Probability considerations in design and formulation of safety factors

Probability Considerations in Design and Formulation of Safety Factors

Considérations des probabilités dans la conception des projets et dans la formulation des facteurs de sécurité

Wahrscheinlichkeitsbetrachtungen beim Entwurf und bei der Ableitung von Sicherheitsbeiwerten

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The analysis of structural safety requires a two-sided activity. On one side is the description of the loading environment and the analysis of load effects; on the other side, we have the description of material properties and the prediction of structural capacity. These may be referred to, respectively, as "stress analysis" and "strength analysis". Results of these analyses then form the basis for design. It is in the consideration of safety and serviceability that the results of stress and strength analyses become meaningful.

Except for the simplest cases, however, the analysis of the loading and its associated load effects, and the analysis of structural capacity necessarily entails a number of factors whose influences on the accuracy of the design calculations are difficult, if not impossible, to assess. Such factors as the unknown inaccuracies arising from the idealization of the loading function and structural system, the assumptions underlying all analyses and failure prediction formulas, and the unknown variances of construction and fabrication, are indeed difficult to evaluate. These difficulties are compounded by the fact that loads and material properties are generally statistical variables; moreover, available data are invariably limited such that estimates of the required statistical parameters are approximate at best. Thus, even if statistical information can be modeled with probability concepts, the difficulties associated with the unknown uncertainties cited above and the general lack of data to properly evaluate the necessary parameters, still remain. That is, the use of statistical and probability models cannot circumvent the above difficulties. These uncertainties can only be treated subjectively through the exercise of engineering judgments, which may be in the form of multiplicative or additive factors. Alternatively, such judgments may be expressed in the form of judgmental probabilities; this serves to express the unknown uncertainties in terms of subjective probabilities, which are, however, unfamiliar and thus confusing in general to engineers at this time. Nevertheless, in appropriate situations, such judgmental probabilities may be a suitable alternative to the conventional form of expressing engineering judgment.

$$P(R < R_p) = p$$

$$P(S > S_q) = q$$

The basic requirement for safety against a specified limit state is then expressed in terms of the characteristic values as,

$$R_p = \gamma \, S_q \qquad\qquad (1)$$

where R_p is the required structural capacity, and γ is the overall safety factor. In more general terms, γ is composed of several components γ_m, γ_s, γ_c, called the 'partial safety factors" [1]. The safety factor $\gamma > 1.0$ (or its constituent partial safety factors) is necessary to take account of the unknown uncertainties and other considerations, as well as the influence of statistical variabilities (e.g., for steel $\gamma_m = 1.15$ whereas for concrete $\gamma_m = 1.50$ are the recommended values of CEB on the grounds that concrete has a wider statistical dispersion of strengths than steel).

It might be observed that using the probability-based nominal values R_p and S_q, the major influences of statistical variabilities have already been accounted for through Eq. (1); on this basis, the calculated design resistance will increase with the degree of statistical dispersion even if the same value of γ were used. The use of larger values for γ in situations where large dispersions are expected must, therefore, be to take care of the eventualities of encountering $R < R_p$ and/or $S > S_q$. These eventualities can and ought to be treated in the context of probability; i.e., the influence of statistical variabilities on γ can be evaluated objectively.

Classical Reliability Theory

Much has been written on the classical reliability theory, beginning with the early papers of Freudenthal [4], Pugsley [5], and Prot and Levi [6]. However, it should be emphasized that relative to structural safety, the classical reliability theory is predicated on the tacit assumption that the statistical distributions of the loading and structural resistance are known precisely, and that there are no other imponderables and uncertainties in the analysis of structural safety. In the premise of the classical theory, structural safety becomes solely a problem of determining the risk associated with the statistical variabilities of the load and strength. The safety of a structure is then measured by the "probability of survival" or reliability, and conversely the "probability of failure" is the calculated risk against an unsatisfactory performance or collapse. That is, if the random load (or load effect) is S, and the structural resistance is R, then assuming no other effects, failure can be defined as the occurrence of the event $(R < S)$; accordingly, in general terms, its probability is

$$P_f = P(R < S) = \int_0^\infty F_R(s) f_S(s) \, ds \qquad\qquad (2)$$

where F_R and f_S are, respectively, the distribution and density functions of R and S. This can be calculated simply if R and S are both normal random variables; i.e.,

Other considerations in design must include the importance and projected use of a structure, and the possible consequences in case of damage or collapse. Also, when treating combined loadings, consideration must be given to the reduced likelihood of encountering two or more extreme loads at the same time.

MODERN BASES OF STRUCTURAL SAFETY

The basic concepts underlying two modern approaches to structural safety are reviewed briefly below: these are namely, the limit-state approach [1]* which is the basis of the European Concrete Committee recommendations for safety [2], and the classical reliability theory [3]. The classical reliability theory offers the correct rationale for the treatment of statistical variables in structural safety consideration, whereas the limit-state format offers the necessary flexibility to account for unknown uncertainties and the simplicity required for conventional design implementation. These features can be combined in a consistent and logical manner to yield a formulation which retains a basic simplicity necessary for practical implementation. This review is presented, therefore, to identify the technical advantages and shortcomings of these methods, for the purpose of showing that capitalizing on the best features of each of these two methods, a third method emerges which is tantamount conceptually to a generalization of the reliability theory incorporating the basic format and intent of the limit-state approach.

Limit-State Approach

Loads and structural material properties are often statistical variables, such that there is no single load nor structural capacity that can be used in design without some risk of encountering some unfavorable state of performance, including collapse, because the no-risk load would be excessively too high whereas the no-risk capacity may require an absurdly massive structure. For purposes of design, it is therefore sensible to specify nominal values of loads and structural capacities on the basis of finite probability levels. In this regard, the consideration of safety would dictate that the nominal value for resistance must be on the low side, whereas the corresponding value of the load must be on the high side of the respective ranges of possible values. This observation naturally leads to the conclusion that the most appropriate nominal values are the "characteristic strength" and "characteristic load" as defined in the limit-state approach.

In general, the characteristic resistance and characteristic load are R_p and S_q which, for normal variates, are

$$R_p = \bar{R}(1 - k_p \delta_R)$$

$$S_q = \bar{S}(1 + k_q \delta_S)$$

where: $\bar{R}$ and $\bar{S}$ are the mean resistance and mean load (or load effect),

δ_R and δ_S are the coefficients of variation of R and S,

k_p is the number of standard deviation σ_R between R_p and $\bar{R}$,

k_q is the number of standard deviation σ_S between S_q and $\bar{S}$.

In more general terms, R_p and S_q are values corresponding to specified probability levels, and can be defined as follows:

* Number in brackets corresponds to reference cited.

$$P_f = \frac{1}{\sqrt{2\pi}} \int_{\frac{\bar{R}-\bar{S}}{\sqrt{\sigma_R^2 + \sigma_S^2}}}^{\infty} e^{-\frac{1}{2}x^2} \, dx \qquad (2)$$

which is easily evaluated using tables of normal probabilities. The probability of survival, or reliability, then is simply

$$P_s = 1 - P_f$$

For specified probability distributions, the probability of failure p_f is related to the safety factor γ, as defined in Eq. (1). For example, if R and S are normal variates, it is clear from Eq. (2) that,

$$\frac{\bar{R} - \bar{S}}{\sqrt{\sigma_R^2 + \sigma_S^2}} = \Phi^{-1}(1 - p_f) \equiv k_{pf}$$

where $\Phi^{-1}(1 - p_f)$ is the value of the standard normal function at a cumulative probability of $(1 - p_f)$. From this equation, we obtain,

$$R_p = \frac{1 - k_{pf}^2 \delta_S^2}{1 - k_{pf}\sqrt{\delta_R^2 + \delta_S^2 - k_{pf}^2 \delta_R^2 \delta_S^2}} \left(\frac{1 - k_p \delta_R}{1 + k_q \delta_S}\right) S_q$$

and the safety factor, therefore, is

$$\gamma = \frac{1 - k_{pf}^2 \delta_S^2}{1 - k_{pf}\sqrt{\delta_R^2 + \delta_S^2 - k_{pf}^2 \delta_R^2 \delta_S^2}} \left(\frac{1 - k_p \delta_R}{1 + k_q \delta_S}\right)$$

where k_{pf} is the number of standard deviation $\sqrt{\delta_R^2 + \delta_S^2}$ that $(\bar{R} - \bar{S})$ is above zero, such that the failure probability is equal to p_f.

Clearly, therefore, k_{p_f} is a function of p_f; for example from tables of normal probabilities, the values of k_{p_f} for specific values of p_f are given in the second column of Table 1, from which we obtain the safety factors given in the third and fourth columns of the same table.

TABLE I: VALUES OF γ WITH p = 0.10, q = 0.01

P_f	k_{pf}	γ for δ_R=.15, δ_S=.20	γ for δ_R=.20, δ_S=.20
10^{-3}	3.090	1.206	1.467
10^{-4}	3.719	1.466	2.152
10^{-5}	4.265	1.788	3.656
10^{-6}	4.753	2.215	10.509

The safety factors formulated using other distribution functions for R and S can be similarly evaluated; as shown in Fig. 1, the safety factor γ varies widely for a given P_f depending on the assumed distribution function.
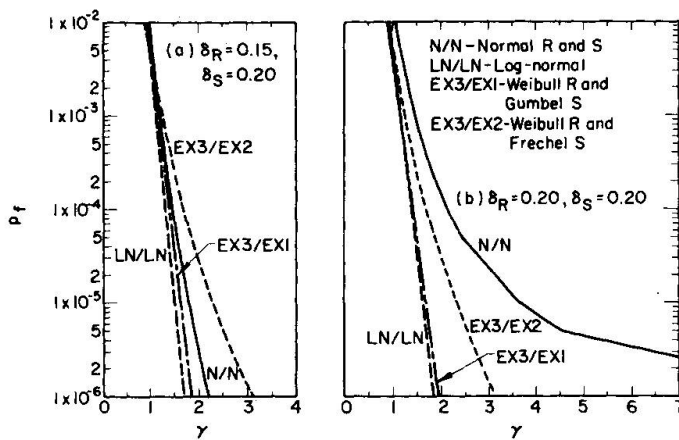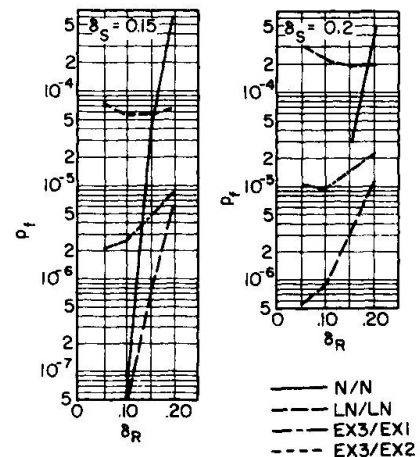

FIG. 1 SENSITIVITY OF γ TO DISTRIBUTION FUNCTIONS (CLASSICAL RELIABILITY BASIS)

FIG. 2 INFLUENCE OF DISTRIBUTION FUNCTIONS ON PROBABILITIES OF FAILURE

There are other formidable difficulties and limitations associated with the classical reliability theory relative to its practical design implementation; practical design situations are invariably shrouded with many uncertainties and unknowns, not all of which are necessarily statistical or probabilistic. These difficulties have been emphasized by Freudenthal [7], which we quote as follows:

1. "the existence of non-random phenomena affecting structural safety which cannot be included in a probabilistic approach,"

2. "the impossibility of observing the relevant random phenomena within the ranges that are significant for safety analysis, and the resulting necessity of extrapolation far beyond the range of actual observation,"

3. "the assessment and justification of a numerical value for the 'acceptable risk' of failure, and"

4. "the codification of the results of the rather complex probabilistic safety analysis in a simple enough form to be usable in actual design."

It should be emphasized that the first three difficulties quoted above are especially significant because in the range of failure probabilities that may be considered acceptable (10^{-4} to 10^{-6} or less) the calculated probabilities of failure are extremely sensitive to the underlying distribution functions of R and S, as illustrated in Fig. 2. These distributions, however, are most difficult to ascertain because of the general lack of data. As expected, this sensitivity is reflected also on the design obtained from a specified failure probability [8], as well as on the safety factor, as shown in Figs. 1 and 3.

In summary, the classical reliability concept is an idealized theory based on assumptions and requirements that are not tenable in practice. Nevertheless, it is a sound and necessary formalism for any rational analysis of structural safety.

EXTENDED RELIABILITY CONCEPT

Basic Principles

From the above review, we recognize that it is desirable to have a method that can overcome the shortcomings but that would retain the rationality of the reliability concept, and possesses the practical flexibility of the limit-state approach. Such a method also should not be too sensitive to the distribution functions of the statistical variables but should reflect the influences of the major statistical variabilities through certain key quantities such as the means and variances (or coefficients of variation) without necessarily knowing the precise underlying distributions. A method developed on the basis of the "extended reliability concept" [8] comes close to fulfilling all of these requirements.
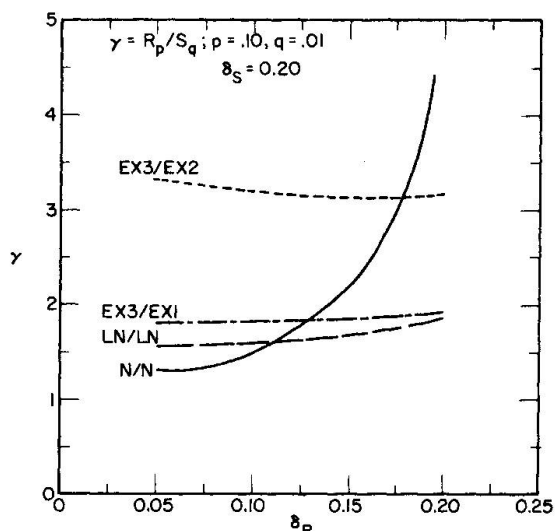

FIG. 3   VARIATION OF γ WITH δ_R BASED ON CLASSICAL RELIABILITY ; $P_f = 10^{-6}$

Following the basic format of the limit-state approach, the unknown uncertainties are covered by a nominal requirement in terms of characteristic values,

$$R_p = \nu \, S_q \qquad\qquad (3)$$

where ν is a "judgment factor," and is necessarily greater than 1.0 to take account of the unknown uncertainties. The factor ν must be determined using engineering judgment in much the same way that the γ-factor is chosen in the limit-state approach. However, in contrast to the factor γ, the factor ν does not include the influence of known statistical variabilities.

Since ν is in reality an ignorance factor, $(R < \nu S)$ must represent a state of unsatisfactory performance or unsafety; therefore, by requiring Eq. (3) alone, the safety of a structure may still be jeopardized if $R < \nu S$, which will occur primarily when $R < R_p$ or $S > S_q$. The logical measure of the occurrence of such eventualities is the probability $P(R < \nu S)$, which can be called the "probability of unsafety", and is clearly a generalization of the classical failure probability. Hence, an additional requirement for structural safety must be,

$$P(R < \nu S) \leq \alpha \qquad\qquad (4)$$

where α is a small probability necessary to insure that the occurrence of $R < \nu S$ is sufficiently rare.

In other words, Eq. (3) is a nominal requirement for safety; however, with this nominal requirement imposed, the remaining question is: "In view of

statistical variabilities, what is the reliability of this nominal design against these latter eventualities?" This reliability is measured by $P(R < \nu S)$, and Eq. (4) accordingly serves to assure a required level of this reliability. Clearly, if R and S are both deterministic, then Eq. (3) is sufficient; whereas, known statistical variabilities should be treated with probabilistic models and Eq. (4) is the appropriate model for this purpose consistent with Eq. (3).

Significantly, it turns out that if the nominal requirement, Eq. (3), is imposed, the risk or probability of unsafety is bounded [8] as follows:*

$$pq < P(R < \nu S) < (p + q - pq) \tag{5}$$

It might be emphasized again that $(R < \nu S)$ will occur primarily when $R < R_p$ or $S > S_q$; but the first part of Eq. (5) says that the probability of such an occurrence is greater than pq. Hence, if Eq. (3) is required, there is no point in specifying the acceptable probability α to be less than pq.

In view of the minimum possible value of the probability of unsafety indicated in Eq. (5), the two requirements for structural safety, i.e. Eqs. (3) and (4), can both be satisfied by the following single requirement:

$$P(R < \nu S) = \alpha; \quad \text{with } \alpha \not< pq \tag{6}$$

Thus, Eq. (6) is the desired basis for design, and the evaluation of safety factors in design. It can be observed that Eq. (6) is similar to the safety requirement of the classical reliability approach. In fact, the probability of unsafety reduces to the classical failure probability if $\nu = 1.0$. In this case it is significant to observe that Eq. (5), which remains valid, means that if $R_p = S_q$ is nominally required the associated probability of failure is also bounded as $p_f > pq$.

Formulation of Safety Factors

Through Eq. (6), specific design formulas can then be derived for given distribution functions of R and S [8]. For example, if R and S are normal variates, Eq. (6) yields

$$R_p = \nu \, \frac{1 - k_\alpha^2 \delta_S^2}{1 - k_\alpha \sqrt{\delta_R^2 + \delta_S^2 - k_\alpha^2 \delta_R^2 \delta_S^2}} \left(\frac{1 - k_p \delta_R}{1 + k_q \delta_S} \right) S_q$$

and the requisite safety factor, γ of Eq. (1), therefore is

$$\gamma = \nu \, \frac{1 - k_\alpha^2 \delta_S^2}{1 - k_\alpha \sqrt{\delta_R^2 + \delta_S^2 - k_\alpha^2 \delta_R^2 \delta_S^2}} \left(\frac{1 - k_p \delta_R}{1 + k_q \delta_S} \right) \tag{7}$$

where $k_\alpha = \phi^{-1}(1-\alpha)$. It might be emphasized that δ_R is the overall measure of variation of the appropriate resistance, which may consist of the variations of several factors or components; e.g., dispersions in material properties and geometrics of structural members, which may be functions of workmanship quality. For example, in the formulas for bending capacity of an under-reinforced

*Eq. (5) really refers to a conditional probability; i.e., the probability of $(R < \nu S)$ given $R_p = \nu S_q$, or $P(R < \nu S | R_p = \nu S_q)$.

concrete beam, $M_u = f_y A_s jd$, δ_R is the coefficient of variation of M_u, which is a function of the variations in f_y, A_s, and d. Similarly, δ_S may also consist of the variations from several contributory factors or components.
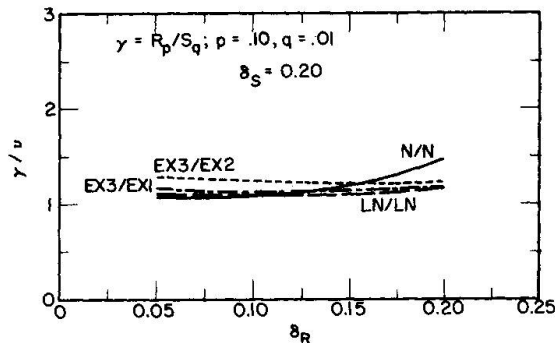

FIG.4   VARIATION OF γ/ν WITH δ_R BASED ON EXTENDED RELIABILITY

Formulas for the safety factor γ corresponding to other probability distributions for R and S can be similarly derived on the basis of Eq. (6). The expressions obtained for these other distribution functions will be different from that of Eq. (7); however, the calculated values of γ for the same ν and coefficients of variation will not differ much. In other words, γ obtained on the basis of Eq. (6) will not be too sensitive to the distribution functions of R and S, as can be seen from the results presented in Fig. 4, which should be contrasted with those of Fig. 3

The formulation typified by Eq. (7) clearly distinguishes the unknown uncertainties from the known statistical variabilities; the unknown uncertainties are handled through a subjective factor ν, whereas observed statistical variabilities are handled by the remaining factor which is a function of the coefficients of variation. This distinction is important. On this basis, it emphasizes that statistical information should not be confused with ignorance and should be handled objectively through appropriate probability models. The vagueness that is unavoidable in the exercise of judgment, which is necessary in the consideration of subjective factors, is however unnecessary when treating information with measured statistical dispersions.

The part of the safety factor necessary to account for unknown uncertainties, i.e. ν, should theoretically remain constant unless the state of ignorance changes; in any event, this part should not change with the measured variability of the observed statistical information. The overall safety factor γ, of course, may change with the degree of statistical dispersion, but this can be done objectively and more consistently with the form suggested by the extended reliability approach.

We observe from Fig. 4 that the variation of the factor γ/ν with δ_R (or δ_S) depends on the distribution functions of R and S. For certain distributions, this factor may even decrease with δ_R (and δ_S) as shown in both Figs. 3 and 4; this is because γ is described in terms of R_p and S_q which are also functions of δ_R and δ_S, respectively. However, it should be emphasized that in spite of this, the resulting designs will always increase with δ_R and δ_S.

For the normal distribution, however, γ/ν is monotonically increasing with δ_R, which is perhaps a desirable property from the standpoint of consistency with conventional thinking. Since the extended reliability approach is somewhat independent of the distribution functions, the normal function therefore may be adopted for general design applications. However, if information or data suggests that other distributions are more appropriate, such distributions can always be used to obtain more precise designs at the expense of more involved computational efforts.

Design Codification

One of the purposes of the proposed extended reliability concept is for the formulation and evaluation of safety factors to be used in a design code.

For this purpose, values of ν must be given for appropriate situations; these values require subjective analysis and may be obtained in much the same way that the partial safety factors are currently obtained, and may similarly be decomposed into several sources of uncertainties; e.g., $\nu = \nu_r \nu_s$, in which ν_r is the judgmental correction necessary to take account of the unknowns in the prediction of resistance; and ν_s is the corresponding factor to include the possible inaccuracies in the analysis of the load and load effects, and the unlikely occurrence of two or more extreme loads at the same time. These factors may each be further broken down into components if necessary to facilitate analysis, as suggested in the limit-state approach [1].

The influences of measured or known statistical variabilities should not be included in the subjective analysis of ν, since these are evaluated through the formula given in Eq. (7).

In its initial implementation, the value of ν may be evaluated on the basis of current designs; i.e., assuming typical values of certain parameters, its value should be such that the same safety factor is obtained as currently used. For example, suppose that based on the recommendations of the CEB, the overall safety factor is $\gamma = \gamma_m \gamma_s \gamma_c = 1.80$, in which the characteristic values are assumed to be based on $p = .05$ and $q = 0.02$, whereas $\delta_R = 0.20$ and $\delta_S = 0.25$; then in order to obtain the same design for this typical case, the judgment factor ν, according to Eq. (7), must be

$$\nu = 1.80 \left(\frac{0.132}{0.400}\right)\left(\frac{1.512}{0.670}\right) = 1.34$$

For subsequent designs of the same or similar types of structures under similar conditions, this value of ν must be held constant, whereas depending on the quality of material and variability of the loadings, the value of the safety factor γ would vary in accordance with Eq. (7).

Other considerations, such as the importance and projected use of a structure, may be taken into account through the specification of the nominal design load S_q; the S_q for an important structure intended for human occupany should correspond to a smaller q than a structure of lesser importance.

SUMMARY AND CONCLUSIONS

Uncertainties in design can be identified to be of two types; namely, unknown uncertainties arising from the lack of perfect knowledge and information, and measured statistical variabilities. The unknown uncertainties can be treated only subjectively through the use of engineering judgments, whereas known statistical information can and should be treated objectively using probability concepts.

The probability-based characteristic load and resistance are suitable nominal design values. In terms of these characteristic values, the unknown uncertainties can be accounted for through a "judgment factor" (or factors) expressed nominally in a conventional format; in these terms, statistical variabilities are also largely accounted for. The remaining concern is then primarily the risk against having a resistance less than the characteristic value or encountering a load greater than the specified characteristic load. However, in view of the nominal requirement, this risk is theoretically limited by a lower bound. Thus, the acceptable risk need not be smaller than the indicated minimum.

The essence of the proposed extended reliability concept can be summarized as follows:

If,
$$R_p = \nu S_q \tag{3}$$

then
$$P(R < \nu S) > pq \tag{5}$$

for all values of ν, including $\nu = 1.0$. Hence, pq is an acceptable risk when Eq. (3) is nominally required, and the appropriate basis for safe design is,

$$P(R < \nu S) = \alpha \tag{6}$$

with $\alpha \nleq pq$. On this basis, reliability-based design procedures (in conventional form) can be developed that are not too sensitive to the assumed distribution functions, thus permitting the adoption of the normal distribution for most practical purposes. However, the approach also allows the use of other distributions if necessary and warranted.

In the context of the above extended reliability concept, a design safety factor derived from Eq. (6) consists of two parts--a subjective part represented by ν, and an objective part for evaluating the influence of statistical information. In this way, the variation of the safety factor with statistical dispersions can be evaluated systematically and objectively.

ACKNOWLEDGMENT

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SUMMARY

    Unknown uncertainties in design are formulated in terms of a
nominal requirement through a subjective judgment factor. In view
of this nominal requirement, the risk against unfavorable per-
formance due to statistical variabilities is theoretically limited.
Hence, a minimum acceptable risk is available to permit the for-
mulation of an extended reliability basis for safe design and
evalutation of safety factors.

RESUME

    Les variables aléatoires inconnues, dans la conception des
projets, sont exprimées sous forme d'une exigence nominale grâce
à un facteur subjectif de jugement. Considérant ce facteur arbi-
traire, la détermination du risque d'un comportement insatis-
faisant créé par les variations statistiques, est théorétiquement
limitée. Ainsi, un risque minimal acceptable est utile afin de
permettre la formulation de bases sérieuses pour une conception
sûre et pour l'évaluation des coefficients de sécurité.

ZUSAMMENFASSUNG

    Mit Hilfe eines subjektiven Beurteilungswertes werden die
unbekannten Unsicherheiten beim Entwurf in Gliedern einer Nenn-
anforderung ausgedrückt. Im Hinblick auf diese Nennforderung
ist das Risiko gegen unerwünschtes, aus statistischer Streuung
hervorgerufenes Verhalten theoretisch begrenzt. Dadurch wird ein
kleinstes, annehmbares Risiko nutzbar für die Formulierung eines
sicheren Entwurfes sowie der Sicherheitsbeiwerte aufgrund eines
erweiterten Zuverlässigkeitsbereiches.

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