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ULTIMATE STRENGTH OF ECCENTRICALLY LOADED COLUMNS

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ABSTRACT

An efficient numerical procedure which enables the ultimate strength of an eccentrically loaded column to be found without developing the load-deflection curve itself is discussed in this paper. Solutions for symmetrical and unsymmetrical uniaxial bending and for the general case of unsymmetrical biaxial bending are considered.

The condition for a maximum value of P for the case of uniaxial bending with equal end eccentricities e is shown to be $\partial e/\partial y_0$, where y_0 is the deflection y at midlength. By constructing an auxiliary curve whose ordinate is $\partial y/\partial y_0$, the problem is reduced to a simultaneous solution of two initial value problems. The equations are solved by numerical integration, proceeding along the z-axis until $\partial y/\partial y_0 = 0$. The corresponding value of z is the half length L/2 of the column and y(L/2) is the eccentricity e.

The initial-value problems for the case of uniaxial bending with unequal eccentricities are also developed and the technique for their solution explained. Finally, the extension to the general case of columns with biaxial eccentricities unlike at the two ends is discussed.

Comparisons of results with results of previous investigations are noted.

1. PREVIOUS WORK

Elastic flexural-torsional behavior of beam-columns is discussed by Bleich,[1] Timoshenko and Gere,[2] and Vlasov.[3] In 1935 Jezek[4] presented an approximate method for determining the ultimate load of a beam-column in uniaxial bending with equal end eccentricities. Galambos and Ketter[5] and Ketter[6] developed a more accurate solution, applicable for unequal end eccentricities, in which the effect of residual stresses is considered. Kabaila and Hall[7] presented a new approach to the solution of the problem for equal eccentricities. Their procedure is extended in this paper to the case of unequal eccentricities, and a technique is developed which results in greater accuracy and a considerable reduction in computational effort.

Investigation of the inelastic behavior of beam-columns under biaxially eccentric load has been undertaken only recently. Birnstiel and Michalos[8] and Harstead, Birnstiel, and Leu[9] developed a general procedure for determining the ultimate load with the same eccentricities at each end. The procedure requires considerable computational effort. Sharma and Gaylord[10] gave a simple approximate solution in which the lateral displacements and twist of the cross section are assumed to vary sinusoidally along the axis of the column. The solution of the nonlinear differential equations is simplified by imposing the equilibrium condition only at midlength of the column. Syal and Sharma[11] presented a numerical technique for the general case in which eccentricities at one end differ from those at the other. However, it is limited to elastic behavior, so that only the load at first yield is obtained. Residual stresses are taken into account in both (10) and (11) and computed loads are in good agreement with test results.

2. UNIAXIAL BENDING WITH EQUAL END ECCENTRICITIES

2.1 Method of Analysis

The curvature ϕ, the axial load P, and the bending moment M at any cross section of the member are related by

$$\phi = f(P,M) \tag{1}$$

In the elastic range, this equation has the familiar form $\phi = M/EI$. Since displacements are small, ϕ can be taken equal to y'', the second derivative of displacement with respect to distance along the column:

$$|y''| = f(P,M) \tag{2}$$

With the coordinate axes shown in Fig. 1, y'' is negative and $M = Py$, so that Eq. 2 yields

$$y'' + f(P,Py) = 0 \tag{3}$$

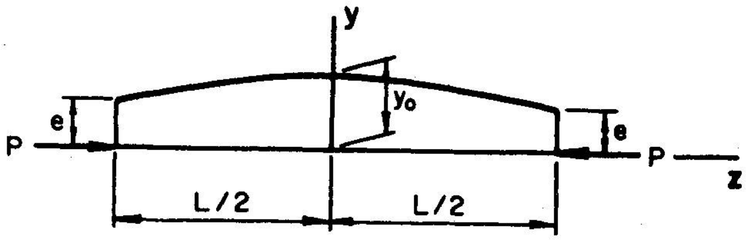


Fig. 1 Eccentrically loaded column

The deflected shape can be found by solving an initial-value problem of Eq. 3. With the following initial values at midlength

$$y(0) = y_0 \qquad y'(0) = 0 \tag{4}$$

the solution is obtained in the form

$$y = y(P, y_0) \tag{5}$$

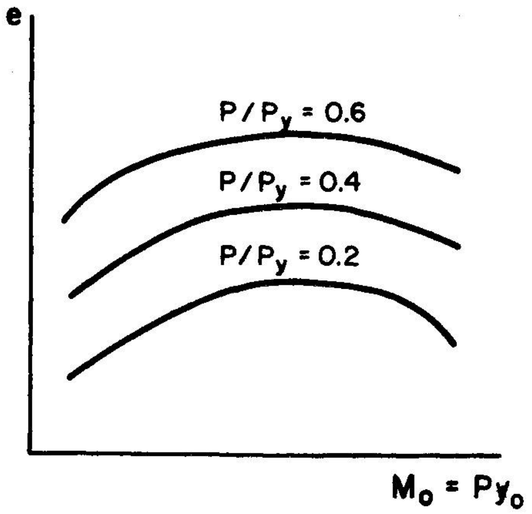
From this equation the end eccentricity e is

$$e = e(P, y_0) \tag{6}$$

Kabaila and Hall[7] plot on the e-M_0 plane ($M_0 = Py_0$) a family of P-curves for a column of given length (Fig. 2). The peak of each P-curve gives the maximum eccentricity for the corresponding value of P. Thus, the ultimate load is identified by

$$\frac{\partial e}{\partial y_0} = 0 \tag{7}$$

In the analysis presented in this paper the ultimate load is determined without developing the P-curves of Fig. 2. This is done by constructing an auxiliary curve whose ordinate is $\partial y/\partial y_0$ (Fig. 3). To develop this curve, Eq. 3 is differentiated with respect to y_0. Thus, for any P-curve of Fig. 2



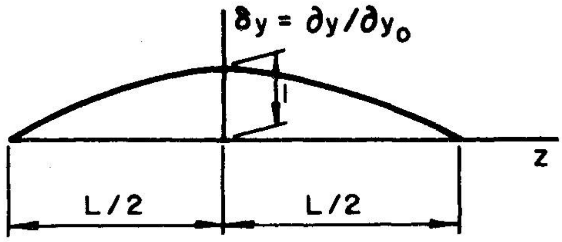


Fig. 2 Plots of P/P_y for column of given length

Fig. 3 Auxiliary curve

$$\frac{\partial y''}{\partial y_0} + \frac{\partial f(P,M)}{\partial M} P \frac{\partial y}{\partial y_0} = 0 \tag{8}$$

which, with the notation $\delta y = \partial y/\partial y_0$ can be written

$$\delta y'' + f'_M (P,M) P\delta y = 0 \tag{9}$$

The auxiliary curve δy is symmetrical about the δy axis (Fig. 3). The initial values are

$$y(0) = 1 \qquad y'(0) = 0 \tag{10}$$

Solutions of Eqs. 3 and 9 with the respective initial values of Eqs. 4 and 10 can be carried out simultaneously by numerical integration, proceeding along the z axis until $\delta y = 0$. This point satisfies the condition expressed by Eq. 7. The corresponding value of z is the half length L/2 of the column and y(L/2) is the eccentricity e.

2.2 Evaluation of P, M, and ϕ

The cross section of the column is shown in Fig. 4a. The stress-strain curve of the steel is assumed to have a plateau at the yield stress σ_y and strain hardening is neglected. The member is assumed

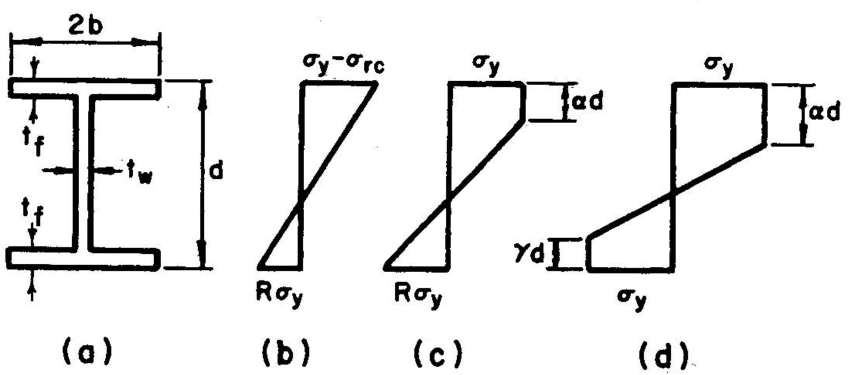


Fig. 4

Fig. 5

to have cooling residual stresses as shown in Fig. 5. This distribution was chosen so that results of the analysis could be compared with those of previous investigations in which it was used (5,10). Ketter, Kaminsky and Beedle[12] developed equations for P, M, and ϕ. Formulas are given for the case of initial yield of the extreme fiber in compression (Fig. 4b), for the compression side partially plastic (Fig. 4c), and for both sides partially plastic (Fig. 4d). For the case shown in Fig. 4d the equations are

$$\frac{P}{\sigma_y} = 2bt_f(R_c + R_t) + t_w(d - 2t_f)R_t + t_w d(\alpha - \gamma) \tag{11a}$$

$$\frac{M}{\sigma_y} = t_f(d - t_f)(2b - t_w) + \frac{t_w d^2}{6} (1 + \alpha + \gamma - 2\alpha^2 + 2\alpha\gamma - 2\gamma^2) \tag{11b}$$

$$\frac{\phi}{\phi_y} = \frac{1}{1 - \alpha - \gamma} \tag{11c}$$

In Eq. 11c $\phi_y = M_y/EI = 2\sigma_y/Ed$. The other symbols are defined in Figs. 4 and 5. Formulas for the other two cases are not repeated here. Figure 6 shows an M-ϕ relationship for a given value of P. Point A corresponds to initial yielding of the extreme fiber in compression, while point B corresponds to the case where yielding is complete through the thickness of the compression flange. For simplification the M-ϕ relation is assumed to be linear between A and B.

Equations 11 can be written in the form

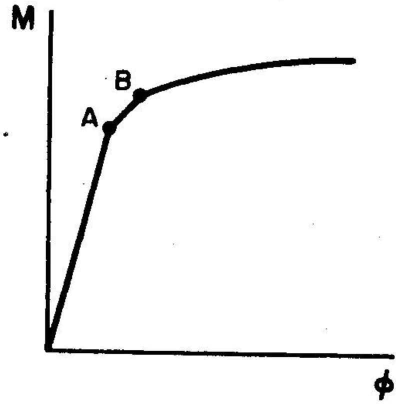


Fig. 6 Moment-rotation curve

$$P = P(\alpha,\gamma) \tag{12a}$$

$$M = M(\alpha,\gamma) \tag{12b}$$

$$\phi = \phi(\alpha,\gamma) \tag{12c}$$

The general forms of the equations for the case shown in Fig. 4c are the same as Eqs. 12 except that R replaces γ. For the case of Fig. 4b, however, the member is elastic and P, M, and ϕ are functions of R only.

2.3 Calculation of y'' and $\delta y''$ for Inelastic Behavior

The numerical integration of Eqs. 3 and 9 requires the evaluation of the curvatures y'' and $\delta y''$ for given values of P and M. Since these cannot be obtained in closed form when the cross section is partially yielded, a numerical procedure which is an extension of the Newton-Raphson method[13] is used.

Expansion of Eqs. 12 using Taylor's theorem and retaining only the linear terms yields

$$dP = \frac{\partial P}{\partial \alpha} d\alpha + \frac{\partial P}{\partial \gamma} d\gamma \tag{13a}$$

$$dM = \frac{\partial M}{\partial \alpha} d\alpha + \frac{\partial M}{\partial \gamma} d\gamma \tag{13b}$$

$$d\phi = \frac{\partial \phi}{\partial \alpha} d\alpha + \frac{\partial \phi}{\partial \gamma} d\gamma \tag{13c}$$

For a given value of P, dP = 0. Therefore, Eq. 13a gives

$$d\alpha = -\frac{\partial P/\partial \gamma}{\partial P/\partial \alpha} d\gamma \tag{14}$$

which upon substitution into Eqs. 13b and 13c gives

$$dM = \left(- \frac{\partial M}{\partial \alpha}\frac{\partial P/\partial \gamma}{\partial P/\partial \alpha} + \frac{\partial M}{\partial \gamma}\right) d\gamma \tag{15a}$$

$$d\phi = \left(- \frac{\partial \phi}{\partial \alpha}\frac{\partial P/\partial \gamma}{\partial P/\partial \alpha} + \frac{\partial \phi}{\partial \gamma}\right) d\gamma \tag{15b}$$

Then, dividing Eq. 15b by Eq. 15a we get

$$\frac{d\phi}{dM} = f_M'(P,M) = \frac{- \frac{\partial \phi}{\partial \alpha}\frac{\partial P/\partial \gamma}{\partial P/\partial \alpha} + \frac{\partial \phi}{\partial \gamma}}{- \frac{\partial M}{\partial \alpha}\frac{\partial P/\partial \gamma}{\partial P/\partial \alpha} + \frac{\partial M}{\partial \gamma}} \tag{16}$$

The following procedure describes the evaluation of y'' and $\delta y''$ for given values of P, M, and δy.

a. Assume $\bar{\alpha}$ and $\bar{\gamma}$ and compute $\bar{P}$ and $\bar{M}$ by Eqs. 12a and 12b.

b. With $dP = P - \bar{P}$ and $dM = M - \bar{M}$, solve Eqs. 13a and 13b to get $d\alpha$ and $d\gamma$. The new values for α and γ are then

$$\alpha = \bar{\alpha} + d\alpha$$

$$\gamma = \bar{\gamma} + d\gamma$$

c. Substitute the values of α and γ from step b into Eqs. 12a and 12b and compare the resulting values of P and M with the given values. If the agreement is not satisfactory, use the new values to start a new cycle. This process is repeated until the desired accuracy is obtained.

d. Substitute the final values of α and γ from step c into Eq. 12c to obtain y''.

e. Substitute the final values of α and γ from step c into Eq. 16 to obtain $f_M'(P,M)$. Use this and the known value of δy to obtain $\delta y''$ from Eq. 9.

2.4 Numerical Integration of Equations 3 and 9

The procedure for determining the column configuration at any station when its configuration at the preceding station is known is as follows:

Step 1. Having P, y_0 (or M_0), and δy_0 at station 0, the corresponding $\overline{y}_0''$ and $\overline{y}_0''$ are found by the procedure outlined in Art. 2.3.

Step 2. Assume y_1'' and $\delta y_1''$ of the next station and use the trapezoidal rule of numerical integration

$$y_1' = y_0' + \frac{h}{2}(y_0'' + \overline{y}_1'') \tag{17a}$$

$$y_1 = y_0 + \frac{h}{2}(y_0' + y_1') \tag{17b}$$

to compute y_1. Similarly, use

$$\delta y_1' = \delta y_0' + \frac{h}{2}(\delta y_0'' + \delta\overline{y}_1'') \tag{18a}$$

$$\delta y_1 = \delta y_0 + \frac{h}{2}(\delta y_0' + \delta y_1') \tag{18b}$$

to compute δy_1. In these equations h is the interval between stations.

Step 3. With y_1 from Step 2 compute $M_1 = Py_1$ and through the procedure described in Art. 2.3 determine y_1'' and $\delta y_1''$.

Step 4. If the values of y_1'' and $\delta y_1''$ of Step 3 do not agree with the assumed values of Step 2 start a new cycle with y_1'' and $\delta y_1''$ as new initial values and repeat the procedure. When agreement between the computed values and those of the previous cycle is satisfactory, return to Step 2.

With the method just described, the solutions of Eqs. 2 and 9 are carried out simultaneously until δy becomes zero. As noted before, the corresponding value of z is the half length of the column and the value of y is the end eccentricity e of P. This procedure was programmed for the IBM 360/75 system of the Digital Computer Laboratory of the University of Illinois at Urbana-Champaign. The procedure converges rapidly, and the solution for a given P and M_0 is obtained with a few seconds of computer time. Computed values for the W8 x 31 column of (5) for a number of cases were found to agree within 3 percent of values interpolated from interaction curves in that reference. Further details are given in (14).

It is of interest to note that y(z) and $\delta y(z)$ become zero at the same station if the initial value of y_0 is such that the cross section is elastic at station 0. Of course, this corresponds to a concentrically loaded column and the given P is the Euler load.

3. UNIAXIAL BENDING WITH UNEQUAL END ECCENTRICITIES

3.1 Method of Analysis

The method of analysis for the case of unequal end eccentricities is similar to the case of equal end eccentricities. The procedure starts at the station at which the column has its maximum displacement y_0 measured from the pressure line (Fig. 7a). In general, the auxiliary curve δy is not symmetrical about the y axis and has an initial slope c (Fig. 7b). To obtain the ordinates of this curve it is convenient to consider it as a combination of two curves δy_1 and δy_2 which are determined by the following initial conditions:

$$\delta y_1(0) = 1 \qquad \delta y_1'(0) = 0 \tag{19a}$$

$$\delta y_2(0) = 0 \qquad \delta y_2'(0) = 1 \tag{19b}$$

The ordinate of the δy curve is then

$$\delta y = \delta y_1 + c\delta y_2 \tag{20}$$

The instability condition $\partial e/\partial y_0 = 0$ of Eq. 7 can be written

$$\delta y(L_i, c) = \delta y_1(L_i) + c\delta y_2(L_i) = 0 \qquad i = 1,2 \tag{21a}$$

which gives

$$c = -\frac{\delta y_1(L_i)}{\delta y_2(L_i)} \tag{21b}$$

where L_i is an end of the column.

The deflected shape y(z) and the values of δy_1 and δy_2 at each station point are found by a straightforward solution of the following initial-value problems:

$$y'' + f(P,M) = 0 \tag{22a}$$

$$y(0) = y_0 \qquad y'(0) = 0 \tag{22b}$$

$$\delta y_1'' + f_M'(P,M)\, P\delta y_1 = 0 \tag{23a}$$

$$\delta y_1(0) = 1 \qquad \delta y_1'(0) = 0 \tag{23b}$$

$$\delta y_2'' + f_M'(P,M)\, P\delta y_2 = 0 \tag{24a}$$

$$\delta y_2(0) = 0 \qquad \delta y_2'(0) = 1 \tag{24b}$$

The following steps describe the procedure for given values of P and y_0:

Step 1. Integrate Eqs. 22a, 23a, and 24a numerically with the prescribed initial values, starting at $z = 0$ and proceeding to the left to any negative value L_1. The procedure for finding y'' and $\delta y''$ at each station point is the same as for the case of equal end eccentricities (Art. 2.3). The corresponding $e_1 = y(L_1)$ is the eccentricity of P at the left end of the column (Fig. 7).

Step 2. Compute c from Eq. 21b.

Step 3. Integrate the same system of equations, with the same initial values, from $z = 0$ to the right until $\delta y(L_2, c) = 0$, using the value of c from Step 2. The corresponding $e_2 = y(L_2)$ is the eccentricity at the right end of the column and $L = -L_1 + L_2$ is the length of the column (Fig. 7), where it is to be remembered that L_1 is negative.

The procedure described above gave results for a number of cases which checked within 3 percent of values according to (15). Further details are given in (14).

4. GENERAL CASE OF BIAXIAL BENDING

The procedure described above was extended to the analysis of columns with biaxial eccentricities unlike at the two ends. The solution is given by

$$\begin{vmatrix} \dfrac{\partial u}{\partial u'_A} & \dfrac{\partial u}{\partial v'_A} & \dfrac{\partial u}{\partial \beta'_A} \\[2ex] \dfrac{\partial v}{\partial u'_A} & \dfrac{\partial v}{\partial v'_A} & \dfrac{\partial v}{\partial \beta'_A} \\[2ex] \dfrac{\partial \beta}{\partial u'_A} & \dfrac{\partial \beta}{\partial v'_A} & \dfrac{\partial \beta}{\partial \beta'_A} \end{vmatrix} = 0 \qquad (25)$$

where u and v are displacements in the x and y directions, respectively, β is the angle of twist, and u'_A, v'_A, and β'_A are first derivatives with respect to z at the end A of the column. Numerical results for a number of cases are reported in (14).

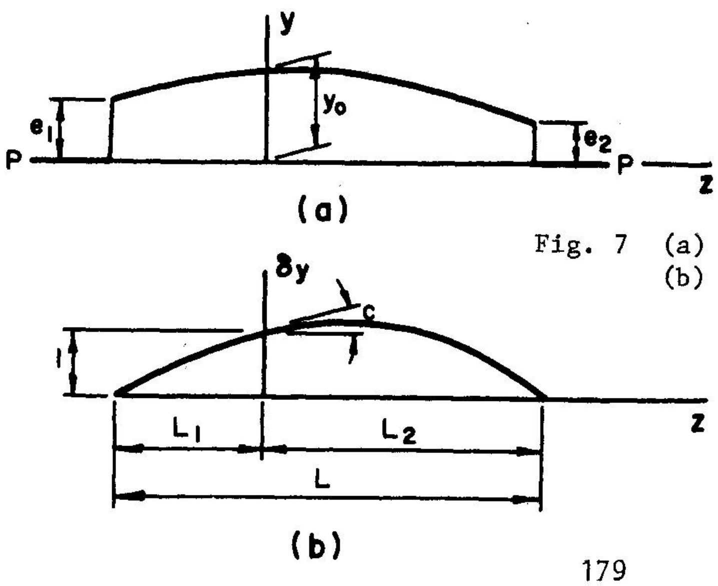


Fig. 7 (a) Column with unequal eccentricities
(b) Auxiliary curve for column in (a)

The concept of an equivalent uniform moment for uniaxial bending with unequal end eccentricities was suggested by Massonnet in 1947, and a formula based on an approximate mathematical investigation was given.[17] Later, other similar formulas were proposed, and the concept was used in codes and specifications. Sharma and Gaylord[10] showed that their interaction curves for biaxially bent columns with eccentricities the same at both ends gave good predictions of the results of four tests[16] on columns with eccentricities unlike at the two ends by using an equivalent uniform moment M_{eq} in each principal plane xz and yz according to

$$M_{eq} = 0.4\,M_1 + 0.6\,M_2 \geq 0.4\,M_2 \qquad (26)$$

In this formula, which was suggested by Austin,[18] M_1 and M_2 are positive when the member bends in single curvature and M_1 is the smaller of the two. Furthermore, results for 15 columns with unlike eccentricities analyzed in (14) by Eq. 25 differed from values given by Sharma and Gaylord's interaction curves with the equivalent uniform moment according to Eq. 26 by not more than 5 percent. Thus, it appears that the extended concept of equivalent uniform moment can be relied upon to simplify the analysis of the biaxially bent column with eccentricities unlike at the two ends.

If twist is neglected Eq. 25 reduces to a second-order determinantal equation. Results for 22 columns analyzed by this procedure differed from values given by the interaction curves of (10) by not more than 8.5 percent. This suggests that the analysis of biaxially bent columns can be reduced with good approximation to one of plane bending in the x and y directions. This conclusion was also demonstrated in (10).

5. ACKNOWLEDGEMENTS

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