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Physical-Mathematical Models and Theoretical Considerations

Modèles physico-mathématiques et considérations théoriques

Physikalisch-mathematische Modelle und theoretische Überlegungen

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1. INTRODUCTION

Theoretical studies of slender compression members tend to fall into two distinct categories, depending upon whether or not effects such as load eccentricity, out-of-straightness, and geometric and material imperfections are taken into account. In one approach, perfectly axial loads are considered to act on an ideal structural system and the critical load, P_{cr}, is determined at which bifurcation of equilibrium occurs. In the other approach, load eccentricity, end moments and other effects are considered, and lateral deflections are found to increase with increasing load. The load typically increases at a decreasing rate, until the peak value P_u is reached. A fall-off in load then occurs, with a further increase in deflection. The two types of analysis are illustrated in Fig. 1.

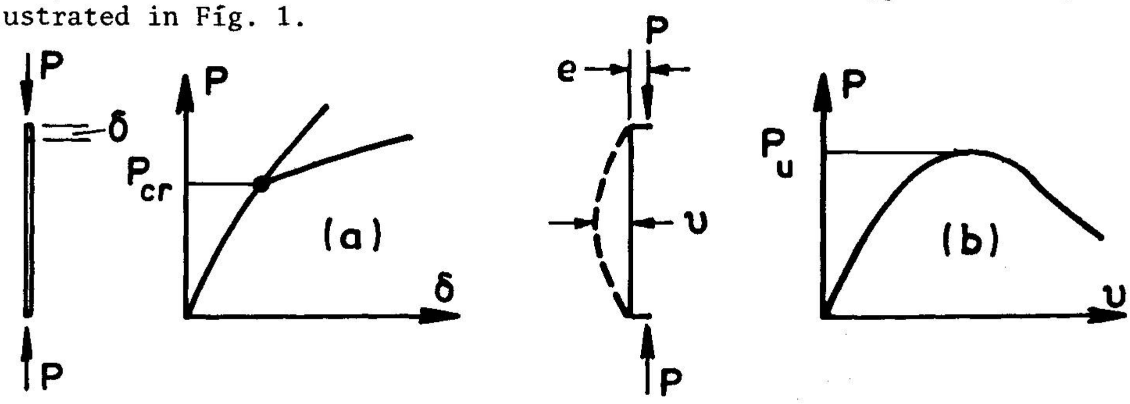


FIG. 1: METHODS OF ANALYSIS

Although information obtained from the idealized bifurcation load concept finds some application in current design procedures (1,9), theoretical studies of reinforced concrete compression members tend to concentrate on the calculations of the load carrying capacity P_u.

The behaviour of slender concrete compression members is complicated considerably by creep and shrinkage effects, and special consideration must be given to time-varying behaviour under sustained loadings. However, concrete compression members tend to be stocky in comparison with metal compression members, and in practice slenderness effects are often of negligible importance. In such cases, analysis and design are considerably simplified, in that load carrying capacity of a "short" member can be equated to the ultimate strength of the section.

Theoretical methods of analysis are here considered for both short and slender members subjected to short-term and sustained loadings. It is to be emphasized that the prime interest is in underlying concepts, and no attempt is made to provide a systematic review of previous research work; this latter task has been undertaken recently by several technical committees for the joint ASCE-IABSE Conference on Tall Buildings, and the reader is referred to relevant reports (22,26).

Theoretical structural analysis proceeds typically from an analytic representation of relevant material properties, to an evaluation of the load deformation characteristics of a cross section, and thence to an analysis of the behaviour of elements and frames. It is convenient to follow this sequence here, and in the following section the behaviour of plain concrete under direct compressive stress is discussed.

2. MATERIAL PROPERTIES

The response of plain concrete to short-term direct compressive stress, shown schematically in Fig. 2a, is distinctly non-linear and anelastic. Simplified representations are required for the analysis of reinforced concrete behaviour, the most frequently used models being linear-elastic (Fig. 2b), non-linear elastic (Fig. 2c), and non linear elastic-plastic (Fig. 2d). More complicated and more realistic models have been developed for special studies of the ductility and behaviour of structural concrete under cylces of loading and unloading (36).

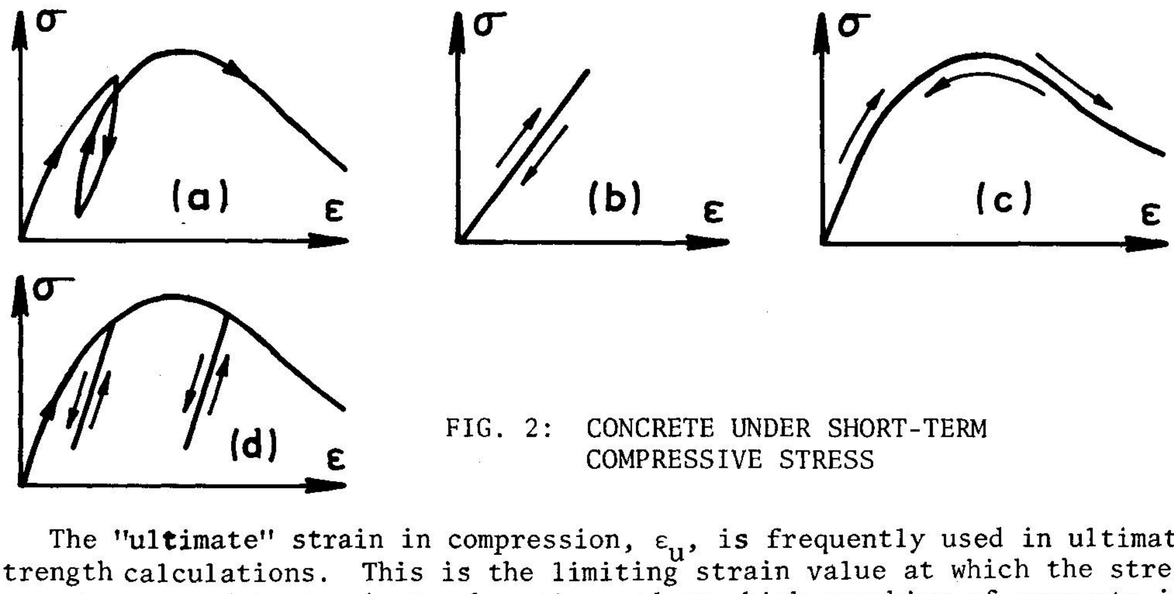


FIG. 2: CONCRETE UNDER SHORT-TERM COMPRESSIVE STRESS

The "ultimate" strain in compression, ε_u, is frequently used in ultimate strength calculations. This is the limiting strain value at which the stress-strain curve is assumed to terminate abruptly, and at which crushing of concrete is supposed to occur in bending. Experimental evaluation of ε_u is a most difficult procedure, since ε_u apparently depends on a wide range of factors, not least important of which are the deformation characteristics of the testing machine. An alternative approach, which may be preferable in fundamental studies, is to allow the stress-strain curve to extend indefinitely, and to identify failure of a piece of material, of a cross-section, or of a structure, as the state reached when the rate of increase of applied load is zero (19, 37, 42). This approach also allows theoretical determination of suitable values of ε_u for use in simplified calculation procedures.

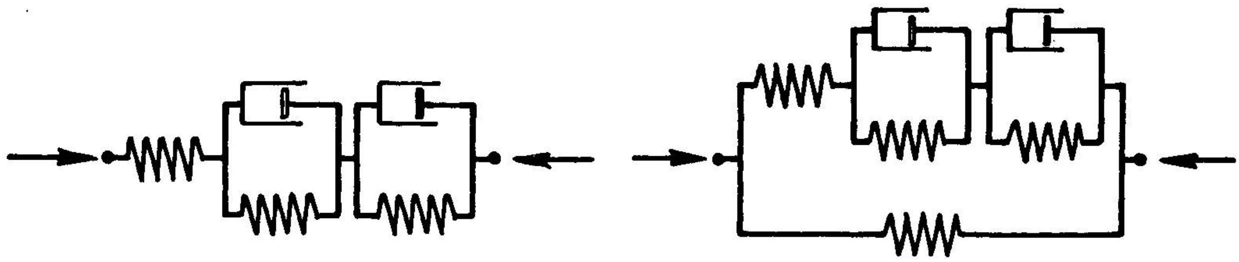
The tensile strength of plain concrete is low in comparison with the compressive strength, and is usually neglected in calculations of the behaviour and strength of compression members.

Analysis of the time-dependent properties of plain concrete is made on the assumption that total strain is made up of instantaneous, creep and shrinkage components which are independent and additive.

$$\varepsilon(t) = \varepsilon_i(t) + \varepsilon_k(t) + \varepsilon_{sh}(t) \qquad (2.1)$$

The shrinkage component is taken to be stress independent, and is evaluated from measurements on unstressed test specimens. Although this phenomenological approach is open to to question and criticism, a practical computational alternative has yet to be proposed.

Linearity of creep with respect to stress level appears to be a reasonably accurate assumption provided the stress remains low; i.e., less than some limiting value σ_c. Values quoted for σ_c range widely, between 25 and 50 percent of the crushing strength. Concrete creep in the linear range is bounded, partially time-hardening and partially recoverable.



(a) Plain Concrete (b) Reinforced concrete

FIG. 3: RHEOLOGICAL MODELS

Rheological models proposed for concrete creep usually contain a number of Kelvin-Voigt elements in series (14). A reasonably simple model of the linear viscoelastic phase is shown in Fig. 3a. The first Kelvin-Voigt element is recoverable, and represents viscoelastic deformation; the second is non-recoverable, and represents remaining creep. A wide assortment of rheological models of concrete is reviewed by Neville (29). However, very few of these have been used in studies of structural concrete behaviour. One exception is the model proposed by Torroja and Paez (38).

The creep component $\varepsilon_k(t)$ is affected by the entire stress history, and both differential and integral expressions have been developed for the analysis of creep strains in the presence of time-varying stress.

One of the simplest and most widely used analytic representations of concrete creep was developed by Dischinger (10) on the assumption of complete time hardening. For a constant sustained stress, $\sigma(t) = \sigma \leq \sigma_c$, a creep function is defined,

$$\phi(t) = \varepsilon_k(t)/\varepsilon_i \tag{2.2}$$

which can be evaluated from a simple creep test. Because of the linearity assumption, $\phi(t)$ is a pure time function, independent of stress level. The creep strain resulting from a constant stress first applied at time τ, $0<\tau<t$, is taken to be

$$\varepsilon_k(t,\tau) = (\phi(t) - \phi(\tau))\ \varepsilon_i \tag{2.3}$$

Eq. 2.3 is an expression of the Whitney parallel creep curve assumption. The rate of creep, $\dot{\varepsilon}_k(t)$, is dependent on current stress level but independent of previous stress history,

$$\dot{\varepsilon}_k(t) = \frac{\sigma(t)}{E_c}\ \dot{\phi}\ (t) \tag{2.4}$$

and the total stress-dependent rate of change of strain is thus

$$\dot{\varepsilon}(t) = \frac{\dot{\sigma}(t)}{E_c} + \frac{\sigma(t)}{E_c}\ \dot{\phi}(t) \tag{2.5}$$

Eq. 2.5 can be formulated in various ways. It is similar in all essential respects to the "rate-of-creep" method.

Perhaps the most widely criticized aspect of the Dischinger creep law is that it predicts complete time hardening; i.e., that predicted creep deformation approaches zero as τ becomes large. Various modifications have therefore been proposed, ranging from a simple lumping of the non-ageing component in with the elastic component (30), to the use of higher order differential equations (23).

Integral expressions for linear creep give more flexible analytic models, but also require much more extensive test data and can lead to a mathematically more complicated analysis of structural behaviour. To allow for partial time-hardening, the creep function of Eq. 2.2 must be taken to depend not only on the time t of the process, but also on the age at loading, τ. The creep strain at time t produced by a stress increment $\Delta\sigma(\tau)$ at time τ is thus

$$\Delta\varepsilon_k(t,\tau) = \frac{\Delta\sigma(\tau)}{E_c}\ \phi(t,\tau) \tag{2.6}$$

An integral expression for the total creep strain at time t is obtained by superposing the strains produced by all stress increments (25),

$$\varepsilon_k(t) = \int_{\tau=0}^{t} d\varepsilon_k(t,\tau) = \frac{1}{E_c}\int_{\tau=0}^{t} \frac{\partial\sigma(\tau)}{\partial\tau}\ \phi(t,\tau)d\tau \tag{2.7}$$

With the creep strain per unit stress (specific creep) expressed as

$$C(t,\tau) = \frac{1}{E_c}\ \phi(t,\tau) \tag{2.8}$$

and evaluated experimentally, the total stress-dependent strain is obtained as

$$\varepsilon(t) = \frac{\sigma(t)}{E_c} + \int_{\tau=0}^{t} \frac{\partial\sigma(\tau)}{\partial\tau} C(t,\tau)d\tau \qquad (2.9)$$

Eqs. 2.5 and 2.9 and various modifications of these equations have been used in studies of compression member behaviour in the linear creep range. Non-linear modifications have also been proposed for the analysis of sustained overload behaviour (15,41).

By considering the stress history to consist of a constant major component equal either to the initial value $\sigma(0)$ or the final value $\sigma(\infty)$, and a variable secondary component, convenient expressions for analysis can be derived from Eq. 2.9 (13). These have been applied to structural concrete by Trost (39).

The analytic treatment of the properties of reinforcing steel is relatively straightforward, and elastic-plastic relations for both tensile and compressive stress are usually used. Curvilinear and multilinear relations are sometimes used to give a more realistic representation of post-elastic behaviour.

3. LOAD-DEFORMATION CHARACTERISTICS OF STRUCTURAL CONCRETE IN COMPRESSION AND BENDING.

Theoretical studies of the behaviour and load carrying capacity of reinforced concrete compression members usually require a preliminary analysis of the load-deformation characteristics of a short segmental length of member. Of particular importance are the rotation and lateral shortening which occur as the result of an eccentrically applied thrust. A simplified representation for the case of short-term loading is the moment-thrust-curvature relation, which allows the curvature K to be determined for any combination of thrust P and moment M acting simultaneously on the cross-section. In more complicated loading cases, where cycles of loading and unloading and periods of sustained loading are involved, there is no simple useful way of representing the load-deformation characteristics, and the deformation history must be evaluated for the particular loading history.

3.1 Axial Loading

In the idealized case of a short-term axial load applied to a steel-concrete section, the curve of load versus axial shortening is obtained from the stress-strain curves for the component materials on the assumption of full strain compatibility, as shown in Fig. 4 (37).

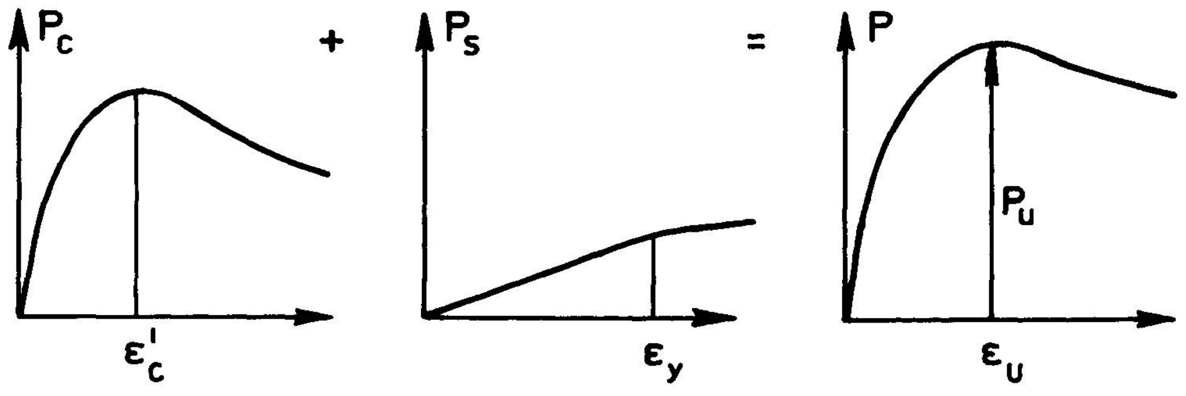


FIG. 4: SHORT-TERM AXIAL LOADING

The load carrying capacity, P_u, is determined by the condition

$$\frac{dP}{d\varepsilon} = 0 \tag{3.1}$$

The failure strain ε which occurs at P_u is also determined by Eq. 3.1.

Some practical difficulty arises in the calculation of P_u, in that concrete properties (including the maximum concrete stress σ_u) in the compression member are likely to differ somewhat from those determined from specimen tests.

Fig. 4 and Eq. 3.1 cease to be valid if premature failure occurs by splitting or spalling of the outer layers of concrete. This might or might not be triggered by outward buckling of the reinforcing bars. Minimum cover and minimum lateral reinforcement requirements usually prevent separation of the outer concrete, even in the case of encased steel sections. However, confinement of the inner concrete core by large amounts of lateral ties and helical reinforcement can have an effect on the overall response to axial thrust. Lateral confinement increases the crushing strength σ_u and also changes the shape of the stress-strain relation. Kent and Park (20) suggest that lateral ties in rectangular sections are not completely effective in confining the concrete, except in the corners, and that the main effect of the ties is to improve ductility in the post-ultimate, unloading stage. Tests by Rüsch and Stöckl (34) indicate that the strength of core concrete is considerably increased by helical reinforcement. However, their test data indicate apparent inadequacies in various physical models which have been proposed for the quantitative analysis of the carrying capacity of helically reinforced columns. Serviceability problems involving premature spalling are not likely to be of practical importance in compression members, provided neither very small nor very large quantities of lateral or helical reinforcement are used.

The effect of sustained axial loading can be represented in terms of a rheological model by placing a linear spring (representing compressive reinforcement) in parallel with the model of plain concrete. This is shown in Fig. 3b. A very significant transfer of force from concrete to steel occurs with time, because of the apparent softening of the concrete. When the section contains very small amounts of reinforcement, light sustained loading, coupled with concrete shrinkage, can produce yielding of the steel. Quantitative analysis of the process can be carried out quite simply on the assumption of full strain compatibility and linear elastic steel behaviour. A differential or an integral expression can be used to represent concrete behaviour. Shrinkage strains contribute significantly to the stress redistribution and should be included in the analysis. The additional shortening of the column, which accompanies the stress redistribution, can be of practical importance, especially in tall buildings.

Variations in stresses and deformations which are produced by a prior history of sustained loading have little direct effect on the carrying capacity of the section, although there may be some slight secondary effect in that the concrete crushing strength can be changed perceptibly by prior sustained stress.

3.2 Short-Term Eccentric Loading

In the case of a short-term thrust P applied with uniaxial eccentricity e_x to a cross section, the moment-thrust-curvature relation is calculated on the assumption of plane distribution of strain over the section. The strain distribution is defined by the extreme values ε_1 and ε_2. Stresses in the concrete and steel can be calculated for a prescribed strain distribution if non-linear elastic models (Fig. 2c) are used for material behaviour. Equilibrium considerations then allow the magnitude of the force P and the eccentricity e_x to be determined for any chosen strain distribution. The curvature is also obtained immediately from the strain distribution.

Although the equilibrium calculations can be made analytically, a finite approach is more suitable for use with a computer, especially when repeated calculations are required, as in the analysis of slender members. The section is partitioned into a finite number of small concrete and steel areas, and equilibrium equations are written in terms of the elemental forces acting on the areas. This finite approach is particularly suitable in the case of biaxially eccentric loading and in the case of sections of irregular shape (40). It will be noted that calculations must be made indirectly if deformations are to be evaluated for a prescribed loading condition. Trial and error calculations are required, for example, to produce the moment-curvature relation for a given constant force P.

The assumption of planar strain distribution in the section appears to be adequate when the concrete stresses remain compressive over the entire section, but becomes questionable when the eccentricity is so large that tensile cracks appear in the concrete. There is then a concentration of deformations in the vicinity of the cracks, and a complex variation in stress distribution occurs along the member length in the region between cracks. Local bond conditions and crack spacing and crack widths thus affect the magnitude and distribution of deformations. In such circumstances, calculated curvatures at a cross section are, at best, idealized average values for a finite length of the member.

In view of this, it may well be more realistic to calculate rotations in short segmental lengths. An advantage of the rotation calculation is that factors representing bond characteristics and crack-spacing can be more readily incorporated in the analysis. When large deformations are to be considered in the post-ultimate, unloading phase of hinge development, consideration of a finite length of member appears to be unavoidable (33).

When cycles of loading and unloading are to be considered, the simplifying assumption of non-linear elastic material behaviour is usually replaced by more realistic stress-strain laws with unloading-reloading paths. The simple concept of a unique moment-thrust-curvature relation then becomes quite inadequate and a step-by-step analysis procedure must be used to determine the deformation history for a prescribed loading history (42).

3.3 Ultimate Strength

In Fig. 5 a typical family of moment-curvature relations is shown. Each curve represents a constant value of P. High ductility is displayed when the value of P is small. This is indicated by the extended flat portion of the curves in which moment remains almost constant for large increases in curvature. As the value of P increases, the flat portion of the curve decreases in extent, and finally disappears. Moment-rotation curves for a short segment of the member would display similar shape characteristics to those of Fig. 5.

The moment carrying capacity at a prescribed load value P is obtained from

$$\frac{dM}{dK} = 0 \tag{3.2}$$

which is analogous to Eq. 3.1. Simplified calculations of ultimate strength are usually made by setting the extreme compressive concrete strain ε_1 equal to a fixed ultimate value ε_u. Values chosen for ε_u usually lie in the vicinity of 0.003. This simplified approach gives quite satisfactory results when the moment-curvature relation displays a flat portion near maximum moment, since the computed moment is then quite insensitive to assumptions regarding deformations.

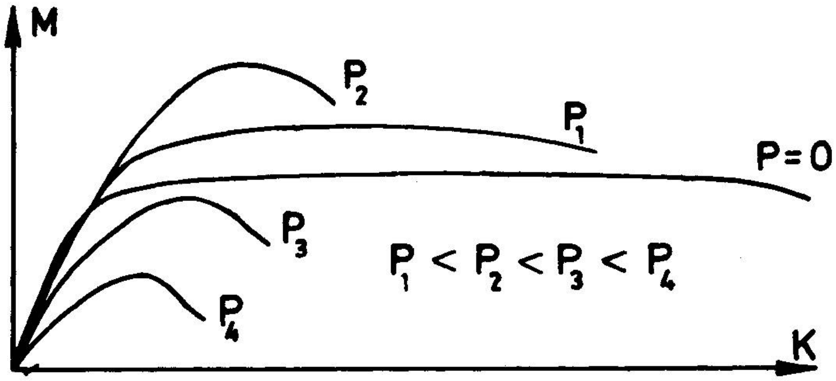


FIG. 5: MOMENT CURVATURE CURVES

It will be noted that the ultimate strain criterion produces a conservative estimate of ultimate strength, in that the moment is calculated for a point on the moment curvature curve, so that the value obtained cannot be greater than that defined by Eq. 3.2.

As the force P becomes larger, the moment-curvature curve peaks more sharply, and the difference between the computed moment and the peak moment is likely to increase. When P is so large that compression exists over the entire section, the value of 0.003 for ε_u leads to significant error and must be reduced.

Various concrete compressive stress blocks are available, which provide reasonably simple and accurate ultimate strength calculation methods. The relation between load and maximum moment, which defines the load carrying capacity of the section, can be represented by an interaction curve, as shown in Fig. 6a.

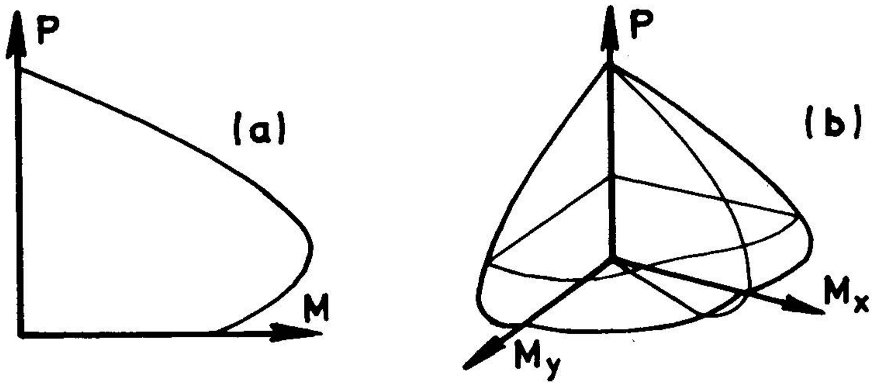


FIG. 6: INTERACTION DIAGRAMS: STRENGTH OF SECTION IN BENDING AND COMPRESSION

In the case of biaxial bending, a three dimensional interaction surface is required to represent load carrying capacity. See Fig. 6b. Calculation of load carrying capacity can also in this case be based on the ultimate strain criterion; however, the procedure becomes tedious, in that various trials must be made to determine the position and inclination of the neutral axis. Various simplified interaction equations and ultimate strength charts are available in the literature which simplify the calculation of ultimate strength in compression and biaxial bending.

3.4 Sustained Eccentric Loading

When a sustained eccentric load acts on a cross section, a transfer of compressive force takes place from the region of concrete under compressive stress to the adjacent compressive reinforcement, as in the case of the axially loaded section. If the loading is constant and the eccentricity is large enough to cause cracking of the section, there is little change in the tensile force in the tension steel, despite the considerable redistribution in compressive stresses. The increase in curvature with time is thus produced primarily by the increased deformations in the compressive region of the beam, and depends very much on the creep properties of the concrete and the quantity and position of the compressive steel.

Theoretical analysis of time-varying stresses and strains under sustained eccentric loading is complicated by the shifting position of the neutral axis of stress, and the shrinkage induced non-coincidence of the neutral axes of stress and strain. It is interesting to note that predicted behaviour is non-linear, even when shrinkage effects are ignored and linear constitutive relations are used to describe material behaviour.

Although theoretical analysis can be carried through to closed form solutions in a few special cases, some form of numerical analysis usually becomes necessary, if only to obtain solutions to non-linear differential and integral equations. In such circumstances, there are advantages in going straight to the step-by-step analysis, in which stresses and strains are evaluated in a finite number of fibres at a finite number of time instants. In contrast to theoretical analysis, the step-by-step analysis avoids unnecessary restrictive assumptions regarding material behaviour, cross section shape, load history, etc. Simplified methods of analysis of creep and shrinkage effects have been proposed for the case of sections in pure bending, but have not been sufficiently developed and tested for sections in combined bending and compression. Thus, use of a distorted concrete stress-strain relation has been suggested for the evaluation of long term response, but has not been thoroughly checked. There is indeed some indication (27) that this method can give misleading results, especially when used in the analysis of creep buckling phenomena.

4. SLENDERNESS EFFECTS

Fig. 7a shows a slender reinforced concrete compression member subjected to end thrusts P at equal end eccentricities e. Although the load and support conditions are much simpler than those usually encountered in practice, the behaviour of this compression member is of considerable practical importance, in that current design procedures often rely on the concept of an "equivalent" pin-ended column (Ersatzstabverfahren) (21). The articulated bar-spring assemblage shown in Fig. 7b might be regarded as a simple physical model of the member in Fig. 7a. It provides a convenient means of studying the essential behaviour of the slender member, while avoiding the mathematical complexity of a complete analysis.

4.1 Lateral Deflections

The moment resisting spring at B allows the assemblage to carry the eccentric end thrusts P. Provided deformations remain small, the rotation $\theta(t)$ in the spring at any time t is related to the lateral deflection $v(t)$ at mid-depth as follows,

$$\theta(t) = 4v(t)/\ell \qquad (4.1)$$

in which ℓ is the system length.

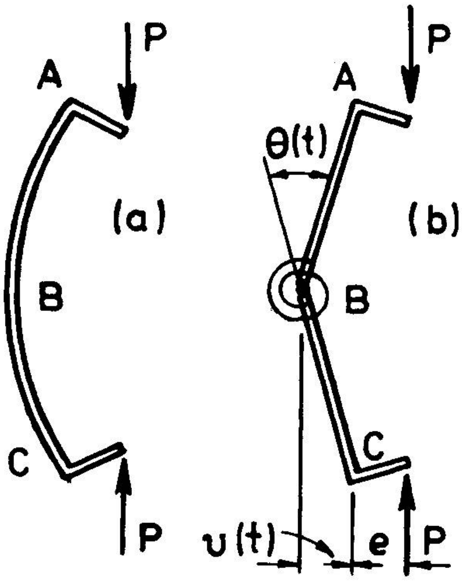


FIG. 7: SLENDER COMPRESSION MEMBER AND BAR-SPRING MODEL

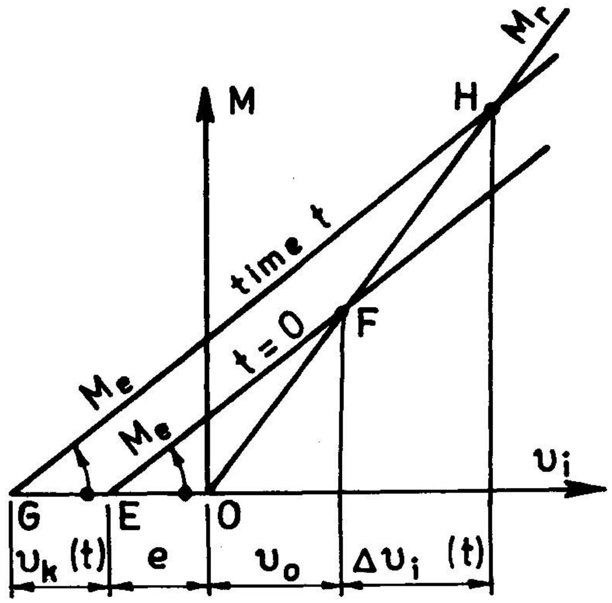


FIG. 8: VISCOELASTIC ANALYSIS OF SECOND ORDER LATERAL DEFLECTIONS.

The response of the assemblage to load is governed by the moment-rotation characteristics of the spring, just as the response of the compression member is governed by the moment-curvature characteristics in critical sections. Initially, the spring is considered to be linear visco-elastic, with the total rotation made up of a creep component, $\theta_k(t)$, and an instantaneous component, $\theta_i(t)$. The latter is related to the internal moment $M_r(t)$ in the spring by the spring constant k,

$$M_r(t) = k\theta_i(t) \tag{4.2}$$

The constitutive relation for the spring is taken to be

$$\dot{\theta}(t) = \frac{\dot{M}_r(t)}{k} + \frac{M_r(t)}{k}\,\dot{\phi}(t) \tag{4.3}$$

in which $\phi(t)$ is a bounded creep function. It will be noted that Eq. 4.3 is analogous to the Dischinger creep equation for concrete in uniaxial compression, Eq. 2.5.

In the case of short-term loading only, the creep component $\theta_k(t)$ is zero, and all reference to time t can be deleted. The internal moment of resistance in the spring is then

$$M_r = 4\,kv/\ell \tag{4.4}$$

and the external moment applied to the spring is

$$M_e = P(e+v) \tag{4.5}$$

By equating internal and external moments

$$M = M_r = M_e \tag{4.6}$$

one obtains the following load-deflection relation,

$$P = 4\,\frac{k}{\ell}\,\frac{v}{e+v} \tag{4.7}$$

If the eccentricity e is set equal to zero, the elastic critical (bifurcation) load is found to be

$$P_{cr} = 4\,k/\ell \tag{4.8}$$

and with the "safety coefficient" ν defined as

$$\nu = P_{cr}/P \tag{4.9}$$

the lateral deflection v is obtained from Eq. 4.7 as

$$v = e\,\frac{1}{\nu-1} \tag{4.10}$$

With the initial first order moment defined as M_o = Pe, Eqs. 4.5, 4.6 and 4.10 can be rearranged to give

$$M = \frac{1}{1-\frac{P}{P_{cr}}}\,M_o \tag{4.10a}$$

Time-varying behaviour is now considered for a constant sustained load P(t) = P The moment and the rotation in the spring both increase monotonically with time, the rates of increase being $\dot{M}(t) = P\dot{v}(t)$ and $\dot{\theta}(t) = 4\,\dot{v}(t)/\ell$, respectively. Substitution of these expressions into Eq. 4.3 and rearrangement gives

$$\frac{\dot{v}(t)}{e+v(t)} = \frac{\dot{\phi}(t)}{\nu-1} \tag{4.11}$$

Integration of Eq. 4.11 and substitution of the initial value $v_o = e/(\nu-1)$, as given by Eq. 4.10, yields the following expression for lateral deflection,

$$v(t) = e\left[\frac{\nu}{\nu-1}\,\exp\left(\frac{\phi(t)}{\nu-1}\right) - 1\right] \tag{4.12}$$

The term v(t) consists of a creep component $v_k(t)$ and an instantaneous, recoverable component $v_i(t)$. The latter is somewhat larger than the initial, instantaneous value v_o, because of the gradual increase in moment, i.e.

$$v_i(t) = v_o + \Delta v_i(t) \tag{4.13}$$

The term $v_i(t)$ can be evaluated from the instantaneous rotation, $v_i(t) = \ell\theta_i(t)/4$, where $\theta_i(t) = P(e + v(t))/k$. Substitution for v(t) and rearrangement gives

$$v_i(t) = e\,\frac{1}{\nu-1}\,\exp\left(\frac{\phi(t)}{\nu-1}\right) \tag{4.14}$$

The creep component $v_k(t)$, which would remain upon sudden unloading of the assemblage, is obtained by subtracting $v_i(t)$ from v(t),

$$v_k(t) = e\left[\exp\left(\frac{\phi(t)}{\nu-1}\right) - 1\right] \tag{4.15}$$

Equations 4.10, 4.12, 4.14 and 4.15 will be recognized as those derived by Dischinger (10), in his classic analysis of slender concrete columns. An analysis of the member shown in Fig. 7a, with Eq. 2.5 taken to represent material properties, would yield similar equations, however with the Euler load

$$P_E = \pi^2 \frac{EI}{\ell^2} \tag{4.16}$$

replacing the bifurcation load P_{cr} given by Eq. 4.8.

A form of Eq. 4.10 is used in the current ACI design provisions (1) to account for slenderness effects in reinforced concrete members under short-term loading, while Eq. 4.15 is used in a slightly modified form in the current German reinforced concrete code, DIN 1045 (9), for estimating creep deflections in slender concrete compression members.

A linear viscoelastic analysis as indicated above obviously ignores many effects of practical importance, and gives a highly idealised account of the actual behaviour of reinforced concrete compression members. In particular, the instantaneous bending stiffness EI in Eq. 4.16 is assumed to be constant and unaffected by the thrust P. The effects of tensile and compressive reinforcement are not taken into account, and the reduction in stiffness which is caused by the formation of tensile cracks is ignored. In practical applications, for example in the design provisions of ACI 318-71 and DIN 1045, approximate semi-empirical expressions are therefore used for the bending stiffness term in order to allow, to some extent, for such short-comings (21, 24).

4.2 Stability Failure

The results of the above analysis are presented graphically in Fig. 8. The straight lines OF and EF are plots of Eqs. 4.4 and 4.5 and show the internal and external moments, M_r and M_e, as functions of the instantaneous deflection v_i. The intersection point F represents the equilibrium state defined by Eq. 4.6. At time t, the instantaneous deflection has increased by $\Delta v_i(t)$ (Eq. 4.13), and there has been a corresponding outward movement of the equilibrium point along OF to H. The external moment is now represented by line GH, which is a plot of the following equation,

$$M_e(t) = P(e + v_k(t) + v_i(t)) \tag{4.17}$$

The creep deflection $v_k(t)$ is seen to have the same effect an an additional increment in eccentricity.

Fig 8 shows clearly that the assemblage, and by analogy also the concrete compression member, remain in a stable state, irrespective of the presence or absence of creep, provided only that the applied load is less than the bifurcation load, $\nu > 1.0$. The equations thus represent a second order deflection analysis only. In fact, of course, stability failure can occur under both short-term and sustained loadings which are much smaller than the bifurcation load. It is important to recognize the deficiencies in the above analysis which must be rectified, in order to obtain an adequate treatment of stability failure.

The long term deflections predicted by Eq. 4.12 are finite and bounded, provided only that $\nu > 1.0$. It is interesting to note that this result is very largely a reflection of the implied assumption of full time hardening in the creep process. If a creep law with partial ageing is adopted, an analysis along the lines suggested by Östlund (31) indicates that deflections become unbounded at time infinity if the sustained load is in the range $P_k \leq P \leq P_E$, where P_k is a limit load less than the bifurcation load P_E.

Reduced modulus expressions for P_k have been derived by Östlund and also by Distefano (11) which take the form

$$P_k = \pi^2 \frac{E_r I}{\ell^2} \tag{4.18}$$

$$E_r = \frac{E_c}{1 + \alpha} \tag{4.19}$$

The parameter α is a measure of the total non-hardening component of creep, which would be obtained in a long term test $(t-\tau\to\infty)$ on an aged $(\tau\to\infty)$ concrete specimen, i.e.

$$\alpha = \lim_{(t-\tau)\to\infty} \lim_{\tau\to\infty} [\phi(t,\tau)] \tag{4.20}$$

The analysis made by Östlund and Distefano can be indicated in very simple terms using the bar-spring model of Fig. 7b. If, analogously to Eq. 2.9, the total spring rotation $\theta(t)$ is expressed as

$$\theta(t) = \frac{1}{k}\left[M(t) + \int_{\tau=0}^{t} \frac{\partial M(\tau)}{\partial \tau} \phi(t,\tau)\, d\tau\right] \tag{4.21}$$

then expressions for $\theta(t)$ and $M(t)$ in terms of deflection $v(t)$ can be substituted to give the following integral equation for lateral deflection.

$$v(t) = \frac{1}{\nu-1}\left[e + \int_{\tau=0}^{t} \frac{\partial v(\tau)}{\partial \tau} \phi(t,\tau)\, d\tau\right] \tag{4.22}$$

Numerical solution of this equation with an appropriate creep function gives the time-increasing deflection of the assemblage. The result of any analysis of the boundedness of the deflection at various loads depends of course on the form of the creep function.

Although this type of analysis gives a more adequate treatment of the creep properties of the concrete, at least in the linear phase, and provides also for the possibility of stability failure at loads smaller than the bifurcation load, the results will still be quite unrealistic in many cases, because the instantaneous response has been assumed to remain linear when the moments and deflections increase without bound. The limit load P_k does not therefore correspond to any physically observable phenomenon and is, at best, an unsafe upper bound value for the sustained load which can be carried indefinitely. The analysis of behaviour under short-term loading is of course equally unrealistic, owing to the assumption of unbounded linear response, and P_E is an upper bound value for the short-term load carrying capacity.

To illustrate the more realistic situation of non-linear instantaneous response, a softening of the spring in the bar-spring assemblage in Fig. 7b is now considered to occur with increasing moment. The curved line OZX in Fig. 9a represents the relation between internal moment M_r and instantaneous deflection v_i. In the case of the compression member in Fig. 7a, a similar type of moment-deflection relation would be obtained from a curved moment-curvature curve.

A sequence of four equilibrium points, corresponding to a sequence of equilibrium configurations at loads P_1, P_2, P_u and P_2 again, with increasing deflection v_i, is shown in Figs. 9a and 9b for the case of instantaneous loading.

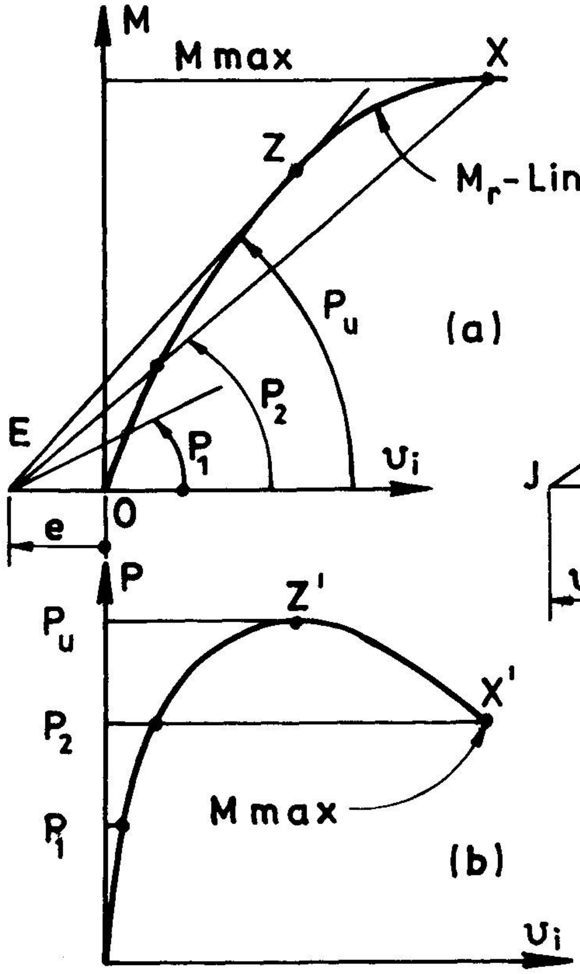


FIG. 9: STABILITY FAILURE SHORT-TERM LOADING

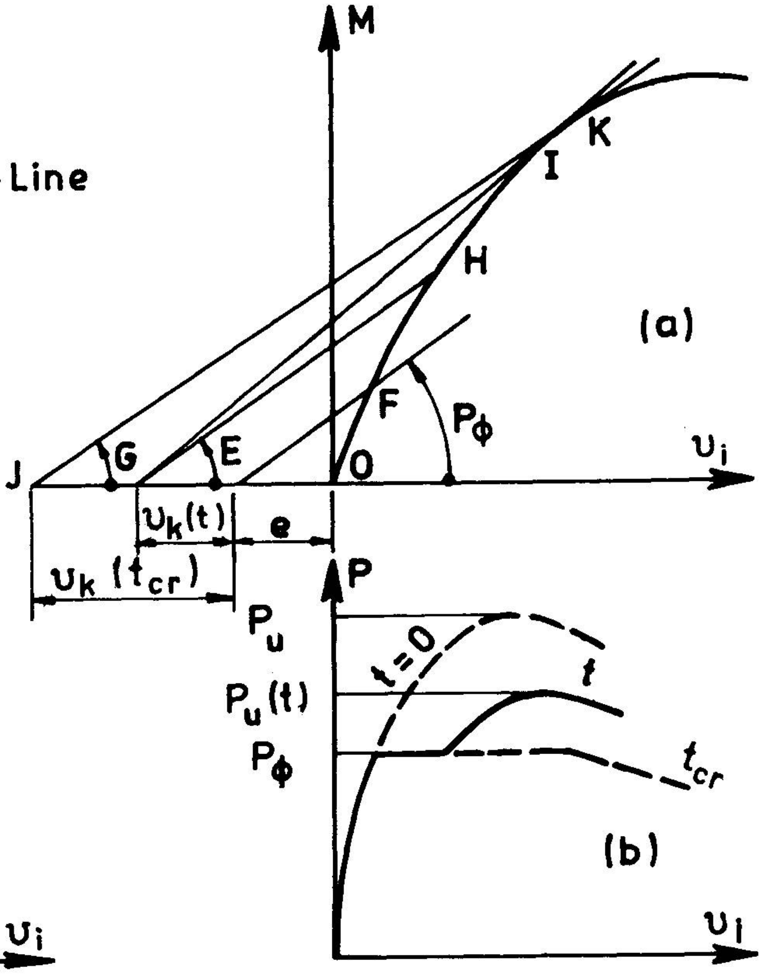


FIG. 10: EFFECT OF CREEP ON LOAD CARRYING CAPACITY

The short-term load carrying capacity P_u is reached when the rate of increase of load with respect to deflection (Fig. 9b) is zero,

$$\frac{dP}{dv_i} = 0 \tag{4.23}$$

In Fig. 9a it can be seen that Eq. 4.23 defines a state of neutral equilbrium for which the rates of increase of external and internal moments, M_e and M_r, are equal,

$$\frac{\partial M_e}{\partial v_i} = \frac{\partial M_r}{\partial v_i} \tag{4.24}$$

Non-linearity of the M_r-v_i relation has a significant effect on behaviour under sustained loading. The situation is illustrated in Fig. 10 for a constant sustained load P_ϕ, applied at eccentricity e. Fig. 10a is similar to one used by Mauch (27) in his discussion of creep effects in concrete columns. As shown previously in Fig. 8, the creep deflection $v_k(t)$ acts as an additional eccentricity. Thus, at time t, the line of external moment in Fig. 10a has moved outward from EF to GH. If the system were suddenly overloaded at time t, a load carrying capacity of $P_u(t)$ would be obtained (represented by the slope of line GI in Fig. 10a), which would be somewhat less than the original load carrying capacity P_u at time zero (represented by the slope of line EZ in Fig. 9a). It should be noted that only the short-term component, $v_i(t)$, of the total deflection v(t) is shown in Fig. 10b.

Two quite different patterns of behaviour are now possible, depending upon the magnitude of the sustained load P_ϕ. For values which are smaller than a critical value P_k, $P_\phi < P_k$, the potential load carrying capacity $P_u(t)$ sinks gradually with increasing time from the initial value P_u to a final asymptotic value P^*_u at time infinity. However, if P_ϕ is sufficiently large, $P_\phi \geqslant P_k$, the potential load carrying capacity sinks until at a critical time t_{cr} it becomes equal to the applied load P_ϕ. At this stage creep buckling occurs. The creep induced neutral equilibrium state is represented by line JK and point K in Fig. 10a. For the limiting case, $P_\phi = P_k$, the critical time approaches infinity.

The patterns of behaviour described here for the bar-spring assemblage also apply to the reinforced concrete compression member and the three loads P_u, P^*_u and P_k are of prime importance in the design process. Because of the bounded nature of concrete creep, design for finite life is neither necessary nor desirable in the case of reinforced concrete compression members. Calculations directed towards the determination of critical times t_{cr} are of secondary importance in practice.

By identifying failure as the state reached when the potential load carrying capacity, $P_u(t)$, becomes equal to the applied external load, $P_\phi(t)$, a general failure criterion can be developed for reinforced concrete columns (42), which is applicable to creep buckling failure (gradual decrease in $P_u(t)$ to the value $P_\phi(t)$), to short-term stability failure (sudden increase in $P_\phi(t)$), and to more complicated situations which cannot be uniquely classified (simultaneous rise in $P_\phi(t)$ and decrease in $P_u(t)$). This failure criterion bears some similarity to the inspection procedure of Hoff and Fraeijs de Veubeke (7).

The simplified method of analysis illustrated in Figs. 9 and 10 can be applied to a reinforced concrete member. The main complication to be considered is the effect of axial thrust on the moment-curvature relation for the cross section of the member. If the deflection curve for the member is approximated by portion of a sine wave (8), a relation between deflection and curvature at mid-depth is obtained in lieu of Eq. 4.1, and the moment-thrust-curvature relation for the critical section at mid-depth (Fig. 5) can be transformed into a moment-thrust-deflection relation which can be represented by a family of moment-deflection curves, as in Fig. 11.

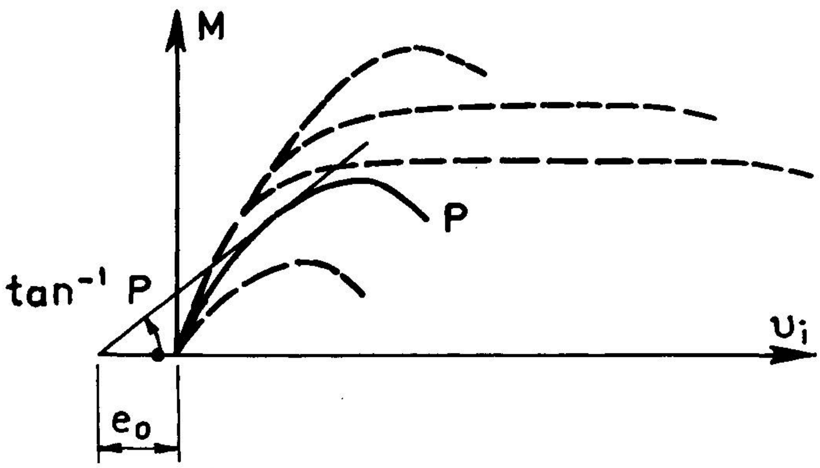


FIG. 11: MOMENT-THRUST-DEFLECTION RELATION.

The relation between external moment M_e and deflection is linear,

$$M_e = P(e + v_i) \tag{4.25}$$

and plots in Fig. 11 as a straight line with slope $\tan^{-1}P$. For a prescribed load P, a simple graphical procedure can be used to establish a critical eccentricity e_o which together with P will produce immediate short-term stability failure. The failure condition is illustrated in Fig. 11, and is defined by the requirement of Eq. 4.24.

It can be seen from Fig. 11 that the maximum moment in the critical section at the instant of stability failure is likely to be considerably smaller than the moment carrying capacity of the section. Only when the eccentricity is very large will the strength of the critical section of a slender member be exhausted at failure. Indeed, stability failure can occur long before the carrying capacity of the section is reached. It will thus be noted that the substitution of an ultimate strength calculation for a stability calculation, although frequently used as a basis for simplified methods of design, lacks theoretical justification and can lead to unconservative results in the case of very slender members with near axial loading.

If a simplified analysis for creep deflections is adopted, for example as in Eq. 4.15, the reduced load carrying capacity P_u^* can also be determined for a given level of prior sustained loading, P_ϕ. Alternatively, the limiting creep buckling load P_k can be determined by trial and error for a given eccentricity e. If the moment-curvature relations can be approximated by bi-linear or tri-linear lines (21), this method of analysis becomes quite simple in application.

An interesting question with respect to creep effects arises in the representation of material properties. Although a creep law which displays less than full hardening ($\alpha>0$) is required to detect unbounded long-term deflections in the presence of linear instantaneous response, the situation is less clear cut when the instantaneous response is markedly non-linear. It could well be that a quite simple analysis of creep deflections, for example as with Eq. 4.15, will then give reasonably realistic values of P_u^* and P_k.

It will be noted that a non-linear creep phase will occur in the final stage of behaviour just prior to creep buckling. Nevertheless, this phase is usually of relatively short duration, since the unloading of compressive stress from concrete to compression steel delays the build-up of concrete stresses. Large increases in the concrete compressive stress apparently do not occur until after yielding of the compression steel (41). It is not therefore unreasonable to ignore the final complicated non-linear phase in a simplified analysis of creep buckling.

4.3 Incremental Analysis

The method of analysis of reinforced concrete compression members considered in the previous section is based on a number of approximations and simplifying assumptions. In particular, a time-invariant moment-thrust-curvature relation has been used to represent the instantaneous response of the mid-depth section at all times, $0<t<\infty$. In fact, however, prior creep under sustained load produces a local redistribution of compressive stresses between concrete and steel and there is the distinct possibility of some or all of the compression steel reaching the yield stress during a period of sustained service loading. Although this does not perceptibly alter the final ultimate strength of the section, it can produce a softening in the short-term response of the section to moment increments, and hence some reduction in the ultimate load $P_u(t)$. Nevertheless, the simplified method gives useful estimates of load carrying capacity and also of lateral deflections under short-term and sustained service loading. It will be noted that the method is restricted to the standard case of a pin-ended member with end loads applied at equal eccentricities, such that the deflected shape can readily be approximated.

If a more accurate treatment of this standard case is required, or if a more complicated support condition and loading history is to be considered, a numerical step-by-step analysis must be used. By introducing into the assemblage of Fig. 7b additional degrees of freedom of deformation in the form of further pins and moment resisting hinges, a physical model is obtained which provides a good conceptual picture of member behaviour, as well as a basis for quantitative analysis. Such bar-spring models were suggested by Biezeno and Grammel (4) and have been used by Kabaila and Hall (18) in studies of elastic frame stability. By choosing non-linear creeping moment springs which match closely the load-deformation characteristics of a short segment of the compression member, a realistic analysis can be made of member behaviour and load carrying capacity for both short-term and sustained loading conditions.

The analysis of time-varying behaviour is effected by a discretization of the time-scale into a finite number of increments. Equations for creep and shrinkage are rewritten in a form suitable for the calculation either of increments in strain on the assumption of constant stress acting over each time interval, or of decrements in stress for constant strains maintained over the time interval (5) . Changes in external loads are usually considered to occur in small increments at specified time instants.

Discretization along the member length can be made either by considering curvatures at a finite number of stations, or by considering rotations in a finite number of short segments. The latter approach is of course equivalent to the use of the multi-spring assemblage. Calculation of the deflected shape is carried out by numerical integration of curvatures or by summation of rotations.

Incremental analysis requires a computation procedure for determining the load-deformation characteristics of a cross section or of a short segment. It is therefore convenient to discretize the section into a finite number of small concrete and steel areas, as discussed in section 3.2 of this report.

The state of the system at a given time instant and load level can thus be prescribed by the deflected position and deformed shape of a finite number of segments along the member, together with stresses and strains in a finite number of steel and concrete elements in each segment. At each step in the analysis, a trial-and-error or search procedure is required to identify the state of the system which satisfies all local and overall equilibrium requirements, all local compatibility requirements and all geometric and static boundary conditions. Incremental analysis has been used by various investigators in studies of a wide variety of effects, including constant and variable sustained load histories, biaxial bending, and frame-column interaction (8, 12, 16, 17, 28, 35, 42).

When computerized, the step-by-step or incremental analysis procedure becomes, in effect, a dynamic simulation of the structural system. It has been described as a numerical reconstruction of a real or conceivable physical test on a structural element (18).

Computerized incremental analysis procedures are of little direct value in any normal design procedure. Their main use is as research tools which, together with experimental investigation, provide a means of identifying and evaluating the various important factors affecting compression member behaviour.

It should be noted that computer-oriented analysis procedures tend to give a detailed, "worm's eye" view of the behaviour of the compression member, and provide little direct information on overall behaviour patterns. Useful information of a general nature might well be obtained through the use of statistical methods in the design and analysis of large computer experiments. Analysis of variance techniques might for example be used on data obtained from a factorial design in order to identify prime variables. Simplified relations involving only prime variables might then be obtained using regression analysis. These and other techniques could well be put to use to take advantage of the many step-by-step analysis procedures which have been developed.

4.4 Frame Action

Reinforced concrete compression members do not usually exist as individual load carrying elements, but as components of structural frames. The behaviour of a compression member cannot really be isolated from overall frame behaviour, and a realistic analysis of the member in effect requires an analysis of the entire frame.

If the non-linear, anelastic, time-varying properties of structural concrete are to be taken into account, frame analysis become a formidable task. The incremental analysis procedure can of course be extended to subassemblages and small frames. Nevertheless, demands on computation time and capacity of the computer installation rapidly become governing factors in any analysis of this type. Investigatory studies using the incremental analysis procedure are primarily of use in evaluating and, where necessary, calibrating simplified methods of analysis and design.

An intermediate approach can be taken, whereby less accurate but more tractable assumptions are made concerning material behaviour and load-deformation characteristics. Thus, standard stiffness methods of frame analysis can be adapted to take into account non-linear elastic behaviour, hinge formation and secondary lateral deflections in columns. Although non-linear methods of computer frame analysis have been suggested as a future basis for design (26), expense and relative inaccessibility prevent their use at the present time.

In simplified methods of analysis and design, the compression member is isolated from the rest of the frame, and interactive effects are represented in an approximate manner, for example by end restraints applied to a pin-ended member. Such methods will be reviewed in more detail in the introductory report to Theme II of this Symposium.

5. CONCLUDING REMARKS

In preceding sections of this report attention has been restricted to deterministic models of concrete compression member behaviour. In the formulation of adequate design procedures account must be taken of large and unavoidable variabilities in structural geometry, material properties and in-service conditions. Attention has recently centered on the use of probabilistic models of structural behaviour as a more appropriate basis for the treatment of structural safety and reliability. Although a promising start has been made in this field (3), largely by Benjamin and his associates, much more work is required. Some difficulties involved in the introduction of probabilistic concepts into compression member design have been discussed recently by Winter (43).

Despite various inadequacies in the present state of knowledge, computer oriented incremental analysis procedures provide reasonably good - but by no means precise - predictions of the behaviour and load carrying capacity of reinforced concrete compression members for a wide range of loading conditions. Such procedures can be applied to members which are either isolated (standard pin-ended case) or are components in structural subassemblages. For practical design calculations, however, simplified methods must be available which account for the most important factors affecting behaviour and strength, while avoiding the complexities of a detailed analysis.

Various simplified procedures for the analysis of individual compression members have been reviewed in this Theme I report. Not surprisingly, the simplest methods also prove to be the least accurate. In order to obtain the simplicity and general applicability required of an adequate design procedure, some degree of accuracy must clearly be sacrificed. Indeed, one of the basic decisions which any code writing body must make, involves the choice of an appropriate and acceptable balance between the conflicting requirements of accuracy and simplicity.

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SUMMARY

In the report the basic theoretical aspects of the behaviour of reinforced concrete compression members are reviewed. Attention is centered on physical-mathematical models which allow load carrying capacity and deformations and deflections under service load conditions to be evaluated. Of particular importance are those theoretical considerations which must be taken into account in the development of practical design procedures.

RESUME

Les aspects théoriques fondamentaux du comportement des pièces comprimées en béton armé sont passés en revue. L'attention est donnée aux modèles physico-mathématiques qui permettent d'évaluer la capacité de charge et les déformations et déflections pour différents cas de charge. L'importance des considérations théoriques devant être prises en considération pour le développement de nouveaux procédés pratiques de calcul est soulignée.

ZUSAMMENFASSUNG

Der vorliegende Bericht enthält die grundlegenden theoretischen Gesichtspunkte über das Verhalten von Stahlbeton-Druckgliedern. Das Augenmerk richtet sich dabei auf physikalisch-mathematische Modelle, welche die Abschätzung der Tragfähigkeit, der Deformationen und Durchbiegungen unter Gebrauchslasten gestatten. Von besonderer Wichtigkeit sind jene theoretischen Ueberlegungen, welche bei der Entwicklung praktischer Bemessungsverfahren in Betracht zu ziehen sind.