.

The probability of failure, p_f, of the structure which has passed the proof-load test can be written in the following well-known form:

$$p_f = \int_0^{\infty} F_R(x)\; f_S(x)dx \tag{4}$$

where $F_R(\cdot)$ is the distribution function of the resistance R of the structure which has passed the proof-load test and $f_S(\cdot)$ is the density function of the load S.

Under further simplifying assumptions, as used in most of previous papers including Ref. 2, that the pertinent resisting strengths (such as yield strength) of the individual structural members and therefore the resistances (load carrying capacities) of the same members are statistically independent of each other as well as of the load S, the distribution functions $F_{R_o}(\cdot)$ and $F_R(\cdot)$ can be written as

$$F_{R_o}(x) = 1 - \prod_{i=1}^{n} \left[1 - F_{oi}\left(c_i x/a_i\right)\right] \tag{5}$$

$$F_R(x) = 1 - \prod_{i=1}^{n} \left[1 - F_i\left(c_i x/a_i\right)\right] \tag{6}$$

where n is the number of members constituting the structure. Eqs. 5 and 6 are to be used respectively in Eqs. 3 and 4. In Eq. 5, $F_{oi}(\cdot)$ is the distribution function of the ("parent") resisting strength τ_{oi} of the i-th member of the structure which has not been subjected to the proof-load yet. Also, $F_i(\cdot)$ in Eq. 6 indicates the distribution function of the resisting strength τ_i of the i-th member of the structure which has passed the proof-load test. Quantities c_i and a_i are such that the load S_i acting in the i-th member can be obtained from the load S as

$$S_i = c_i S \tag{7}$$

and the resistance of the same member can be computed as

$$R_i = a_i \tau_i \tag{8}$$

For example, τ_i and a_i are respectively the yield strength and the cross-sectional area of the i-th member if a truss structure is considered.

As was discussed in detail in Ref. 2, the following points are to be noted in deriving Eqs. 4, 5 and 6; (1) the definition of structural failure is in accordance with the weakest link hypothesis, that is, the failure will take place if at least one of the components fails, (2) the assumption that the member strengths are statistically independent to each other is a conservative one, (3) p_f in Eq. 4 indicates the probability of structural failure due to a single application of the load S. Also, in deriving Eq. 7, the effect of the dead load is neglected for simplicity. Any method of structural analysis can be employed to obtain Eq. 7 including the finite element method.

By applying the proof load mS_d, each member is subjected to a force $c_i mS_d$. Therefore, if the structure (and therefore all the members) survives the proof load, a lower bound $c_i mS_d$ is established for the resistance of the i-th member. Because the force and the stress are related by Eq. 8, this in turn establishes a lower bound

$$\tau_{mi} = c_i mS_d / a_i \tag{9}$$

for the parent resisting strength τ_{oi} of the i-th member. Then, the distribution function $F_i(\cdot)$ of the ("truncated") resisting strength τ_i of the same member of the structure having passed the proof-load test can be shown to be

$$F_i(x) = \frac{F_{oi}(x) - F_{oi}(\tau_{mi})}{1 - F_{oi}(\tau_{mi})} H(x - \tau_{mi}) \tag{10}$$

where $H(\cdot)$ is the Heaviside unit step function.

Eq. 10 indicates that the distribution function of the (truncated) resisting strength of the structure which passed the proof load test is obtained from that of the parent strength by "truncating" it at the lower bound established by the proof load (and normalizing it).

The standard design requires that the nominal resistance $a_i\tau_{ai}$ be equal to the nominal applied load c_iS_d:

$$a_i\tau_{ai} = c_iS_d \quad \text{or} \quad a_i\tau_{pi}/\nu_i = c_iS_d \tag{11}$$

where τ_{ai} = the allowable stress, τ_{pi} = the specified minimum resisting strength and ν_i = the safety factor of the i-th member (these quantities are functions not only of the material but also of the mode of failure, e.g. in bending, in tension, in stability, etc.).

From Eq. 11, it follows that

$$c_i/a_i = \tau_{pi}/(\nu_iS_d) \tag{12}$$

The right hand side of Eq. 12 consists of quantities specified in the design code. Therefore, Eq. 12 makes it possible to replace c_i/a_i in Eqs. 5, 6 and 9 by known quantities.

Eqs. 2 and 4 (together with Eqs. 3, 5, 6, 9, 10 and 12) can now be used in Eq. 1 to compute the relative expected cost if $F_{oi}(\cdot)$ and $f_S(\cdot)$ are known. The optimum intensity of the proof load is then obtained as that value of m which minimizes the relative expected cost EC*.

3. Example

In the following, the assumptions are made that (1) the allowable stresses (or both the specified minimum strengths and the safety factors) and (2) the distribution functions $F_{oi}(x)$ of the parent strengths are identical for all the members; $\tau_{oi} \equiv \tau_o$ and

$F_{oi}(x) \equiv F_o(x)$. These assumptions are made purely for simplicity. The analysis presented in the preceding section can easily accommodate the situations in which this is not the case; e.g. consider different allowable stresses specified for tension and compression members and also consider the fact that in reality, different distribution functions of the parent strengths are needed for tension and compression members.

The immediate consequences of these assumptions are that (1) c_i/a_i in Eq. 12 and hence τ_{mi} in Eq. 9 become independent of the subscript i; $c_i/a_i = \tau_p/(\nu S_d)$ and $\tau_m = m\tau_p/\nu$, and (2) the truncated strength distribution $F_i(x)$ also becomes independent of i; $F_i(x) \equiv F_\tau(x)$.

In the present paper, the parent strength distribution is assumed to be distributed according to the Weibull distribution:

$$F_o(x) = 1 - \exp\left[-(x/\tau_c)^b\right] \tag{13}$$

where τ_c is the characteristic strength and b is a positive constant.

From Eqs. 10 and 13, it follows that

$$F_\tau(x) = \left\{1 - B\exp\left[-\left(\frac{x}{\tau_c}\right)^b\right]\right\} H\left(x - \frac{m\tau_p}{\nu}\right) \tag{14}$$

with

$$B = \exp\left[\left(\frac{m\tau_p}{\nu\tau_c}\right)^b\right] \tag{15}$$

Therefore, Eqs. 5 and 6 can be respectively written as

$$F_{R_o}(x) = 1 - \exp\left[-\left(\frac{x}{R_c}\right)^b\right] \quad x > 0 \tag{16}$$

$$F_R(x) = 1 - B^n\exp\left[-\left(\frac{x}{R_c}\right)^b\right] \quad x \geq mS_d \tag{17}$$

and from Eq. 2,

$$q_o = B^n - 1 \tag{18}$$

where $R_c = hS_d/n^{1/b}$ with $h = \nu\tau_c/\tau_p$ is the characteristic resistance of the structure which has not proof-load-tested yet. The parameter b is a measure of dispersion of the distributions of τ_0 and R_0; the coefficients of variation in terms of their characteristic values are 0.46, 0.33, 0.25 and 0.21 respectively for $b \doteq 2$, 3, 4 and 5.

For the distribution function $F_S(x)$ of the load S, the first asymptotic distribution function of largest values is assumed. However, since only the upper tail portion of the distribution is significant, the following exponential form is used as an approximation for larger values of the load;

$$1 - F_S(x) = r \exp\left[-a\left(x-kS_d\right)\right] \qquad x > kS_d \tag{19}$$

where "a" is a positive constant and $kS_d (0 < k < 1)$ is the lower bound above which such an approximation is valid and r is such that the probability that S will be larger than kS_d is r.

The final expression for the probability of failure is

$$p_f = ra \int_{mS_d}^{\infty} \left\{1 - B^n \exp\left[-\left(\frac{x}{R_c}\right)^b\right]\right\} \exp\left[-a\left(x - kS_d\right)\right]dx \tag{20}$$

Although this integral cannot be evaluated in closed form unless $b = 1$ or 2, an asymptotic approximation can be obtained by expanding the first term of the integrand and integrating term by term as long as $\lambda \gg 1$ where $\lambda = sh/n^{1/b}$ with $s = (1-k)^{-1}\ell n(r/q)$. The result is

$$p_f \doteq Ar \exp[-s(m-k)] \tag{21}$$

with

$$A = (2ms + 2)/\lambda^2 \qquad (b=2) \tag{22a}$$

$$A = \left\{3(ms)^2 + 6(ms) + 6\right\}/\lambda^3 \qquad (b=3) \tag{22b}$$

$$A = \left\{4(ms)^3 + 12(ms)^2 + 24(ms) + 24\right\}/\lambda^4 \qquad (b=4) \tag{22c}$$

$$A = \left\{5(ms)^4 + 20(ms)^3 + 60(ms)^2 + 120(ms) + 120\right\}/\lambda^5 \qquad (b=5) \tag{22d}$$

where ms should be smaller than λ and q is the probability that the load S will be larger than S_d. The result does not contain the parameter "a" (Eq. 19) explicitly. It however, appears in the preceding equations implicitly since $a = s/S_d$.

The validity of such asymptotic approximations is checked by comparing the result using Eq. 21 with that of the exact integration for $b = 2$. The agreement is more than satisfactory.

A number of sets of parameters are considered for numerical examples. Among these, the result for the case where the structure consists of 7 members $(n=7)$, $b = 4$, $q = 0.02$, $r = 0.1$, $k = 0.6$ (thus $s = 4.03$) and $\nu = 1.67$, is presented. The specified minimum strength τ_p is defined so that the probability of the parent strength τ_o being less than τ_p is p. Therefore, from Eq. 13, $\tau_c/\tau_p = [-\ell n(1-p)^{-1/4}]$. For the present example, $p = 0.1$ is used (hence $h = 5.15$). The assumption that $q = 0.02$ implies that the design load with a return period of 50 years is considered if the distribution $F_S(x)$ is that of the annual largest load.

The result is illustrated in Fig. 1 where the relative expected cost EC* is shown as a function of $\mu = m/h$. The value μ indicates a magnitude of the proof load relative to hS_d at which the loads (the stresses) acting within the individual members are equivalent to their characteristic values $a_i\tau_c(\tau_c)$. Since the optimum proof-load is the one at which EC* becomes minimum, Fig. 1 indicates that the proof-load becomes optimum when $\mu = 0.2$, 0.38, 0.55 and 0.67 (or $m = 1.03$, 1.95, 2.83 and 3.45) respectively for $\gamma = 10^{-1}$, 10^{-2}, 10^{-3} and 10^{-4}. The locus of those points at which EC* assumes minimum values (Curve 1) is also plotted as a function of μ in Fig. 1. Since $b = 4$, the coefficient of variation with respect to the characteristic value of the parent strength τ_o is 0.25. Therefore, these optimum proof loads truncate the strength distribution at 3.2σ, 2.5σ, 1.8σ and 1.3σ below its characteristic value respectively for $\gamma = 10^{-1}$, 10^{-2}, 10^{-3} and 10^{-4}. Also plotted in Fig. 1 is the probability of failure as a function of μ. The probability decreases monotonically as μ increases; the reliability increases as a larger proof load is applied.

The above result indicates that, for this particular example, performing the proof-load test may not be justified if γ is of the order of 10^{-1} because (1) the optimum proof stress is more than 3σ away from the characteristic strength and therefore not much improvement in statistical confidence in reliability estimation is expected and (2) if one increases the magnitude of the proof load beyond the optimum value to achieve such improvement, the prohibitive cost is likely to be incurred due to possible loss of the (candidate) structure(s) which is rather expensive (larger value of γ). However, if γ is of the order of 10^{-2} or less, performing the proof-load test appears justified from the point of view of improving (1) the statistical confidence in the reliability estimation (since the points of truncation are at most 2.5σ away from the characteristic value) and (2) the reliability itself. However, the optimum magnitude of the proof-load increases considerably as γ decreases. This may present some difficulty in performing the proof-load test.

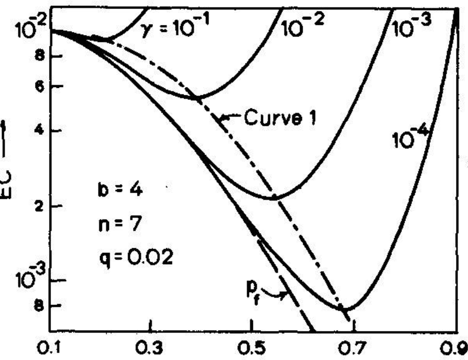

Fig. 1 $\mu \longrightarrow$

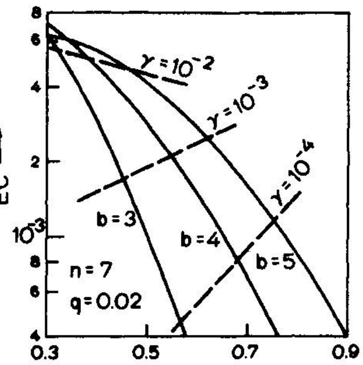

Fig. 2 $\mu \longrightarrow$

Since the preceding observation is based on (1) the computation associated with a particular set of parameters, (2) the particular form of the expected cost and (3) the specific form of strength and load distributions, and sensitivity of these items on the result will be an interesting subject of future study. For example, Fig. 2 shows the loci of the optimum points (such as Curve 1 in Fig. 1) for b = 3, 4 and 5 plotted on the same diagram, indicating the effect of b and γ on the optimum proof-load.

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SUMMARY

The interrelationship among the probability of structural failure, the expected cost of structure and the proof-load testing is established and used for a general reliability analysis. The optimum proof-load is defined for structures designed under a conventional design code and conditions are examined under which the proof-load testing is advantageous economically as well as from the viewpoint of improving both the reliability itself and the statistical confidence in such a reliability estimate.

RESUME

On examine la relation entre la probabilité de ruine, le prix évalué de la construction et les essais de charge. La charge d'essai optimale est définie pour les structures conçues d'après les normes conventionnelles. Puis on examine les conditions sous lesquelles les essais de charge sont aussi bien avantageux économiquement qu'utiles pour la sécurité et pour la certitude de la sécurité évaluée.

ZUSAMMENFASSUNG

Aufgezeigt wird die Beziehung zwischen der Bruchwahrscheinlichkeit, dem Erwartungswert der Kosten sowie der Prüflast und für die Zuverlässigkeitsrechnung verwendet. Die optimale Prüflast wird für nach alten Vorschriften entworfene Bauwerke definiert. Sodann werden die Bedingungen untersucht, für welche das Prüflastverfahren sowohl wirtschaftlich als auch im Hinblick auf die Zuverlässigkeit selbst und das Vertrauen in eine solche Zuverlässigkeitsschätzung vorteilhaft ist.

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The probability of failure and safety of structural section loaded with a multi-dimensional force-combination

La probabilité de rupture et la sécurité d'un élément de structure chargé avec une combinaison multidimensionnelle de forces

Die Versagenswahrscheinlichkeit und die Sicherheit eines mit einer vieldimensionalen Kraftkombination belasteten Bauteiles

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The problem of reliability has been discussed in several papers during recent years and, as we know, many methods have been developed to solve this question. These solutions usually aim at determining the probability of failure of the observed structure.

As far as the writer is aware, the calculation methods are all quite approximate, and the mathematical difficulties have prevented the development of more exact solutions.

However, the character of the problem, means that there is a need for a mathematically satisfactory design method. The purpose of research in this subject is to take rational account of the irregularities of material, dimensions, loads and calculations, e.g. by a number called "safety factor".

If the calculation method by which the safety factor is determined is very approximative, we are actually obliged to use a complementary factor to eliminate all the unreliabilities which are included in the calculation of this factor. This is, of course, not desirable.

The development of the computers in the last years has made it possible to solve more complicated mathematical problems and to reach a higher degree of exactness of results than before. The following work presents an attempt to solve the probability of failure of a structural element using a computer, by a method which the writer supposes to be general enough and to contain a number of approximations, which gives a sufficient exactness for practical purposes.

This paper is to a great extent partial abbreviation of a larger study, supposed by the Scandinavian Building Institutes. The study forms part of a joint Scandinavian project and will be published by the State Building Research Institute in Denmark.

1. The necessity of a general kind of frequency-function.

A central problem in the calculation of the probability of failure is the combining of several known fr.f. (frequencyfunctions) which are connected with each other by some known function. The result of such combinations is a new fr.f., which cannot generally be determined exactly. On the other hand, also the form of the initial distributions is in most cases unknown and to be estimated from the sample.

In addition to these aspects it is necessary to avoid the errors caused by small samples. We will return this later.

To comply with the requirements mentioned above, the following fr.f. has been chosen for use in the one-dimensional case:

$$(1) \qquad f(x) = e^{\sum_{k=o}^{n} a_k \cdot x^k}$$

The parameter a_o will be determined so that

$$F(\infty) = \int_{-\infty}^{+\infty} f(x) \cdot dx = 1$$ where x represents an arbitrary quantity, which has an influence on the probability of failure, e.g. a property of a material, a dimension of a structure or a load.

Without paying more attention to the following question, we need only mention that, e.g.,

- the normal distribution
- the log-normal distribution
- the first asymptotic distribution of the extreme value
- the Weibull-distribution

all converge toward (1) with increasing n.

For the distribution function we use

$$(2) \qquad F(x) = e^{-e^{\sum_{k=o}^{n} a_k{}^k}}$$

and in the multi-dimensional cases anologically to (1) and (2)

$$\begin{cases} (3) \quad f(x) = e^{\sum\limits_{k_1=0}^{n_1} x_1^{k_1} \sum\limits_{k_2=0}^{n_2} x_2^{k_2} \cdots \sum\limits_{k_r=0}^{n_r} x_r^{k_r} \cdot a_{k_1 k_2 \ldots k_r}} \\ \\ (4) \quad F(x) = e^{-e^{\sum\limits_{k_1=0}^{n_1} x_1^{k_1} \sum\limits_{k_2=0}^{n_2} x_2^{k_2} \cdots \sum\limits_{k_r=0}^{n_r} x_r^{k_r} \cdot a_{k_1 k_2 \ldots k_r}}} \end{cases}$$

With increasing $n_1 \ldots n_r$-values in (3) and (4) we can estimate multi-dimensional samples with arbitrary moments and also define distributions with very varying forms.

2. Estimation of the parameters of the various distributions.

For large samples we use either of two estimation methods, both well known from the statistical literature. The simpler is the method of moments, introduced by K Pearson, and the more developed is the method of maximum likelihood introduced by R.A. Fisher. In this connection, it is not sensible to explain either of these methods.

For small samples we use the following, more complicated method of estimation.

We first assume that the parent population has a general fr.f. $f(x, a_o \ldots a_m)$ where the parameters $a_1 \ldots a_m$ are assumed to be unknown. The parameter a_o is a function of $a_1 \ldots a_m$ so that $F(\infty) = 1$. The sample values of x are $x_1 \ldots x_n$.

We then study the situation after one value of the sample, x_1 has been found. In this case the fr.f. of a parameter combination can be represented by

$$(5) \quad g_1(a_o \ldots a_m) = \frac{f(x_1, a_o \ldots a_m)}{\int\limits_{R_m} f(x_1, a_o \ldots a_m) \cdot da_1 \ldots da_m}$$

The result has been found by examining a conditional frequency function of $a_1 \ldots a_m$, relative to the hypothesis $x = x_1$. We assume then that before any values of the sample are known the fr.f. $f(x, a_o \ldots a_m)$ is represented by an m + 1-dimensional fr.f. where m-dimensional marginal distribution in the space $a_1 \ldots a_m$ is rectangular.

If we then assume that we take n values from the same unknown population of the form $f(x, a_o \ldots a_m)$, we again get a conditional distribution

$$(6) \qquad g_n(a_o \ldots a_m) = \frac{\prod_{k=1}^{n} f(x_k, a_o \ldots a_m)}{\int_{R_m} \left(\prod_{k=1}^{n} f(x_k, a_o \ldots a_m) \right) da_1 \ldots da_m}$$

Function (6) represents the combined distribution of parameters $a_o \ldots a_m$ on the basis of the sample $x_1 \ldots x_n$. If we now define the distribution of the value x_{n+1}, we evidently obtain a fr.f. of this value:

$$(7) \quad h(x) = \frac{\int_{R_m} f(x, a_o \ldots a_m) \cdot \prod_{k=1}^{n} f(x_k, a_o \ldots a_m) \cdot da_1 \ldots da_m}{\int_{R_m} \prod_{k=1}^{n} f(x_k, a_o \ldots a_m) \cdot da_1 \ldots da_m}$$

The formula (7) can now be applied to arbitrary types of distributions. It has the advantage that the mistakes which can be made using the method of moments or the method of maximum likelihood with small samples can be avoided.

4. Capacity of a structural element.

The failure of a structural element can be defined by one or several inequalities (9), assuming that this element is loaded with a k-dimensional combination of forces and moments.

These inequalities can be illustrated in a k-dimensional space R_k so that the different types of failure each form a k-dimensional set of points in R_k, which have an infinite volume and are formed as sectors.

These sets are limited in relation to each other by k-1 dimensional hyper-surfaces, and each set is divided into two subsets, the first containing all the points which cause failure and the second containing all the combinations by which failure does not occur.

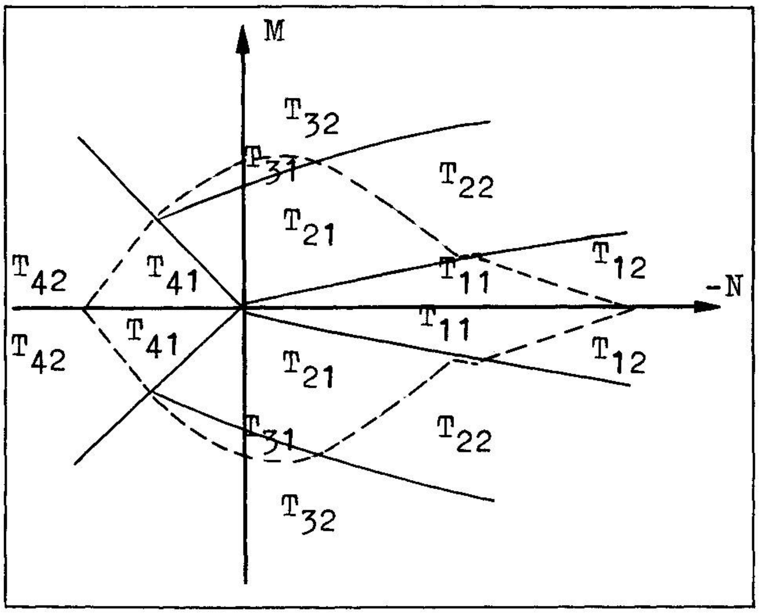
We get the equations:

$$(8) \quad \begin{cases} \sum_{j=1}^{r} T_j = R_k \text{ where } T_i \cdot T_j = 0 \text{ when } j \neq i \\ T_j = T_{j1} + T_{j2} \quad \text{where } T_{j1} \cdot T_{j2} = 0 \end{cases}$$

$$(8)\quad \begin{aligned} &\sum_{j=1}^{r} T_{j1} = U_1 \\ &\sum_{j=1}^{r} T_{j2} = U_2 = U_1^* \end{aligned}$$

where the set $\sum_{j=1}^{r} T_{j2} = U_2$ represents the points in the space $R_k(S_1 \ldots S_k)$ which cause failure of the element and the complementary set U_1, the points where no failure is produced. Here $S_1 \ldots S_k$ represent the external forces.

These parts are also represented in fig. 1, which shows an example of the different possibilities of failure by a rectangular reinforced concrete element.



Usually on the basis of empirical studies and statics we can write

$$(9)\quad \begin{cases} g_1(x_1 \ldots x_n, S_1 \ldots S_k) \leqq 0 \\ \ldots \\ g_r(x_1 \ldots x_n, S_1 \ldots S_k) \leqq 0 \end{cases}$$

where every inequality gives one type of a condition of failure. Here $x_1 \ldots x_n$ represent the internal properties of the element and $S_1 \ldots S_k$ the external forces. Anyhow, every inequality requires a group of supplementary conditions which separate the different types of failure from each other.

In this way from (8) and (9) we get as the complete condition of failure

$$(10)\quad \begin{cases} (g_1 \leqq 0 \wedge g_{11} \leqq 0 \wedge \ldots \wedge g_{1m_1} \leqq 0) \\ \vee(g_2 \leqq 0 \wedge g_{21} \leqq 0 \wedge \ldots \wedge g_{2m_2} \leqq 0) \\ \ldots \\ \vee(g_r \leqq 0 \wedge g_{r1} \leqq 0 \wedge \ldots \wedge g_{rm_r} \leqq 0) \end{cases}$$

We have already been able to define the fr.f. of the factors $x_1 \ldots x_n$. These can usually be considered as independent, and so we can write:

$$(11) \quad f(x_1 \ldots x_n) = f_1(x_1) \ldots f_n(x_n)$$

Using the quantities $S_1 \ldots S_k$ as parameters for every combination of $S_1 \ldots S_k$ we get the probability of failure through the integration:

$$(12) \quad h(S_1 \ldots S_k) = P(42) = \int_{(10)} f(x_1 \ldots x_n) \cdot dx_1 \ldots dx_n$$

where P(10) indicates the probability that (10) is valid and the region of the integration signifies the part of the space R_k where the inequalities (10) are valid.

Without further consideration of the question of the integration above, it may be noted that there are simplifying methods to solve the integral (12) so that it is not necessary to operate in n dimensions.

In this way we have been able to determine the function (12) to represent the probability of failure of the known structural element as a function of the k-dimensional combination of forces. The next problem is to define the fr.f. of the external forces which load this element.

4. Transformation of the loads into forces and moments.

By the determination of the probability of failure there is a fundamental difference between the invariable and variable loads, since the variable loads are considered as inconstant with time, and the invariable loads are considered to retain their size during the life time of the construction. The difference in the calculation is that the forces and moments caused by the invariable loads are of direct importance, while the variable loads and the forces caused by them are not of interest in themselves, but only the corresponding extreme values appearing during the lifetime of the construction.

By both types of loads we have to change the fr.f. of the loads into fr.f. of the forces. This will be done in both cases in a similar way, which will be presented below.

In most cases the mutual dependence of the loads and the forces can be given in the following form:

$$(13) \quad \begin{cases} a_{11} \cdot q_1 + \ldots + a_{1m} \cdot q_m = S_1 \\ \ldots \\ a_{k1} \cdot q_1 + \ldots + a_{km} \cdot q_m = S_k \end{cases} \quad \text{or } A \cdot q = S$$

The parameters $a_{11}\ldots a_{km}$ can usually be considered as constants. If this is not the case, the solution will have a complementary complication, which will be explained later. In principle we have three different cases; $m < k$, $m = k$, $m > k$. We assume here that the rank of matrix A is m, or in the last case k.

Without the deduction of the following formulas, we have as the fr.f. of $S_1\ldots S_k$ in the three different cases:

$m = k$

$$f_s(S_1\ldots S_k) = \left[f_{q_1}(q_1 = c_{11}\cdot S_1+\ldots+c_{1k}\cdot S_k)\ldots f_{q_k}(q_k = c_{k1}\cdot S_1+\ldots+c_{kk}\cdot S_k)\right]\cdot \frac{1}{\begin{vmatrix} a_{11}\ldots a_{1k} \\ \ldots \\ a_{k1}\ldots a_{kk}\end{vmatrix}} \tag{14}$$

Here c is a reciprocal matrix of A.

$k < m$

$$f_s(S_1\ldots S_k) = \int_{R_{m-k}} f_{q_1}(q_1=c_{11}\cdot S_1+\ldots+c_{1m}\cdot q_m)\ldots f_{q_k}(q_k = c_{k1}\cdot S_1+\ldots+c_{km}\cdot q_m)\cdot f_{q_{k+1}}(q_{k+1})\ldots f_{q_m}(q_m)\cdot \frac{1}{\begin{vmatrix} a_{11}\ldots a_{1k} \\ \ldots \\ a_{k1}\ldots a_{kk}\end{vmatrix}}\, dq_{k+1}\ldots dq_m \tag{15}$$

$k > m$

$$f_s(S_1\ldots S_k) = \left[f_{q_1}(q_1=c_{11}\cdot S_1+\ldots+c_{1k}\cdot S_k)\cdot\ldots\cdot f_{q_k}(q_k= c_{k1}\cdot S_1+\ldots+c_{kk}\cdot S_k)\right]\cdot \frac{1}{\begin{vmatrix} a_{11}\ldots a_{1k} \\ \ldots \\ a_{k1}\ldots a_{kk}\end{vmatrix}} \tag{16}$$

The difference between (14) and (16) is that the fr.f. given in (16) is limited in the degenerate part of the space R_m, where $S_{k+1}\ldots S_m$ have the values:

$$\begin{cases} S_{m+1} = c_{m+1,1}\cdot S_1+\ldots+c_{m+1,m}\cdot S_m \\ \ldots \\ S_k = c_{k1}\cdot S_1+\ldots+c_{km}\cdot S_m \end{cases} \tag{17}$$

5. Definition of the probability of failure by a structural element.

We have now in R_k two different fr.f. for external forces which have been found in the way explained in 4. We also have the fr.f. of those internal quantities of the element, which are independent:

$$(18)\quad f_y(y_1 \dots y_m) \cdot dy_1 \dots dy_m = fy_1(y_1) \dots fy_m(y_m) \cdot dy_1 \dots dy_m$$

The exact solution of the probability of failure, which is our goal, can be obtained by integrating all the possibilities by which the sum of the forces produced by the variable and invariable loads at some time during the life-time of the construction exceeds the capacity of the structural element.

This probability can be found by the following formula:

$$(19)\quad P(y<S) = \int_{R_m} f_y(y) \int_{R_k} f_{S_g}(S_g) \cdot \left\{ 1 - \left[\int_{T_k} f_{S_p}(S_p)\, dS_{p1} \dots dS_{pk} \right]^N \right\} \cdot dS_{g1} \dots dS_{gk} \cdot dy_1 \dots dy_m$$

In this formula the set T_k gives the k-dimensional set defined in the following way:

T_k is the set of combinations which form the complementary set to (10), actually the set U_1 in (8). The difference is, however, that $x_1 \dots x_n$ have been changed into $y_1 \dots y_m$ by gradual integration, and the values $S_1 \dots S_k$ in (9) are represented by $S_{g_1} + S_{p_1}, \dots S_{g_k} + S_{p_k}$.

The value N gives the relation between the life-time of the construction and the interval which has been used to define the d.f. of the variable loads in an arbitrary moment.

We assume that T_k is a set of points which fulfil the following reguirement:

$$(20)\quad g(g_1(S_{g_1} + S_{p_1}, \dots, S_{g_k} + S_{p_k}),\ g_2(y_1 \dots y_m)) > 0$$

Writing

$$(21)\quad f_{S_{pe}}(S_{p_1} \dots S_{p_k}) = N \left[\int_{(20)} f_{S_p}(S_{p_1} \dots S_{p_k}) \cdot dS_{p_1} \dots dS_{p_k} \right]^{N-1} \cdot f_{S_p}(S_{p_1} \dots S_{p_k})$$

Through a rather complicated deviation, we get the probability of failure (19) in the following relatively simple form:

$$(22)\quad P(y<S) = \int_{R_k} f_{S_g+S_{pe}}(S_g + S_{pe}) \cdot h(S_g + S_{pe}) \cdot d(S_g + S_{pe})$$

where $-f_{S_g+S_{pe}}(S_g + S_{pe})$ is the k-dimensional fr.f. of the sum of forces caused by invariable loads and the extremevalue of variable loads.

$-h(S_g + S_{pe})$ is the function from (12).

6. Definition of the probability of failure by a structure.

To define the probability of failure by a structure is a much more complicated question than the reliability of a single element of this structure. Work on this branch has already begun, and some of the main aspects, which seem to be important, are as follows:

- whether the material of the structure is brittle or tough
- the number of different possibilities of structure failure
- the number of critical sections by different types of failure
- the interdependence of the capacity of these sections.

7. Determination of the method of design the structural element.

In 5. we have been able to find a method of determining the probability of failure of a structural element. However, this does not give us the necessary information, as to what methods we should use to determine the right dimensions of this element. Because we strive for a certain, suitable probability of failure $P_1(S_q > S_y)$, we write (22) in the form

$$(23) \quad P_1(S_q > S_y) = \int_{R_k} f_{S_q}({}^{S}q/\alpha^k) \cdot h(S_q) \cdot {}^1/\alpha^k \cdot dS_q$$

and solve the value which corresponds to the probability $P_1(S_q > S_y)$ which has been chosen in the beginning of the calculation. For this value we can usually use $10^{-6} - 10^{-8}$.

The value α gives us the possibility to see, what nominal values $x_1 \ldots x_n$, $q_1 \ldots q_m$, $p_1 \ldots p_m$ we have to use in the calculation to find structures, which have the probability of failure $P_1(S_q > S_y)$. After this we maybe have the possibility of finding such methods of calculation, which are simple enough to use for an engineer who does not know the statistical basis of these methods, and at the same time achieve the same probability of failure in various parts of the structure. This should also be our goal.

Symbols:

- x - quantities, which have influence on the probability of failure.
- q - loads
- S - forces and moments loading the structural element
- S_g- forces and moments loading the structural element, caused by invariable loads.
- S_p- forces and moments loading the structural element, caused by invariable loads.
- S_{pe}-forces and moments loading the structural element, caused by extreme values of variable loads.
- S_q- forces and moments loading the structural element, caused by total load.
- S_y- forces and moments representing the capacity of the structural element.
- α - a scale coefficient

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SUMMARY

A method to determine the probability of failure by different structural elements is presented, based on the use of a computer. It treats a general case where the element is loaded with a multidimensional combination of forces and moments. The paper has four mainthemes: Estimation of the parameters of the various distributions; Capacity of a structural element; transformation of the loads into forces and moments; and definition of the probability of failure.

RESUME

On présente une méthode pour déterminer la probabilité de rupture causée par différents éléments de structure et basée sur l'emploi d'un ordinateur. La méthode traite le cas général de l'élément chargé par une combinaison multidimensionelle de forces et de moments. Cet article a quatre thèmes principaux: l'estimation des paramètres de différentes distributions, la résistance d'un élément de structure, la transformation des charges en forces et en moments et la définition de la probabilité de rupture.

ZUSAMMENFASSUNG

Man hat eine Methode für die Bestimmung der Versagenswahrscheinlichkeit bei verschiedenen Konstruktionselementen dargelegt. Die Theorie fusst auf der Anwendung elektronischer Rechenmaschinen. Ein allgemeiner Fall, wo das Element mit einer multidimensionalen Kombination von Kräften und Momenten belastet ist, wird behandelt. Der Artikel ist in vier Hauptthemen aufgeteilt: Schätzung der Parameter der verschiedenen Verteilungen, die Tragfähigkeit des Konstruktionselementes, die Transformation der Lasten in Kräfte und Momente und die Bestimmung der Versagenswahrscheinlichkeit.

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II

Factors of Safety for Structural Design

Coefficients de sécurité pour le calcul des constructions

Sicherheitsfaktoren für den Entwurf

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Introduction

Sufficient statistic information on the probability of deviations from mean values of both stress and strength still lacking for the next future, the structural engineer needs a clear and simple method of evaluating adequate factors of safety in design practice.

Factors of Safety Composed of Partial Coefficients

As outlined earlier /1/ the factor of safety has to prevent actual stress from becoming equal to actual strength. Possible deviations from the mean values of both stress and strength assumed in the design calculations can be accounted for by partial coefficients /2/ considering all influences of any importance.

1. Influences on stress:
   (a) loads,
   (b) design calculations,
   (c) adaptability of structure,
   (d) type of failure.

2. Influences on strength:
   (a) strength of construction material,
   (b) workmanship,
   (c) section-size of member,
   (d) type of load.

The partial coefficient characterizing the uniformity of a value has been defined as the possible deviation from the mean for a certain probability /1/.

Table I: Partial Coefficients for Structural Design

Group No.	Influence group	Partial coefficient
1(a)	Loads	
	Standardized (dead, live, snow load; wind and water pressure; temperature changes; earth quake acceleration)	1,o
	Non-standardized (earth and ice pressure; air-blast from weapons)	1,2
1(b)	Design calculations	
	Interpolated from measurements	1,1* (1,o)
	Extrapolated from measurements	1,2* (1,1)
	Not based on measurements	1,3* (1,2)
	*) valid for the probable loading combination only, to be reduced for the most unfavorable loading combination to values in brackets	
1(c)	Adaptability of structure	
	Linear systems	
	(1) statically determinate	1,1
	(2) statically indeterminate	1,o
	Plane and spatial systems	o,9
1(d)	Type of failure	
	With warning (preceding deformations)	1,o
	Without warning (brittle failure or instability)	1,1

Table I: continued

	Progressive failure	1,2
	Catastrophic consequences	1,3 to 1,5
2(a)	Construction material	
	Steel and aluminum	o,9
	Timber and plastics	o,8
	Concrete	
	(1) ready-mixed	o,7
	(2) mixed-in-place	o,6
2(b)	Workmanship	
	Excellent	1,o
	Average	o,9
	Poor or unknown	o,8
2(c)	Section-size of member	
	Big	1,1
	Average	1,o
	Small	o,9
2(d)	Type of load	
	Static Dynamic Vibration Impact Fatigue Effect of (1) time (2) temperature	Introduced as reference or response strength

With the above values for the partial coefficients K the factor of safety is calculated with formula

$$S = \frac{K_{1a} \times K_{1b} \times K_{1c} \times K_{1d}}{K_{2a} \times K_{2b} \times K_{2c}} \qquad (1)$$

Numerical Example

A completely worked example for the reinforced concrete skeleton of a multi-story office-building will explain how the factor of safety is calculated from partial coefficients.

Loads standardized	$K_{1a} = 1{,}o$
Design calculations extrapolated from measurements	$K_{1b} = 1{,}2$
Linear system statically determinate (columns)	$K_{1c} = 1{,}1$
Plane system (flat slab)	$K_{1c} = o{,}9$
Failure with warning deformations (flat slab)	$K_{1d} = 1{,}o$
Progressive failure (columns)	$K_{1d} = 1{,}2$
Strength of ready-mixed concrete	$K_{2a} = o{,}7$
Strength of reinforcement	$K_{2a} = o{,}9$
Workmanship average	$K_{2b} = o{,}9$
Section-size average	$K_{2c} = 1{,}o$

(a) Members mainly in bending stress (flat slab)

$$S = \frac{1{,}o \times 1{,}2 \times o{,}9 \times 1{,}o}{o{,}7 \times o{,}9 \times 1{,}o} = 1{,}7 \text{ for concrete}$$

$$S = \frac{1{,}o \times 1{,}2 \times o{,}9 \times 1{,}o}{o{,}9 \times o{,}9 \times 1{,}o} = 1{,}3 \text{ for reinforcement,}$$

(b) Members mainly in direct stress (columns)

$$S = \frac{1{,}o \times 1{,}2 \times 1{,}1 \times 1{,}2}{o{,}7 \times o{,}9 \times 1{,}o} = 2{,}5 \text{ for concrete}$$

$$S = \frac{1{,}o \times 1{,}2 \times 1{,}1 \times 1{,}2}{o{,}9 \times o{,}9 \times 1{,}o} = 2{,}o \text{ for reinforcement.}$$

References

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SUMMARY

Departing from a definition of its mission the factor of safety is composed of partial coefficients taking into account all possible influences on both stress and strength. Numerical values of the partial coefficients are given for design purposes. The method described is illustrated with a typical numerical example.

RESUME

Partant de la définition de sa mission, le facteur de sécurité est composé de coefficients partiels prenant en considération toutes les influences possibles aussi bien sur les contraintes que sur les résistances. Des valeurs numériques sont données pour les coefficients partiels applicables dans la pratique. La méthode décrite est illustrée par un exemple numérique caractéristique.

ZUSAMMENFASSUNG

Von der Definition seiner Aufgabe ausgehend, wird der Sicherheitsfaktor aus Partialkoeffizienten zusammengesetzt, die alle möglichen Einflüsse sowohl auf die Beanspruchungen als auch auf die Festigkeiten berücksichtigen. Zahlenwerte der Partialkoeffizienten für Entwurfszwecke werden mitgeteilt. Die beschriebene Methode wird mit einem typischen Zahlenbeispiel erläutert.

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