II

Analysis and Design of Reinforced Columns under Biaxial Loading

Calcul et dimensionnement de colonnes en béton armé soumises à une charge biaxiale

Berechnung und Bemessung von Stahlbetonstützen unter zweiaxiger Belastung

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INTRODUCTION: In an actual building framework, many columns are frequently subject simultaneously to bending moments about both major axes in addition to an axial compressive load, especially for corner columns. This type of loading is commonly called "biaxial loading" or "biaxial bending". The biaxial bending moments may be resulted from the space action of the entire framing system or from an axial compressive load biaxially located with respect to the major axes of the column cross section.

The mathematics of such columns is quite involved, even for the special case of relatively short columns for which the effect of lateral deflections on the magnitudes of bending moments is negligible. For the most part, analysis and design of such columns have in the past been directed toward the study of ultimate strength of reinforced concrete short columns [see for example Chap. 13, Ref. 10]. For square columns, detailed ultimate strength interaction diagrams relating axial load, and biaxial bending moments have recently been reported [7]. For the case of long columns, the present design procedure of biaxially loaded columns does not differ from uniaxially loaded columns. The 1971 ACI Building Code [1], for example, recommends to calculate the moment magnifier separately and apply to the moment about each axis independently. The long columns are then designed according to the given axial compressive load and the magnified biaxial moments. For a short column, the moment magnification factor is taken as unity.

Although this procedure has been used extensively in design computations, it does not give accurate indications of the true load-carrying capacity of a biaxially loaded column. To determine the ultimate strength of such a column, it is necessary to perform an elastic plastic stability analysis that considers the entire range of loading up to ultimate load. This is described in the present paper.

The first part of the paper discusses a rigorous method for performing elastic-plastic stability analysis of reinforced concrete columns subject to axial load combined with biaxial bending. Three classes of problems are considered: short columns, long columns under symmetrical loading, and long columns under unsymmetrical loading conditions. The ultimate strength interaction curves for symmetrical and unsymmetrical loading cases are presented in forms of charts suitable for direct analysis and design. The important factors influencing the behavior of these curves are discussed such as strength of materials, percentage of reinforcement and the magnitude of compression load.

A design method, based on the C_m factor method, is then developed. Preliminary verification indicates that maximum load carrying capacity predicted by C_m factor method for unsymmetrically loaded cases from the exact symmetrically loaded solutions is in good agreement with calculated exact unsymmetric theoretical solutions. The analytical results are also compared with the current ACI (318-71) design method for the biaxial bending case. For a standard cross section considered herein with a moderate axial compression, it is found that the ACI method gives overconservative results.

SCOPE AND ASSUMPTIONS: The columns are assumed to be isolated, simply supported, prismatic and made of rectangular cross section as shown in the insets of Fig. 3. The range of variables considered in the numerical solutions are summarized as follows:

A square cross-section with a = b = 24 in. is considered. The positioning of the reinforcement is chosen as being close to a practical average as shown in the inset of Fig. 2. Three percentages of reinforcement, A_s/ab = 1.25%, 3.25% and 8.33% are used. The stress-strain relationship is assumed to be linearly elastic-perfectly plastic [Fig. 1(b)] and Young's modulus E_s is taken at 29,000,000 psi. Three types of steel, f_y = 40, 60 and 80 ksi, are used.

Three types of concrete are considered, having compressive strengths $k_1 f'_c$ = 3, 4.2 and 5 ksi. The characteristic stress-strain curve assumed for the concrete in compression is shown in Fig. 1(a) [7]. The tensile strength and creep effect of concrete are neglected. The initial modulus of elasticity is taken as E_c = 57600 $\sqrt{f'_c}$ (normal weight concrete) [1]. The concrete strain ϵ'_c when concrete stress is $k_1 f'_c$ is taken at 0.002. The values of $\gamma_2 = 4$ and $\gamma_1 = E_c \epsilon'_c / k_1 f'_c$ are used [see Fig. 1(a)]. Details of the description of the stress-strain curve are given elsewhere [7,9].

Analyses are carried out for two values of crushing strain ϵ_o = 0.003 (ACI [1]) and ϵ_o = 0.004 (note: CEB recommends 0.0035 [3]). The column is subjected to three levels of axial compressive load, $P/f'_c ab$ = 0.1, 0.5 and 1.0. The slenderness ratios, L/a considered are 0, 10, 20, 30 and 40.

The following two additional assumptions are made in the solutions: (1) plane sections remain plane after bending; and (2) lateral-torsional twisting of the column is neglected. The failure of the column is always caused by the crushing of concrete due to excessive bending curvature. In performing numerical calculations, it is further assumed that the axial compressive load P is applied first and maintained at a constant value as the biaxial bending moments increase or decrease proportionally.

In presenting the families of interaction curves the parameters L/a, $P/f'_c ab$, $M_x/f'_c ab^2$ and $M_y/f'_c a^2b$ are chosen. The computed interaction curves are used here as a basis for (1) comparing with the C_m method; and (2) comparing with 1971 ACI moment magnifier method. The development of simple approximate interaction formulas is given elsewhere [2]. All the numerical computations are carried out in the CDC 6400 electronic digital computer.

MOMENT-CURVATURE-THRUST RELATIONSHIPS: The relations between the bending moments M_x, M_y and axial force P, and the bending curvature φ_x, φ_y and axial strain ϵ_o at corner 0 [Fig. 2] are of prime importance in the analysis of a long reinforced concrete column. These relationships used in the present calculations were determined by a separate program. In this program a moment vs. curvature curve was developed for a constant axial compressive load with the other moment being held constant. Typical moment-curvature relations for a square section with $P/f'_c ab$ = 0.5 are shown in Fig. 2, which has been computed from the stress-strain curves given in Fig. 1. The moment-curvature curve is obtained numerically by dividing the cross section into a large number of rectangular finite elements. Assuming linear strain distribution over the section, the strains of the elements are

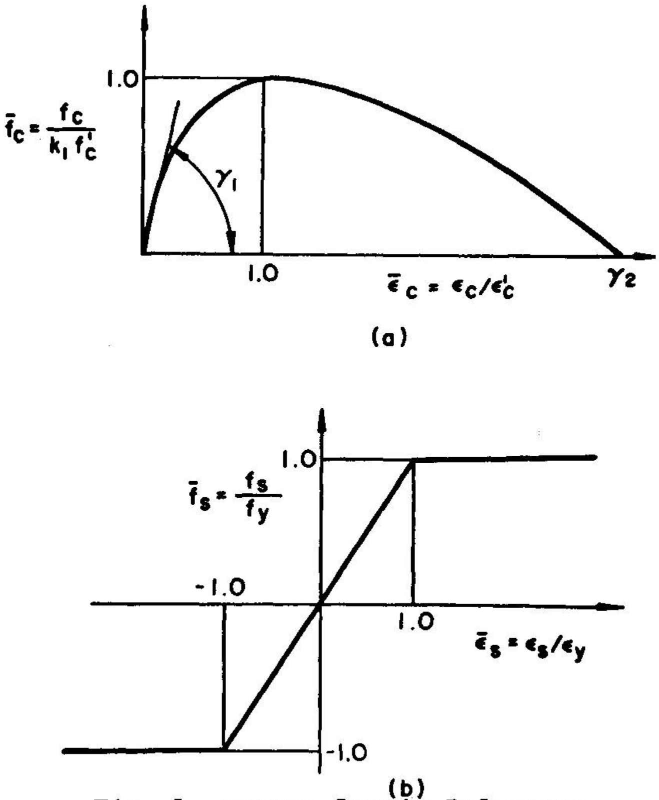

Fig. 1 Stress-Strain Relations

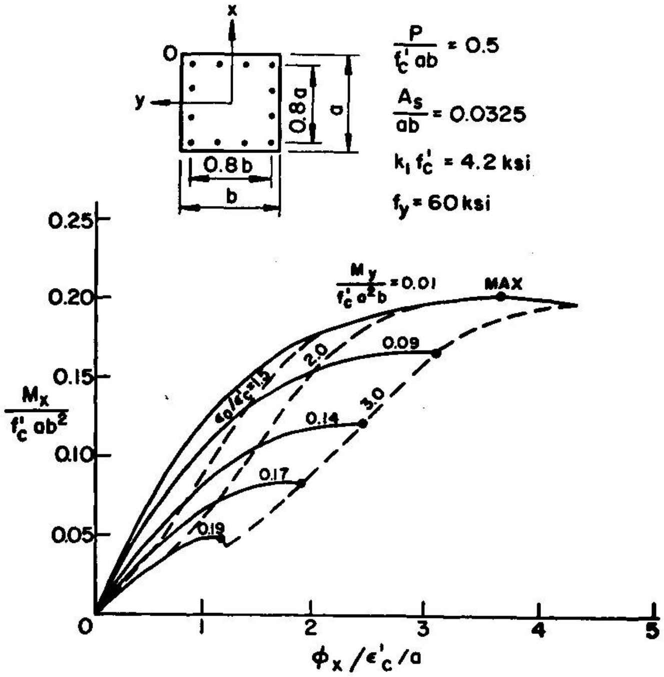

Fig. 2 Moment-Curvature Relations: Standard Case

related to the curvatures of the section. The stresses are related to the applied bending moments through the condition of equilibrium. The relationship between the applied moments and the resulting curvatures can therefore be found through the stress-strain relations shown in Fig. 1. The details of the method and the computer program are described in Ref. [7,9] for reinforced concrete sections and in Ref. [6] for steel H-sections.

In the numerical analysis, the square cross section was divided into 100 (10x10) and 400 (20x20) elements. The increase in accuracy obtained by using the finer grids was only 0.1%. A partitioning of the concrete cross section into 100 elements and steel areas into 12 elements distributed uniformly around the sides of the section are used herein. The strain and stress in each element were computed as the average value at its centroid. The allowable error in P/f'_c ab was 0.002.

METHOD OF SOLUTION: The desire response of a given column (with known length and cross sectional properties) subjected to a specified axial compressive load is the relation between an end moment and a lateral deflection. Once the complete load-deflection curve is obtained, the maximum biaxial moment that can be carried by a reinforced concrete column can be easily determined from the peak of the curve.

The numerical integration process used here is the Newmark's method [5]. The column lengths were divided into 9 segments. This degree of subdivision of column length was checked against a sub-division of 15 and 20 segments. The refined analysis leads to only an improvement of less than 1% in the results, when compared with the 9-segment solution, while the computational time was increased by more than twice.

The starting point in the Newmark's method is to assume a reasonable initial deflected shape: Herein, the elastic deflection is used with the flexural rigidity EI being computed from the approximate EI formula given by 1971 ACI Code [1]. The details of Newmark's method are described in Ref. [5].

NUMERICAL RESULTS: The interaction diagrams giving the combinations of axial compressive load and biaxial moment corresponding to maximum or ultimate load conditions are shown in Figs. 3 through 11 for symmetrically loaded cases and in Figs. 12 to 15 for unsymmetrical loading cases. The interaction curves shown in

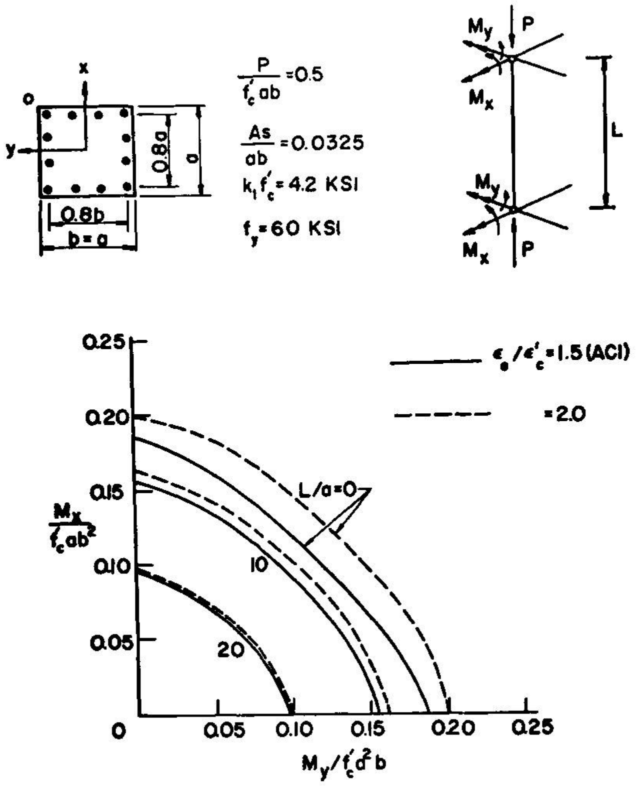

Fig. 3 Interaction Curves: Standard Case, Long Column

Fig. 3 and the moment-curvature relations shown in Fig. 2 are considered here as the standard case. Each diagram is for a particular reinforced concrete column with a given compression load. Since these interaction curves are nondimensionalized, they can be directly used in analysis and design computations and also in checking the validity of the existing design approximations. This will be described later.

Referring now to the symmetric loading cases [Figs. 3 to 11], plotted on each interaction diagram for a slender ratio are two sets of interaction curves corresponding to two different values of concrete crushing strain $\varepsilon_o/\varepsilon'_c = 1.5$ (solid lines) and 2.0. As can be seen, an increase in concrete crushing strain from $\varepsilon_o = 0.003$ to 0.004 significantly increases the ultimate strength of a biaxially loaded column, when the slenderness ratio L/a is less than 20. This is almost true for all the variables investigated here.

The important factors influencing the ultimate strength of a long column are the magnitude of compression load P (Figs. 3, 4 and 5), concrete quality $k_1 f'_c$ (Figs. 6 and 7), steel quality f_y (Figs. 8 and 9), and percentage of reinforcement A_s/ab (Figs. 10 and 11). The variations of these factors with respect to the standard case may be obtained by comparing Figs. 4 to 11 with Fig. 3. The shape of the interaction curves is obviously a function of these factors considered. This is discussed further in Ref. [2] where a mathematical equation is used to approximate the interaction curves.

COMPARISON WITH EQUIVALENT MOMENT METHOD (C_m-METHOD): Since the determination of the ultimate strength of reinforced concrete columns subject to biaxial bending requires lengthy computations, design aids in the form of interaction curves such as those shown in Figs. 3 to 15, or tables are needed for design in practice. However, even for the symmetrically loaded case, the range of important variables which could be considered is very large, and some restriction is necessary to keep the total number of design aids to a practical level. It is obvious that further presentation of interaction curves for various ratio of combinations of unsymmetric biaxial bending moments about each axis is too large in number to be practical. Hence, to cover adequately and comprehensively unsymmetric biaxial bending within a manageable compilation, the ultimate strength of columns to unsymmetric biaxial bending conditions must be related to its symmetric counterpart. The equivalent moment or C_m method is adopted herein to achieve this.

The C_m-method has been used extensively in the in-plane beam-column design. Direct modification of the equation recommended by AISC [4] for in-plane cases is now extended to the biaxial cases in the following manner.

$$C_{mx} = 0.6 + 0.4\,\frac{M_{bx}}{M_{ax}} \geq 0.4, \qquad C_{my} = 0.6 + 0.4\,\frac{M_{by}}{M_{ay}} \geq 0.4 \qquad \text{(1a,b)}$$

where M_{ax}, M_{ay}, M_{bx} and M_{by} are the end moments, and M_{ax} and M_{ay} being the larger ones. The equivalent symmetric bending moments in the two axes are

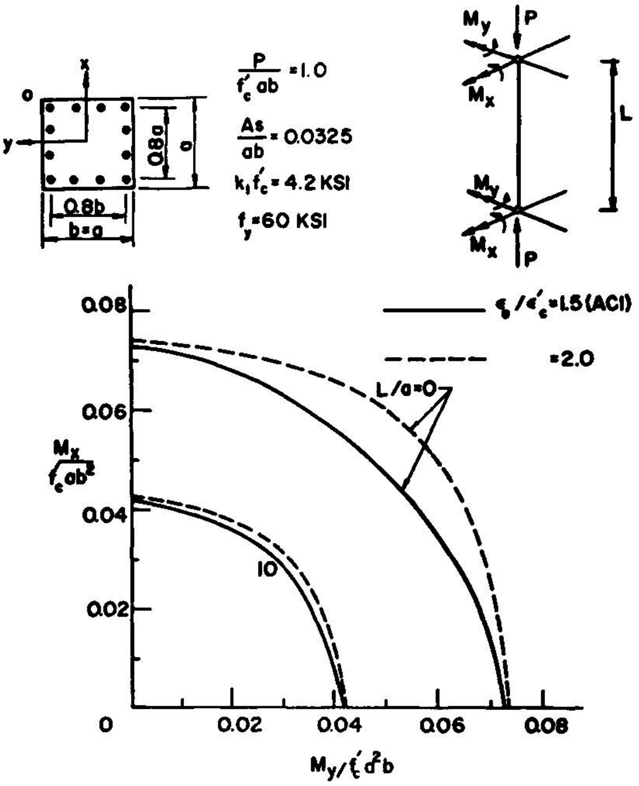

Fig.4
Interaction Curves:
Maximum Axial
Compression Force Effect

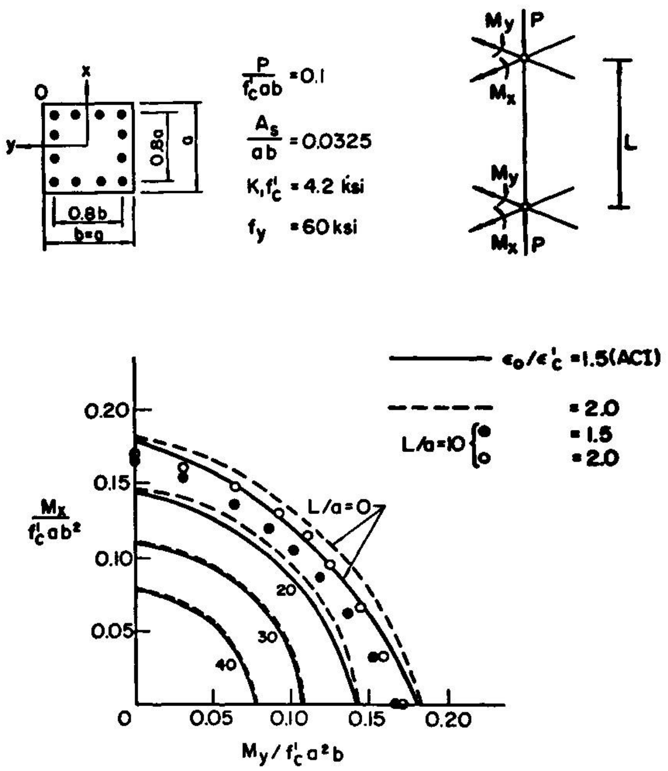

Fig.5
Interaction Curves:
Minimum Axial
Compression Force Effect

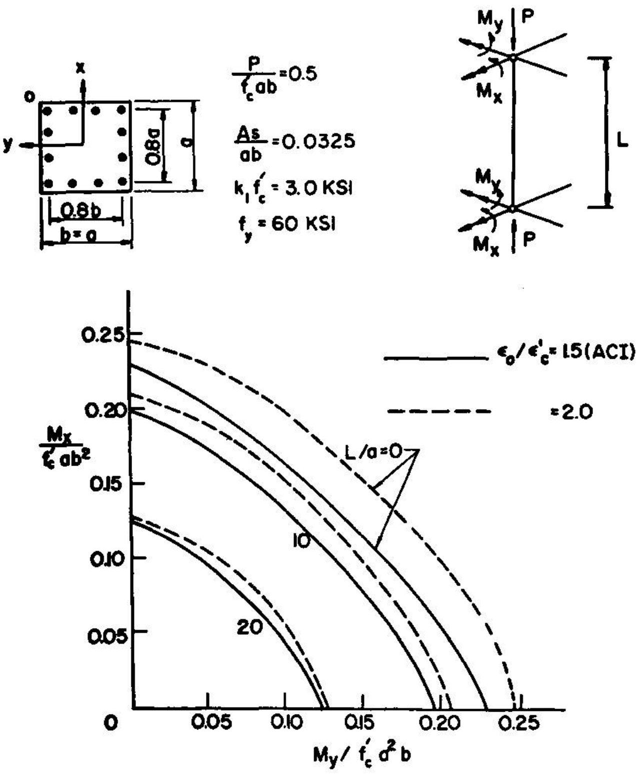

Fig.6
Interaction Curves:
Minimum
Concrete Quality Effect

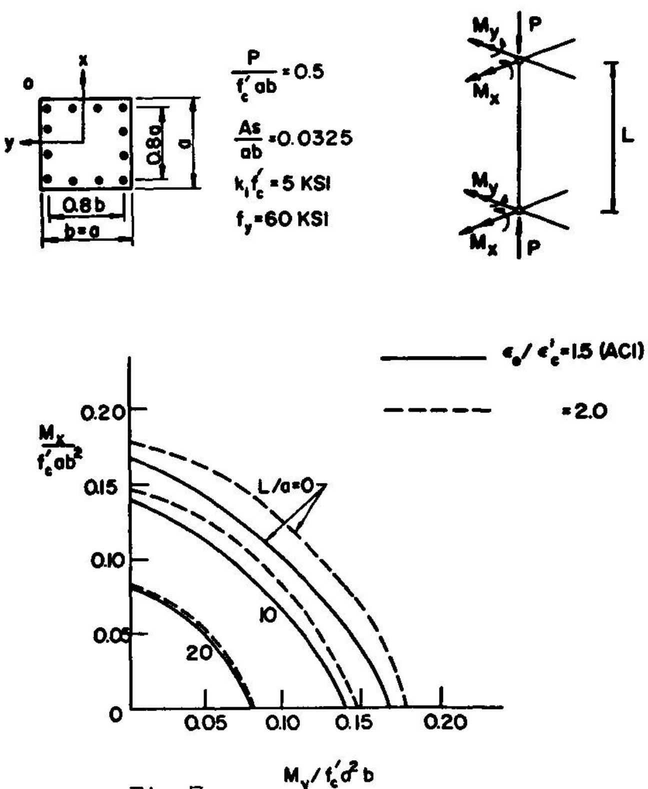

Fig.7
Interaction Curves:
Maximum
Concrete Quality Effect

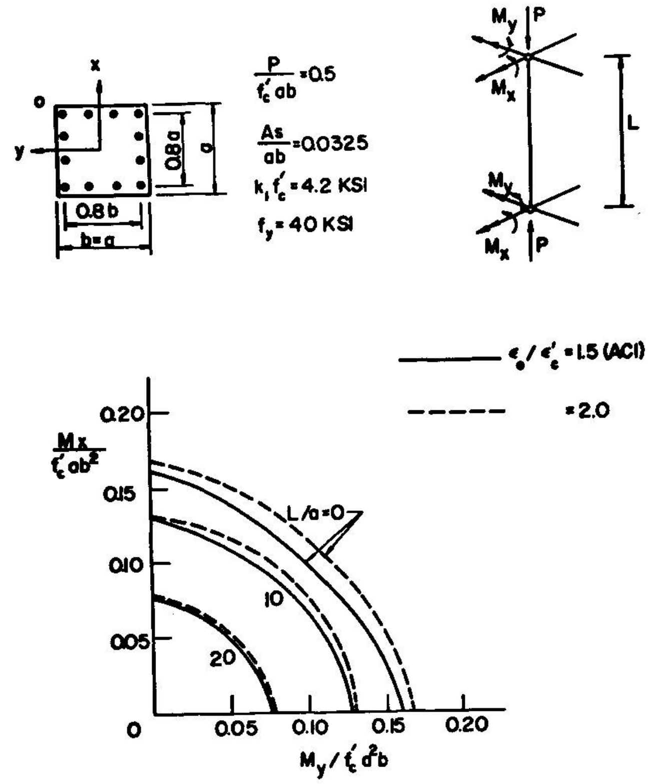

Fig. 8 Interaction Curves: Minimum Steel Quality Effect

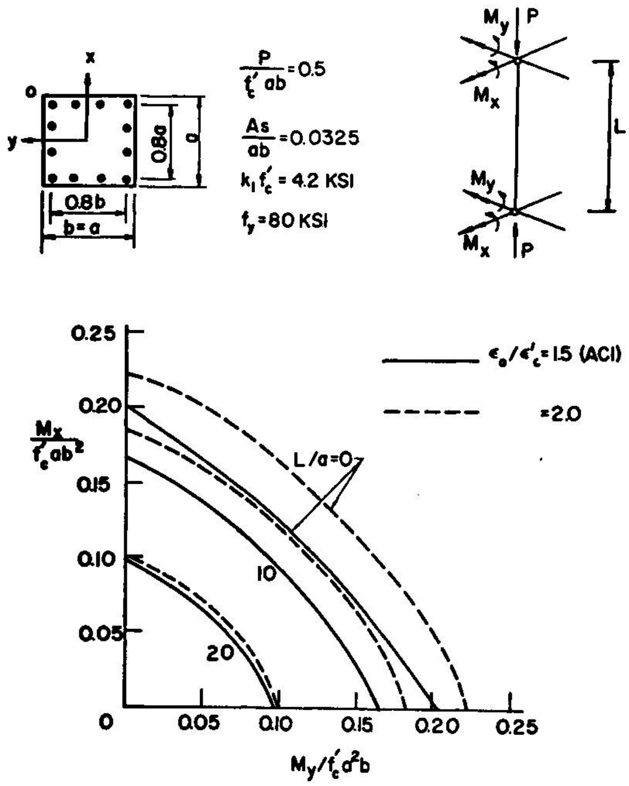

Fig. 9 Interaction Curves: Maximum Steel Quality Effect

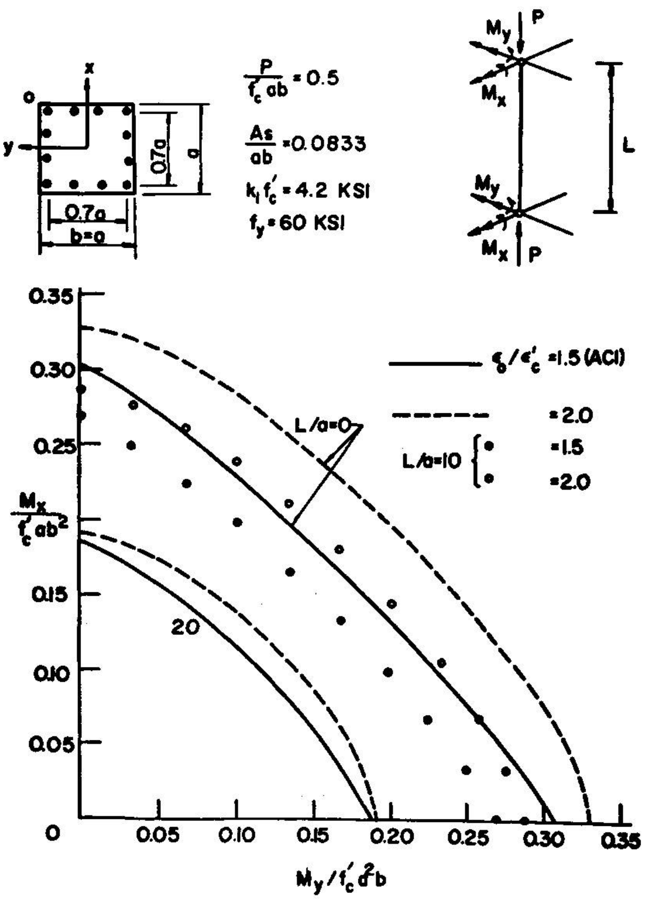

Fig. 10 Interaction Curves: Maximum Percentage of Steel

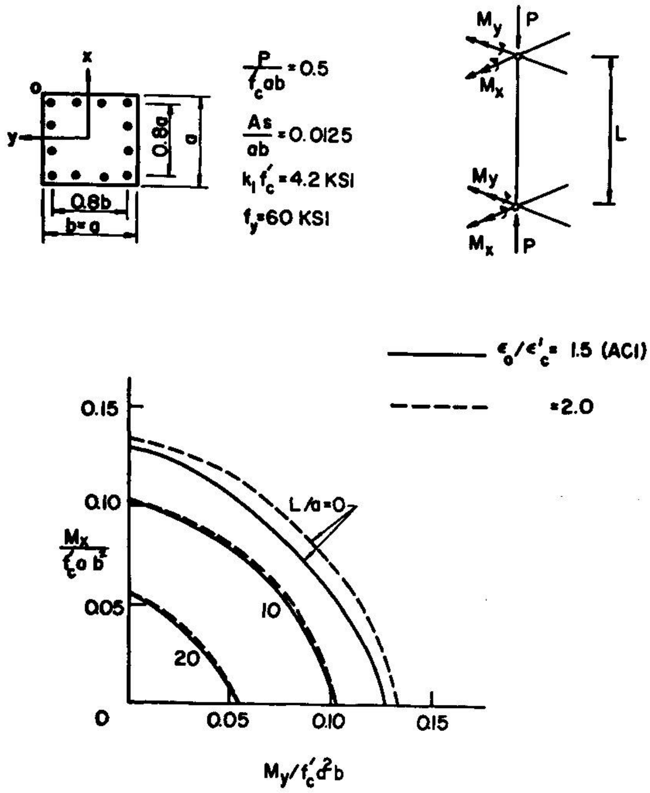

Fig. 11 Interaction Curves: Minimum Percentage of Steel

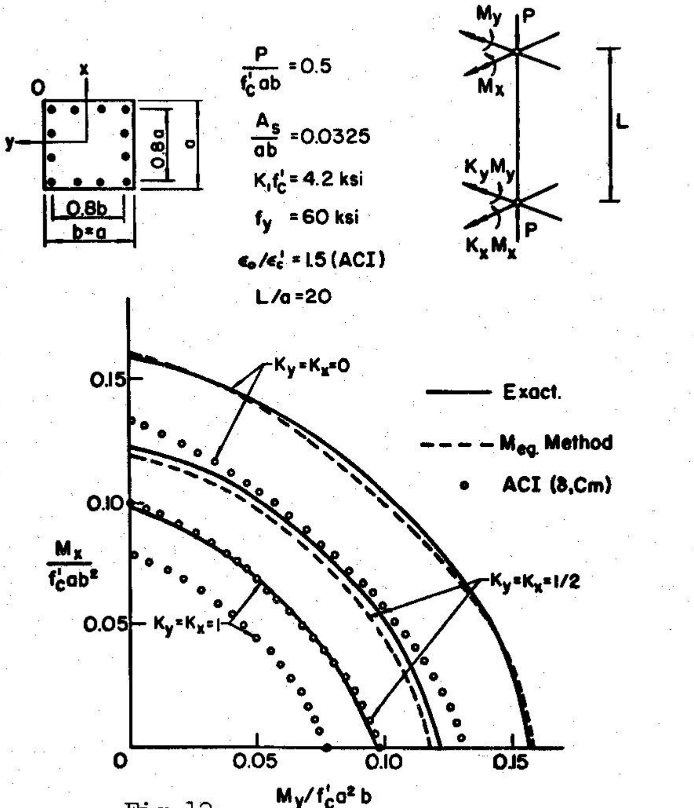

Fig.12
Comparison of Exact Solutions
with Equivalent Moment Method and ACI
Moment Magnifier Method

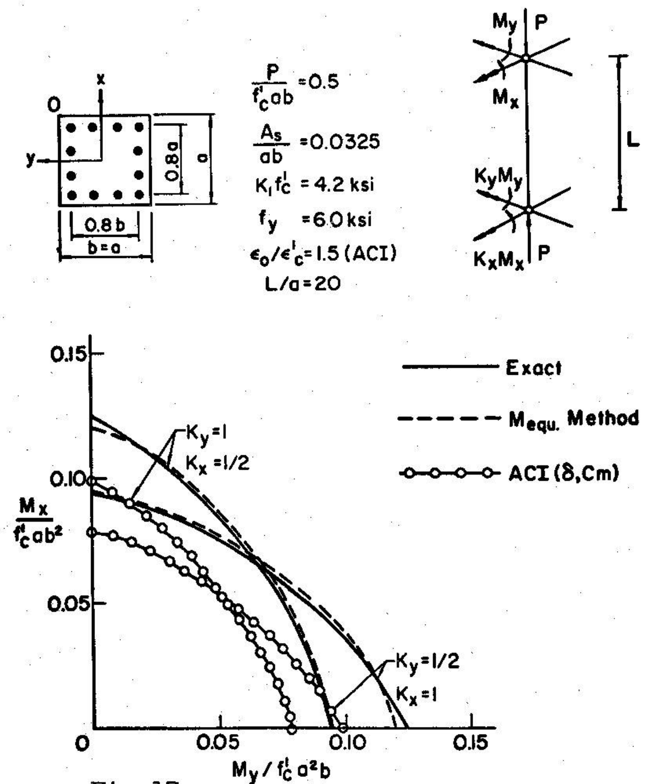

Fig.13
Comparison of Exact Solutions
with Equivalent Moment Method and ACI
Moment Magnifier Method

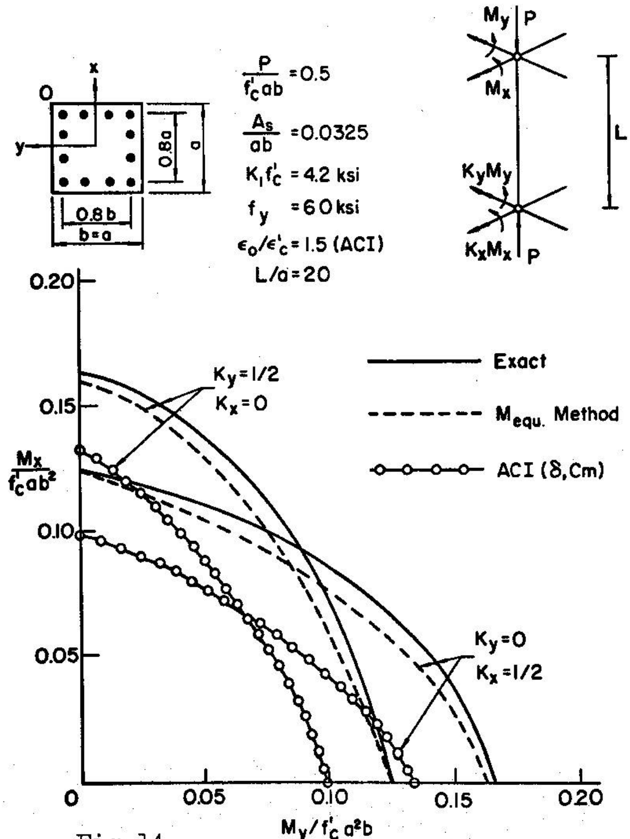

Fig.14
Comparison of Exact Solutions
with Equivalent Moment Method and ACI
Moment Magnifier Method

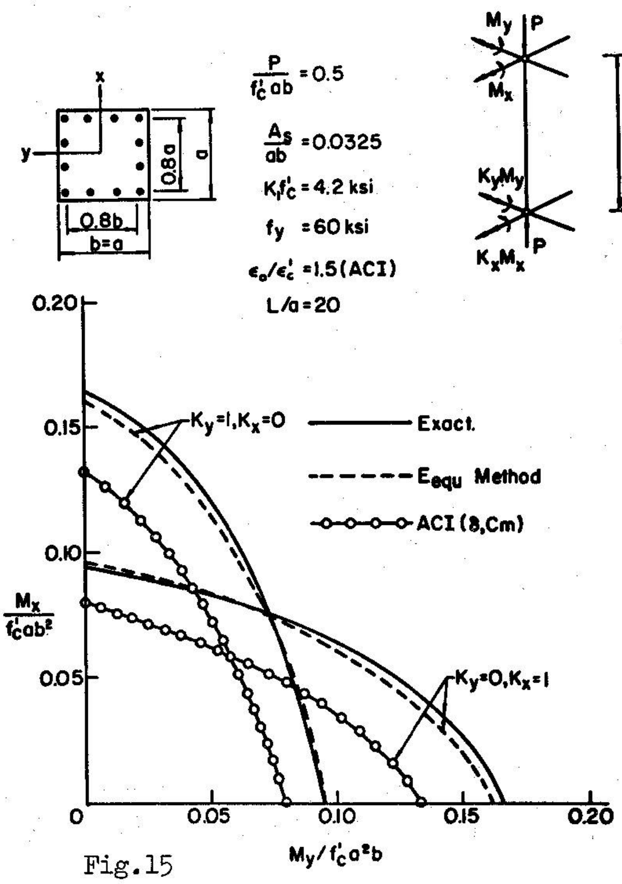

Fig.15
Comparison of Exact Solutions
with Equivalent Moment Method and ACI
Moment Magnifier Method

$$(M_x)_{eq} = C_{mx} M_{ax}, \quad (M_y)_{eq} = C_{my} M_{ay} \tag{2a,b}$$

these equivalent moment values should be used in Figs. 3 to 11 when the interaction curves for the case of unsymmetrically loaded columns are sought from the corresponding interaction curves developed for the case of symmetrically loaded conditions. These results (dotted curves) are compared with the exact solutions in Figs. 12 to 15 for several ratios of combinations of unsymmetric biaxial bending moments $K_x = M_{bx}/M_{ax}$ = 0, 1/2 and 1 and $K_y = M_{by}/M_{ay}$ = 0, 1/2 and 1. In all cases good agreement is observed. It may be concluded from this study that the strength of unsymmetrically loaded biaxial columns may be obtained directly from Figs. 3 to 11 using the equivalent moment concept. This observation is in agreement with previous investigations on steel beam-columns [8].

COMPARISON WITH MOMENT MAGNIFIER METHOD (δ-METHOD): The formula which will be used in the comparison is Formula (10-4) given in Chap. 10 of the 1971 ACI Code [1]. In Formula (10-4), the capacity reduction factor Φ is taken as 1.0 and the creep reduction factor β_d is taken as 0. The value EI is computed from Formula (10-7) of the Code. The C_m factor as given in Eqs. (2) [or Formula (10-9) of ACI Code] is used for the case of unsymmetrical loading conditions. The maximum loads determined by the ACI formula (10-4) are compared in Figs. 12 to 15 (open circles) with the present "exact" solutions.

The moment magnifier method combined with the C_m factor is seen to give extremely conservative results for the standard case investigated. This is probably due to the fact that a more precise formula for estimating the initial stiffness is required, when a column is subjected to biaxial bending conditions.

CONCLUSIONS: Ultimate strength interaction relations for reinforced concrete columns subjected to compression combined with biaxial bending have been developed for short as well as long columns under symmetric and unsymmetric loading conditions. The results are presented in Figs. 3 to 15 in the form of interaction curves relating the axial compression, maximum biaxial moment and slenderness ratio.

The maximum biaxial moments determined by the ACI moment magnifier method have been compared with the present analytical results and are found to give overconservative results for the standard cross section considered here with a moderate axial compression. It is also found that maximum load carrying capacity predicted by C_m factor method for unsymmetrically loaded cases from the symmetric cases is in good agreement with calculated theoretical values.

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SUMMARY

Several design criteria for reinforced concrete columns subjected to compression combined with biaxial bending are discussed. The load carrying capacity of the column is presented in terms of interaction diagrams. Three classes of problems are considered: short columns, long columns under symmetrical loading and long columns under unsymmetrical loading conditions. The analytically obtained results are compared with the current ACI (318-71) design formula and also with the C_m method for the case of unsymmetrically loaded conditions.

RESUME

On examine plusieurs critères de dimensionnement pour les colonnes soumises à une charge axiale et à une flexion biaxiale. La charge que peut supporter la colonne est présentée sous forme de diagrammes d'interaction. On distingue trois catégories de problèmes: colonnes courtes, colonnes longues avec charge symétrique et colonnes longues avec charge asymétrique. On compare les résultats du calcul avec les formules de dimensionnement ACI (318-71) et avec la "C_m method" dans le cas d'une charge asymétrique.

ZUSAMMENFASSUNG

Es werden einige Bemessungskriterien für Stahlbetonstützen unter Normalkraft und zweiaxiger Biegung diskutiert. Die Tragfähigkeit der Stützen wird in Form von Interaktionsdiagrammen dargestellt. Dabei werden drei Problemgruppen berücksichtigt: kurze Stützen, lange Stützen unter symmetrischer Belastung und lange Stützen unter unsymmetrischen Belastungsbedingungen. Die rechnerisch erhaltenen Ergebnisse werden mit den Bemessungsformeln der gültigen Normen ACI (318-71) sowie - für den Fall unsymmetrischer Lastbedingungen - auch mit der C_m-Methode verglichen.

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