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THE PROBABILISTIC CHARACTERISTICS OF MAXIMUM COLUMN STRENGTH

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ABSTRACT

This paper presents a novel method for computing the maximum strength of centrally loaded columns, whereby the probabilistic nature of the column strength factors has been taken into account. A quasi-steady probabilistic solution procedure has been used to reformulate the deterministic, incremental relationships that express the behavior of the inelastic column, so as to account for the random nature of the column strength parameters.

The contents of the paper may be outlined briefly as:

1) Evaluation of the statistical characteristics of the column strength parameters (geometric properties, yield stress, residual stresses, and initial out-of-straightness).
2) The method of probabilistic evaluation of the maximum column strength.
3) Analysis of the random variation of the maximum strength, with particular reference to probability density functions, confidence intervals, and the influence of each of the random column strength factors on the variation of the strength.

1. INTRODUCTION

Numerous theories and attempts at presenting the most rational solution to the problem of defining the strength and behavior of the centrally loaded column have been formulated throughout the years, but until fairly recently none of these studies have taken into account the stochastic characteristics of the column strength parameters. As is common to almost all physical phenomena, the factors that influence the strength and the behavior of the column exhibit unpredictable - random - variations, and an added fraction of realism therefore is introduced when the problem is treated within the realm of probability theory.

Whereas conceptually obvious, it is inconceivably complex to incorporate all factors and their variability into a practical solution, and some simplifications therefore have to be made in order to arrive at a practicable column model. This does not imply that the study and its results will be of lesser value, but rather that it represents a step in the direction of improving the method of column strength determination.

The investigation that is presented in this paper constituted a major phase of a research program that was conducted at Lehigh University. Of major concern in the study was the problem of defining the variation of the column strength, and how best it could be accounted for. In the probabilistic study that will be presented here, the variation of the relevant strength parameters has been considered explicitly in the calculation of the maximum strength. A direct analysis of the separate and joint effects of the variables therefore is made possible. To the best of the authors' knowledge, this represents the first time that the concept of probability theory has been applied towards the solution of the problem of an inelastic, initially curved column, for which the basic relationships are given in the form of incremental, iterative equations. The mathematical method of solution therefore may provide a theory that can be used in other areas of similar nature.

A complete evaluation and discussion of the theory and the results of the probabilistic study of the maximum column strength is provided by Reference 1.

2. PROBABILISTIC ANALYSIS OF COLUMN STRENGTH

The deterministic modeling of any problem implies the use of the fundamental concept of a one-to-one correspondence between the dependent and the independent variables (i.e. in a deterministic sense), and any variation of the pertinent variables is omitted from consideration. A probabilistic model, on the other hand, takes the variability explicitly into account, and the resulting solution thereby becomes expressed as a number of values, of which some are more likely to occur than others. The concept of the probability of the occurrence of an event thus is naturally introduced, whereby the multi-valued solution of the problem may be expressed either as a probability density function or a distribution function.

One of the first attempts at solving the inelastic column problem was provided by Chung and Lee[2], who presented a tangent modulus based approach. A similar technique was utilized by Rokach[3] in an effort to compare his data with the results provided by a number of European column test results. Whereas the analysis made by Augusti and Baratta[4] did incorporate the initial out-of-straightness as a random variable and therefore gave a maximum strength solution, their omission of the residual stresses in the column seriously reduced the usefulness of the study.

2.1 The Random Variation of the Column Strength Parameters

The probabilistic treatment of the maximum strength of a column is essentially a study of a structure which exhibits a random non-linear behavior. It therefore is necessary to establish the mathematical laws that reflect the random nature of the pertinent factors, prior to the formulation of the equations that govern the maximum strength. This has been done in the present study by expressing the column strength parameters in terms of probability density functions or distribution functions, and their characteristic quantities. The form of the functions have been assumed, but the available data from several experimental investigations have confirmed the validity of the assumptions made.

The detailed evaluation and analysis that have led to the determination of the probability density functions that illustrate the random nature of the column strength factors will not be presented here, due to the limitations on the length of the paper. Exhaustive developments of these relationships are given in Ref. 1, however, and therefore only the type of functions used will be outlined. These are the following:

1. For the cross-sectional properties of wide-flange and box shapes: Normal (Gaussian) distribution.
2. For the mechanical properties of the steel (only the yield stress is considered as a random variable): Type I asymptotic extreme value distribution for largest values.

3. For the residual stress in any element in the cross section: Normal (Gaussian) distribution.
4. For the initial out-of-straightness of the column: Type I asymptotic extreme value distribution for the smallest value.

It should be noted that although the residual stress in an element in the cross section is assumed to vary normally and independently of all of the other elements in the shape, the overall residual stress distribution has to satisfy force and moment equilibrium in the shape, prior to the application of any external load. The random nature of the overall residual stress distribution, as evidenced by the different patterns in rolled, welded, universal mill, and flame-cut shapes, has not been studied. This distribution is greatly influenced by the manufacturing method, and therefore has a most significant effect on the column strength. The random variations of the residual stresses that are considered are thus indicative only of deviations about the mean residual stress pattern.

Figures 1 and 2 give examples of the above described developments. Figure 1 shows the probability density function for the initial out-of-straightness, and Fig. 2 shows the derived probability density function (three-dimensional response surface) for the yield load of the rolled wide-flange shape W8x31 of steel grade ASTM A36. It should be noted that the specification[5] maximum allowable out-of-straightness, L/1000, has been assumed representative of the $97\frac{1}{2}$ percent probability level, such that values larger than this occur with a probability of 2.5 percent. The minimum out-of-straightness is 0 (zero), which is assumed to occur with a probability of 1 percent.

2.2 Probabilistic Evaluation of Maximum Column Strength

The column maximum strength may in principle be expressed by the following equation, where P_{max} denotes the maximum strength:

$$P_{max} = f(\sigma_y, \sigma_r, B, b, t, d, w, e_L, L) \tag{1}$$

where σ_r denotes the residual stress; b, t, d, and w are geometric descriptors of the cross section of the column; and e_L is the initial out-of-straightness. The other factors have been defined previously. The function given by Eq. (1) represents a multidimensional probability density function, or a response surface, since the parameters involved may be treated as random variables.

The probabilistic characteristics of the pertinent factors have already been established. The modulus of elasticity, E, is treated as a constant, and the column length, L, is also a deterministic quantity. The column is thought of as being subjected to a deterministic load, P, which remains as such from the onset of the loading and until the maximum capacity is reached. The load-deflection analysis of the column therefore will result in the determination of a semi-probabilistic load-deflection curve with the load as a random variable if the deflection is the input-value, and vice versa. The concept of semi-probabilistic load-deflection curves is schematically illustrated in Fig. 3.

The maximum strength that is found by the solution of the incremental, iterative expressions becomes a fixed value for every given set of values of the strength parameters for a given column. The random variation of the column strength factors provides for a random variation of the strength, thus leading to the determination of the probabilistic characteristics of the strength (see Fig. 3b) for a given column and length. Solving the same problem with different values of the length eventually leads to a set of column curves that illustrates the total variation of the strength. Such a set of column curves is defined as the column curve spectrum.

The complete evaluation of the probabilistic, incremental/iterative equations will not be presented, but a detailed development is given in Ref. 1. As an illustration, however, the total stress in an element i of the cross section is given by:

$$\frac{\tilde{\sigma}_i}{\tilde{\sigma}_y} = \frac{\tilde{\epsilon}_i}{\tilde{\epsilon}_y} = \frac{1}{\tilde{\epsilon}_y}\left[\tilde{\epsilon}_{ri} + \tilde{\epsilon}_p + \tilde{\theta}.\tilde{\epsilon}_{si}\right] = \Phi\ (\epsilon) \qquad (2)$$

where the tilde (~) denotes a random variable. For example, with the distribution characteristics for $\tilde{\sigma}_y$ given, those of $\tilde{\epsilon}_y$ can easily be found, since

$$\tilde{\sigma}_y = \tilde{\epsilon}_y \cdot E \qquad (3)$$

where E is a deterministic quantity. Similar principles apply for the solution for $\tilde{\epsilon}_{ri}$, $\tilde{\theta}$, and $\tilde{\theta}.\tilde{\epsilon}_{si}$. For $\tilde{\epsilon}_p$ a probability density function is used, as opposed to a specific value in the deterministic approach. This is a very significant computational advantage, since the range of ϵ_p -values is taken into account at the same time. The time-consuming and error-prone repetition of the calculations that result from incorrectly assumed ϵ_p -values is thereby eliminated.

Equation (2) forms the basis for the development of the probabilistic characteristics of the elemental stress and strain, and can be extended to incorporate all elements in the cross section. This in turn is used to determine the properties of the (random) internal force and moment, which leads to the solution for the equilibrium external load.

Various numerical methods may be used to determine the random variation of the maximum strength. The use of a Monte Carlo approach was investigated, but was discarded as an inefficient and expensive solution procedure. It may prove advantageous if purely theoretical values are used for the column strength parameters. For all practical purposes, however, the complete distribution of the maximum strength is not needed, since basically the upper and lower bounds, and a central distribution parameter such as the mean, will provide the information necessary.

3. THE VARIATION OF MAXIMUM COLUMN STRENGTH

Large amounts of data have been produced in this investigation, and only a few representative examples are shown and discussed here. The information presented is thus but a small part of what is available, but it nevertheless illustrates and emphasizes all of the important findings of the study.

Figure 4 shows the column curve spectra for the major and minor axis bending of a typical light rolled wide-flange shape (W8x31, steel grade ASTM A36). Each spectrum reflects the variation of the strength of the shape, when all of the column strength parameters vary between their respective extreme values. The spectra therefore illustrate the 95 percent confidence intervals for the maximum strength of the shape, such that there is only a probability of 5 percent that the strength of a randomly chosen W8x31 (A36) column will fall outside the interval. The upper limit of each spectrum is indicative of columns with an initial out-of-straightness of L/10,000, and the lower limit of columns with e_L = L/1000.

In order to detect and analyze the effects of the variability of the other column strength parameters, column curve spectra were prepared, for which the initial out-of-straightness was kept constant. Figure 5 shows the resulting spectra for the W8x31 (A36) shape, with e_L maintained at its mean value of L/1470.

Within the limitations and assumptions imposed by the study, the data presented in Fig. 5 show that the influence of the variability of the yield stress and the cross-sectional properties, and of the ± - variations of the residual stresses in any particular shape with a specific manufacturing method, is relatively small for the variation of the maximum strength. The two column curve spectra both indicate maximum strengths that lie within a range of 3 to 7 percent (from the upper to the lower limit), depending on the magnitude of the slenderness ratio. Extended analysis of the data, furthermore, show that this variation almost in its entirety may be attributed to the variation of the yield stress. In these analyses, means and coefficients of variation of the maximum strength were computed for various slenderness ratios, maintaining the yield stress at its minimum, mean, and maximum values. For each value of the yield stress, the corresponding mean values of the maximum strength were clearly different, although the differences were very small; and the coefficients of variation were extremely small (between 0 and 0.6 percent). No systematic influence of the varying residual stresses and cross-sectional properties was found. These statements are true for all slenderness ratios, and also for other values of the out-of-straightness.

The reason for the lack of influence of the residual stress variation about the mean residual stress distribution in the shape, is partly due to the over-riding influence of the initial out-of-straightness which strongly governs the behavior and strength of the column. It is also due to the fact that any residual stress distribution has to be in equilibrium. The effects of the geometric properties are probably almost completely over-ridden by the variation of the yield stress.

The conclusions arrived at above are basically true for all the rolled and welded wide-flange and box shapes that have been studied. However, the yield stress becomes more important as the range between its maximum and minimum values increases. For heavy rolled shapes also, the variability of the cross-sectional dimensions has a certain effect.

Figure 6 shows the major axis column curve spectrum for the W8x31 (A36) shape, together with the curves depicting its dispersion characteristics. Due to the influence of the initial out-of-straightness, which is distributed according to an extreme value density function (see Fig. 1), the maximum column strength for any given slenderness ratio also will be distributed as such. This is indicated in Fig. 6, and Fig. 7 illustrates the probability density function for the maximum strength of the W8x31 (A36) column with a non-dimensional slenderness ratio of 0.9, bent about the major axis. It was found that a Type I (Gumbel, largest value) asymptotic extreme value distribution fits the data very well; and the results for all slenderness ratios and for the other types of columns investigated also confirm this finding.

The data presented in Fig. 8 are analogous to those of Fig. 6, but represent the column curve spectrum for the minor axis bending of the W8x31 shape. The skew distribution of the maximum stength prevails, although it may be noted that it is significantly more pronounced for the intermediate and high slenderness ratios, when compared to the data in Fig. 6. This is a commom property for many column curve spectra for minor axis bending of wide-flange shapes.

The results that have been given here are indicative of the most important findings of the probabilistic study of the maximum strength of centrally loaded steel columns. Detailed and extensive data for a number of column types and shapes, in different steel grades, are provided in Ref. 1. The findings have furthermore been utilized in the development of a set of multiple column curves, which, it is believed, will provide significant improvements in the method of assessing the design strength of real columns.

4. SUMMARY AND CONCLUSIONS

Some of the most significant findings of the study presented here may be summarized briefly:

1. A probabilistic method for the solution of the problem of defining the maximum strength of centrally loaded, initially curved, pinned-end, prismatic steel columns has been developed. This represents the first time that a structure exhibiting a random non-linear behavior, for which the basic relationships are expressed as incremental, iterative equations, has been treated within the context of probability theory.
2. The random variation of the strength of a particular column, given its manufacturing method, almost entirely may be attributed to the random variation of the initial out-of-straightness. The random variation of the yield stress has a small effect, but this increases with the increasing yield stress and its range of variation.
3. The random variation of the residual stresses about their mean, and of the cross-sectional properties, do not contribute significantly to the random variation of the maximum column

strength. The probabilistic nature of the overall residual stress distribution has not been studied, and the pattern of residual stress in the shape therefore remains one of the most significant column strength parameters.

4. Due to the overall importance of the initial out-of-straightness, the maximum strength of a specific column will be distributed in a skew fashion. It has been found that a Type I asymptotic extreme value distribution is a good representation of the random column strength variation.

5. ACKNOWLEDGEMENTS

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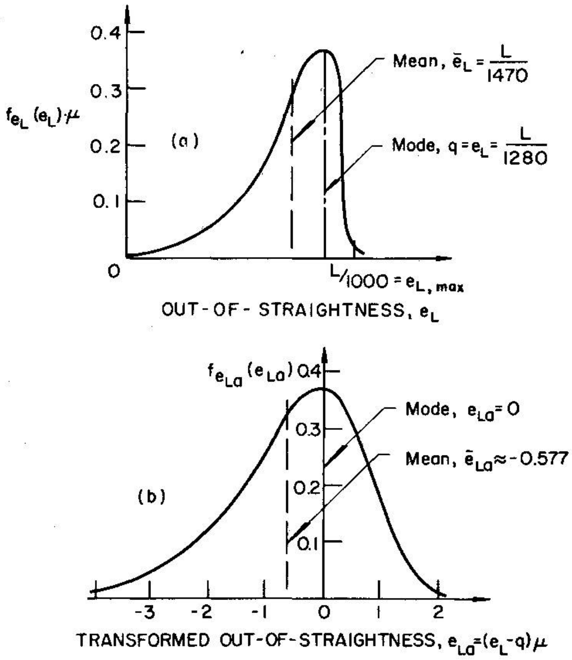

Fig. 1 The Type I Asymptotic Extreme Value Distribution Representing the Probability Density Function for the Initial Out-of-Straightness of the Column

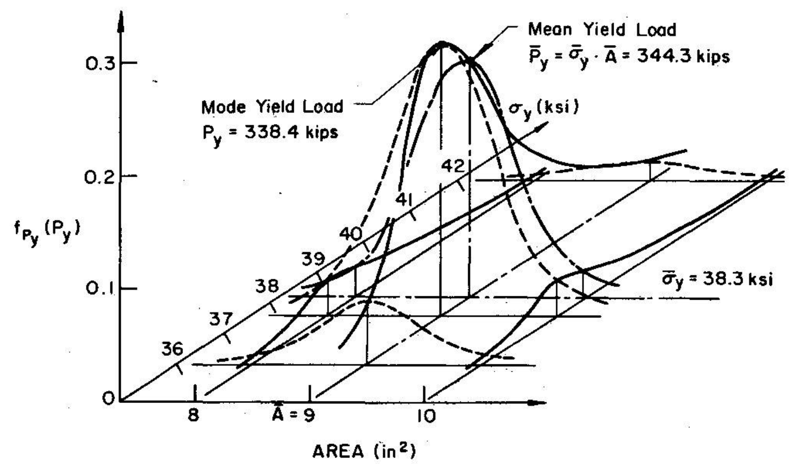

Fig. 2 The Derived Probability Density Function for the Yield Load of the Rolled Wide-Flange Shape W8x31 of Steel Grade ASTM A36

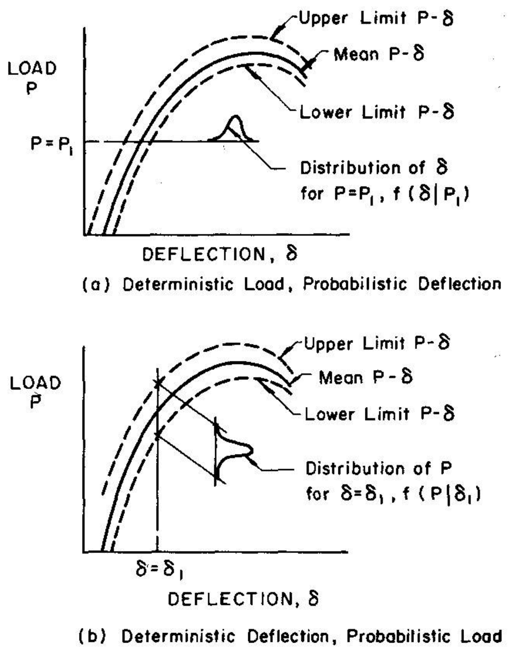

Fig. 3 **A Schematic Illustration of the Concept of Semi-Probabilistic Load-Deflection Curves**

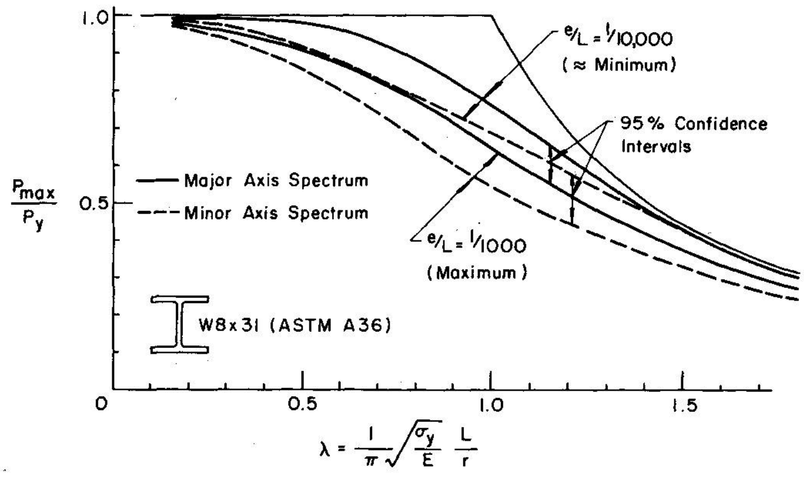

Fig. 4 **The Column Curve Spectra for Major and Minor Axis Bending of the Rolled Wide-Flange Shape W8x31 (A36)**

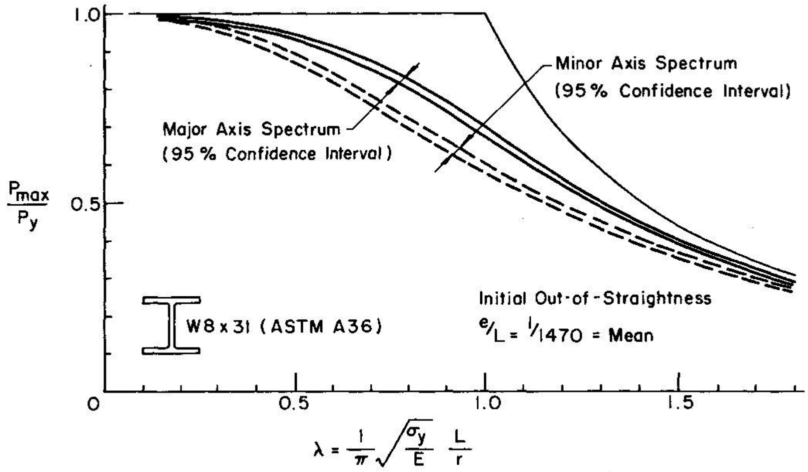

Fig. 5 The Column Curve Spectra for Major and Minor Axis Bending of the Rolled Wide-Flange Shape W8x31 (A36), with the Initial Out-of-Straightness Kept Constant (=L/1470)

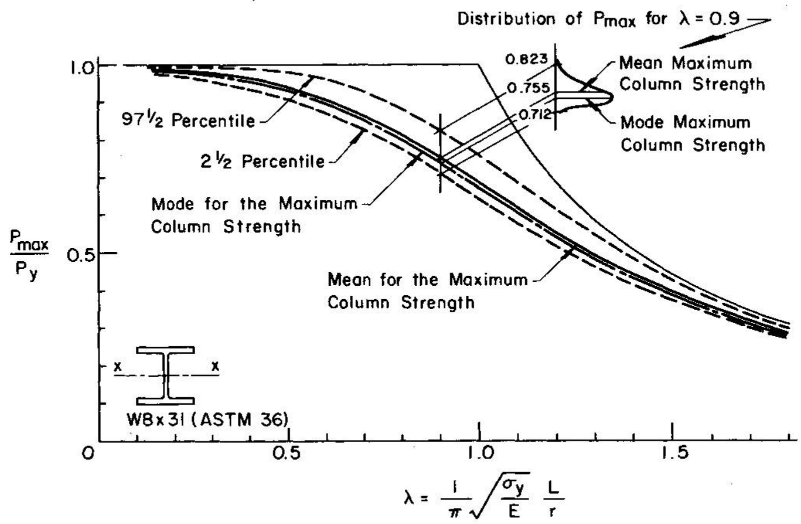

Fig. 6 The Dispersion Characteristics of the Major Axis Column Curve Spectrum for the Rolled Wide-Flange Shape W8x31 (A36)

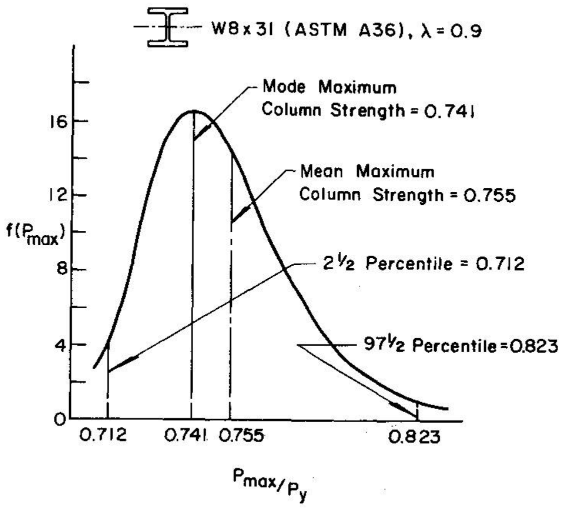

Fig. 7 The Probability Density Function (Type I Asymptotic Extreme Value Distribution) for the Maximum Strength of a Column W8x31 of Steel Grade ASTM A36, with Non-Dimensional Slenderness Ration of 0.9 (L/r = 90)

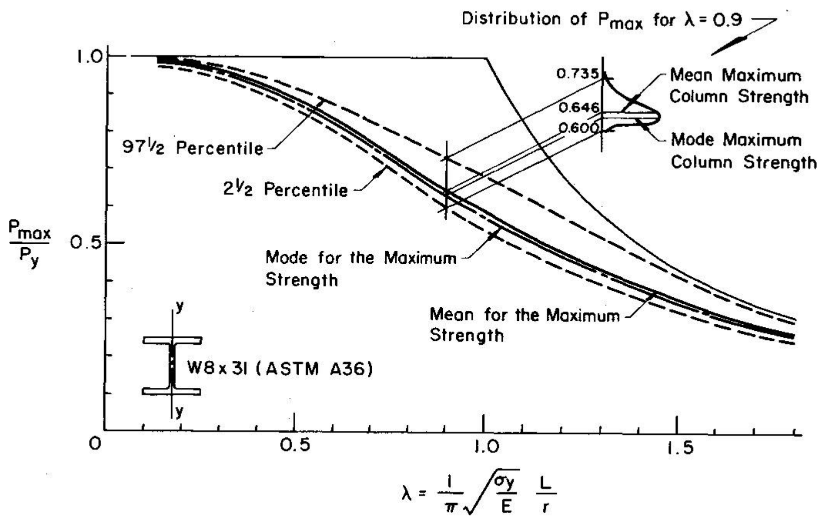

Fig. 8 The Dispersion Characteristics of the Minor Axis Column Curve Spectrum for the Rolled Wide-Flange Shape W8x31 (A36)