A probability model for low-cyclic fatigue

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A Probability Model for Low-Cyclic Fatigue

Un modèle de probabilité pour la fatigue à basse fréquence

Ein Wahrscheinlichkeitsmodell für nieder-zyklische Ermüdung

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Due to the random properties of material, the relationship ($\mathcal{S}$, $\mathcal{N}$) describing the dependence between the ultimate number of cycles, $\mathcal{N}$, and the ultimate load-effect, $\mathcal{S}$, is also random. Its statistical treatment makes no difficulties for middle and high ranges of load repetitions (Fig.1 b,c), however for the low-cyclic fatigue loading no method was available until now. On the other hand, a good knowledge of the ($\mathcal{S}$, $\mathcal{N}$)-relationship is necessary whenever reliabilistic considerations are to be applied to the low--cyclic problems.

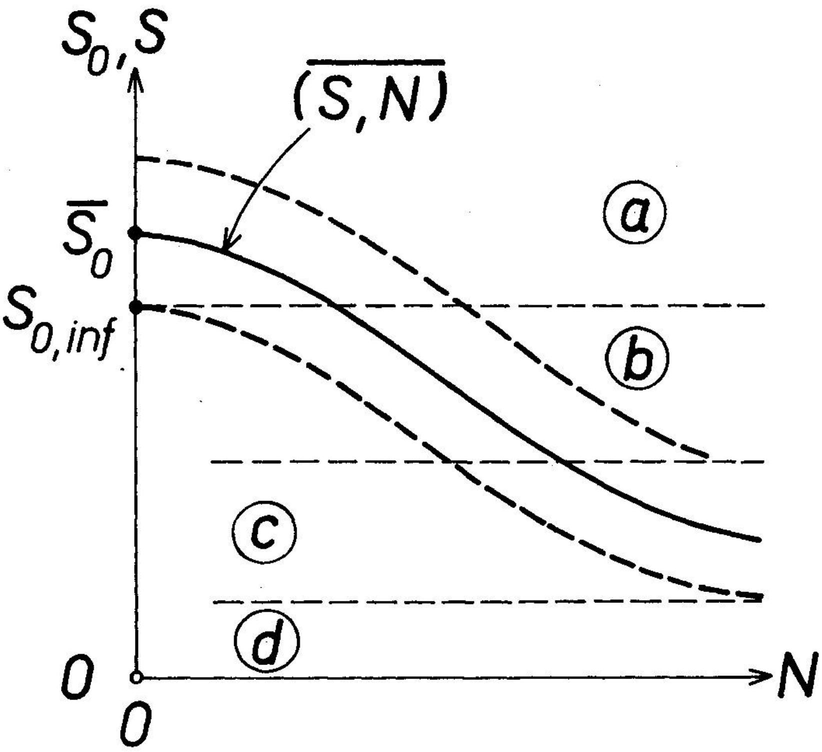

Fig.1. Fatigue ranges: (a) low-cyclic,
(b) middle-cyclic,
(c) high-cyclic,
(d) no fatigue range

The problem of fatigue is essentially two-dimensional since in the / $\mathcal{S}$, $\mathcal{N}$ / space two random variables are dealt with. According to the theory of interaction diagrams /1/ the quantiles of the ($\mathcal{S}$, N)-relationship might be found by investigating $\mathcal{S}$ under given N . Due to the nature of the phenomenon such a procedure is of course impossible and, consequently, testing of N at preliminarily selected levels of the ultimate load-effect, $\mathcal{S}$, is the only way to reach the results.

Whereas for middle and high cyclic loading only one random variable, N , enters the solution, two random variables appear in the range of low-cyclic loading. Referring to Fig.1a the low--cyclic fatigue range may be defined probabilistically as that range where under the first loading ($N = 0$) a certain probability of failure already exists. Evidently, testing a sample of specimens, if the level of fatigue loading, $\mathcal{S}_f$, is chosen so that

$$\mathcal{S}_f > \mathcal{S}_{0,inf}$$

(where $\mathcal{S}_{0,inf}$ is the lowest possible ultimate load-effect, $\mathcal{S}_0$, under single loading), certain amount of specimens fails before the $\mathcal{S}_f$ level is reached at all. Consequently, no repeated loading is possible for the failed specimens whereas the surviving specimens can be subjected to various number of load repetitions.

The ultimate number of cycles, N , is defined for a specimen as the number attached to the last completed loading cycle. It is known that fractions of cycles cannot be measured (which is particularly due to the non-linearity of the stress-strain relationship in the low-cyclic fatigue domain); the ultimate number of cycles, N , is integer. Thus, the random variable N is discrete, defined by probability mass function $p(N)$, Fig.2. On the other hand, the random variable $\mathcal{S}_0$ is obviously continuous defined by probability density function φ ($\mathcal{S}_0$).

Sumarizing these observations: the random behaviour of the ultimate load-effect at the low-cyclic loading must be modelled by a statistical distribution which consists of a continuous and a discrete portion. The random variable changes its character: in the first portion it is the ultimate load-effect (strength of material, ultimate strain, ultimate moment, etc. – according to the type of the problem), in the discrete portion it is the ulti-

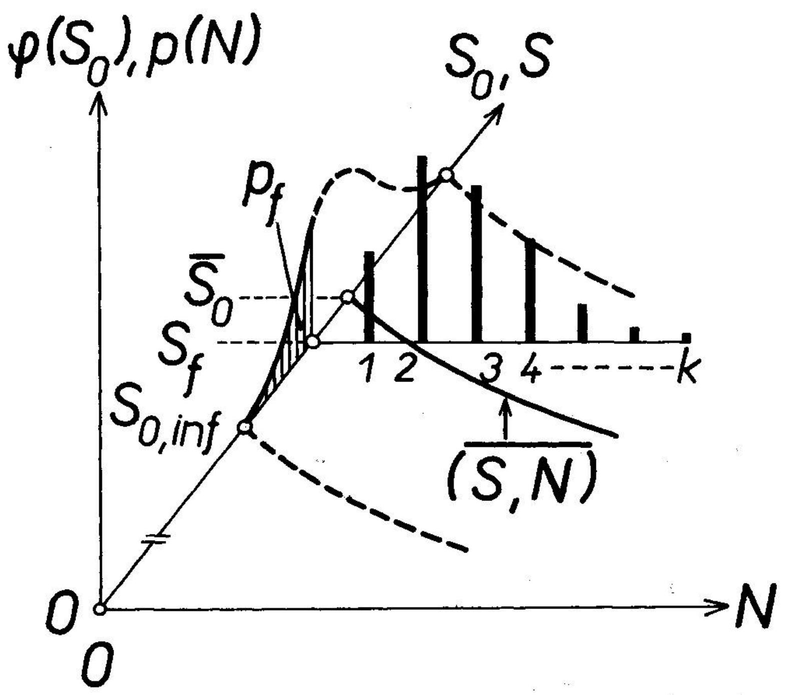

Fig.2. The broken probability distribution of the low-cyclic fatigue phenomenon

mate number of cycles, N. It is evident that both portions are mutually complementar, forming together a complete probability distribution. Let it be called "broken distribution".

The usual aim of a statistical treatment is to establish descriptors of the distribution such as the mean, variance, etc., and various quantiles, particularly the median, 0.05 quantile (characteristic strength) etc. To make this possible also with a phenomenon described by the broken distribution, the following simple procedure proves to be feasible:

Both variables, S_0 and N, are transformed by dividing individual realisations through the respective value at the break-point of the distribution, i.e. by $S_0 = S_f$ and $N = 1$, respectively. Obviously, in the continuous portion it is

$$0 \leqq S_0 / S_f \leqq 1,$$

in the discrete portion (since $N/1 = N$)

$$1 \leqq N \leqq k$$

where k is the highest observed (or possible) value of N.

The n-th general moment of the distribution is

$$\mu_n = \int_0^1 \left(\frac{S_0}{S_f}\right)^n \varphi\left(\frac{S_0}{S_f}\right) d\,\frac{S_0}{S_f} + \sum_{N=1}^{k} N^n p(N),$$

hence the mean is

$$\mu_1 = \int_0^1 \frac{S_0}{S_f}\,\varphi\left(\frac{S_0}{S_f}\right) d\,\frac{S_0}{S_f} + \sum_{N=1}^{k} N\,p(N)$$

and the n-th central moment

$$\nu_n = \int_0^1 \left(\frac{S_0}{S_f} - \mu_1\right)\varphi\left(\frac{S_0}{S_f}\right) d\,\frac{S_0}{S_f} + \sum_{N=1}^{k} (N-\mu_1)^n\,p(N).$$

The variance or other moment parameters can be found by usual procedure.

The above formulae can be easily adapted to formulae giving sample moments.

If quantiles for specified probabilities p are to be determined, the continuous portion of the broken distribution is used for $p \leqq p_f$, and the discrete portion for $p > p_f$; here it is

$$p_f = p\,[S_0 \leqq S_f].$$

The type of probability distribution describing the continuous portion of the broken distribution follows from the distribution of the ultimate load-effect under single loading; e.g. normal, log-normal or any other suitable distribution may be selected. Nothing can be said at present however, about the distribution type of the ultimate number of cycles, N . In some solutions, it may prove practical to substitute the discrete distribution of N by a continuous function. Further research is here necessary.

/1/ TICHÝ, M., and VORLÍČEK, M.: Statistical Theory of Concrete Structures. Irish University Press, Shannon- Academia, Prague, 1972.

SUMMARY

For the statistical treatment of the low-cyclic fatigue problem a broken probability distribution can be used. It consists of a continuous portion referred to the ultimate number of cycles. Moments and other descriptors of the probability distribution can be obtained by means of transforming both variables.

RESUME

Pour l'étude statistique du problème de la fatigue à basse fréquence on peut utiliser une répartition probabiliste non continue. Ceci consiste en une partie continue relative à la charge ultime et une partie discrète relative au nombre de cycles à la rupture. Les moments et autres valeurs décrivant la répartition probabiliste peuvent être obtenus au moyen d'une transformation des deux variables.

ZUSAMMENFASSUNG

Zur statistischen Behandlung zyklischer Ermüdungsprobleme mit kleiner Amplitude kann eine diskrete Wahrscheinlichkeitsverteilung benutzt werden. Sie besteht aus einem durchgehenden Teil, der sich auf den Bruchlasteffekt bezieht, und einem diskreten Teil, der sich auf die maximale Anzahl der Zyklen bezieht. Momente und andere Indikatoren der Wahrscheinlichkeitsverteilung lassen sich durch Transformieren der beiden Variablen bestimmen.

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