Ductility of reinforced concrete columns

Autor(en): **Bo, Gian Mario / Capurro, Marco**

Objekttyp: **Article**

Zeitschrift: **IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen**

Band (Jahr): **30 (1978)**

PDF erstellt am: **26.05.2024**

Persistenter Link: https://doi.org/10.5169/seals-24197

DUCTILITY OF REINFORCED CONCRETE COLUMNS

Gian Mario Bo

Associate Professor
Istituto Tecnica delle
Costruzioni
Politecnico di Torino

Marco Capurro

Associate Professor
Istituto Scienza delle
Costruzioni
Università di Genova

SUMMARY

A method is presented for calculating the rotation of the end plastic hinge in a reinforced concrete column subject to axial load, bending moment and shear. The method is based on the assumption that, in the regions adjacent to that to which ultimate bending moment is applied, the bond between tension metal reinforcement and concrete is completely inoperative. Besides, the plastic deformation of the tension steel is assumed to be linearly variable in the portion where slipping occurs from zero up to maximum value in the ultimate bending moment section.

The length of the plastic zone is then obtained on the basis of equilibrium considerations, while the overall rotation of the plasticization zone is calculated by taking the internal work to be equal to the external work.

1. INTRODUCTION

In a framed building structure subject to strong earthquakes, over stresses are generally produced in the members and particularly in the columns. The mode of failure of the structure under seismic loads and dead loads plus a fraction of vertical service loads depends on the capacity of deflection of the columns in the inelastic range. Hence, calculating the ultimate deflection of each single column, as permitted by the rotation of plastic hinges, is a fundamental step in the analysis and design of buildings exposed to seismic risk. A typical pattern for the ultimate behaviour of a simple oscillator subject to lateral load is shown in fig. 1.

The evaluation of the ultimate deflection η_u at the top of a column and more generally of any structural member subject to constant axial load and bending moment varying along the axis, requires the knowledge of the moment curvature diagram of the member and turns out to be rather complicated for R.C. columns M- diagrams as available in the literature [1] may be usefully employed; in this case, however, the effects of confining of concrete due to transversal reinforcement are neglected, thus obtaining values of the deflection η_u significantly lower than the experimental ones [2] [3] . A different procedure consists of obtaining the ultimate curvature or the ratio of the ultimate to the perfectly elastic curvature at the clamped-end cross-section of the column [4].

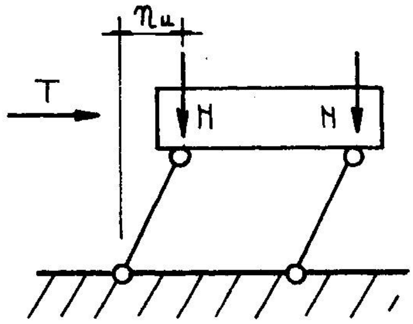

fig. n. 1

The structural member is assumed in this case to be divided into two sharply distinguished zones, in one of which the material behaves elastically, while in the other, close to the clamped end, plastic behaviour is widely predominant.

The length of the equivalent plastic zone is usually taken to be proportional to the depth and span of the column according to binomial relations [5] .

According to [2] we have, for example:

$$\Delta_p = 0.50h + 0.2 \sqrt{h}\left(\frac{z}{h}\right) \tag{1}$$

From the knowledge of curvatures and the extension of the plastic hinge, values of the capacity of rotation φ may be easily calculated and hence the ultimate deflection is obtained. This method, however, applies only to a restricted range of dimensions of the rectangular cross-section and no extrapolation of the semiempirical relation (1) seems to be reliable to cover different shapes of cross-sections (such as cave or lamellar section and so on).

In the present work a theoretical approach is put forward allowing the designer to calculate the amount of rotation at the plastic hinge irrespectively of the hypothesis of plane cross-sections which has been commonly adopted so far in all R.C. calculations. In the regions adjacent to the clamped end, the bond between concrete and tension steel is assumed to have failed because of local overstress phenomena and cracking, so that slippling of longitudinal bars is permitted and consequently, over some fraction of the column length, the tensile stress in the reinforcement is, at a good estimate, constantly equal to the yield stress of steel.

The proposed method is more advantageous than the others described above insofar as it does not rely solely on experimental data and consequently it could lead to a further extension of its range of applicability.

2. BASIC ASSUMPTION

Let us consider a R.C. rectangular column with symmetrical single-layer reinforcement.

This column will be subjected to bending and axial force N lower than the balanced value N_b, with non-vanishing shear force T (fig. 2).
The problem is examined while the rotation of the plastic hinge is taking place.
At the clamped-end cross-section $M=M_{ult}$ is assumed and the strains ε'_{br}, ε_{au} are calculated according to the hypothesis of plane cross-sections. In the vicinity of the clamped end the tension steel will undergo yielding with $\sigma_a = \sigma_{as}$ = const and will be strained plastically from zero plastic strain (at the top of the plastic region) to ε_{au}, at a rate that, grossly estimated, may be assumed to be constant. Throughout the region affected by the plastic hinge no assumptions concerning compatibility of strain for compression concrete and tensile reinforcement are made. The plastic region may be considered to extend up to the cross-section where, the tensile stress in steel altogether equalling σ_{as}, the amount of plastic strain ε_a approaches zero.

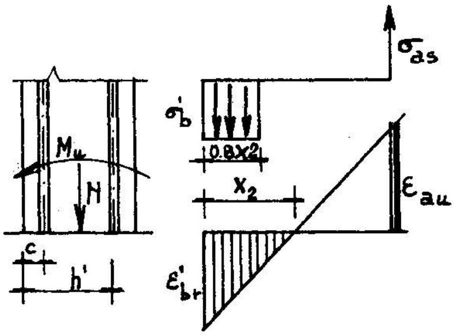

fig. n. 2

In this cross-section the position of the neutral axis, on account of the hypothesis of free slipping of tensile steel in the plastic zone, may be assumed to coincide with the centroid of tensile reinforcement.

This would indeed be a drastic assumption, as some residual bonding effect is expected to work even in a widely cracked zone. More conservatively, the plastic region may be considered as ending below the cross-section where onset of yielding in the tensile steel takes place, the concrete still behaving elastically and in full bond with the tension bars; here the law of plane cross-sections is again held to apply.

Under all these assumptions the amount of rotation of the plastic hinge may be calculated and an estimate of its effective extension may be provided.

3. EVALUATION OF THE EXTENSION OF THE PLASTIC ZONE

Consider a column with rectangular cross-section as illustrated in fig. 3. Let:

h = be the full depth of cross-section;

$h'=h-c$ = the reduced depth, with c = the distance of the reinforcement from the edge;

b = the width of the cross-section;

x = the position of the neutral axis with respect to the compression edge; $\xi=x/h'$

$F_a=F'_a$ the area of both tension and compression reinforcement;

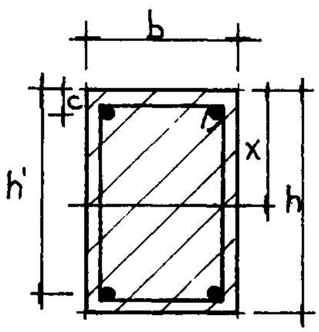

fig. n. 3

$\mu_a=\mu'_a=F_a/bh'$ the percentage of reinforcement;

$\bar{\mu}_a=\bar{\mu}'_a=\frac{\sigma_{as}}{\bar{\sigma}'_b}\mu_a$ the mechanical percentage of reinforcement;

σ_{as} being the yield stress of steel; $\varepsilon_{ae}=\frac{\sigma_{as}}{E_a}$ the first yielding strain in steel;

$\bar{\sigma}'_b$ = the crushing stress of concrete; $\bar{\varepsilon}'_{br}$ the ultimate strain of concrete;

$n=\frac{E_a}{E_b}$ the ratio of elastic moduli;

$n_d=\frac{N}{\sigma_{as}F_a}$ = the non dimensional axial load;

T = the shear force

The assumptions discussed in the foregoing section lead to stress and strain distributions as illustrated in fig. 4.

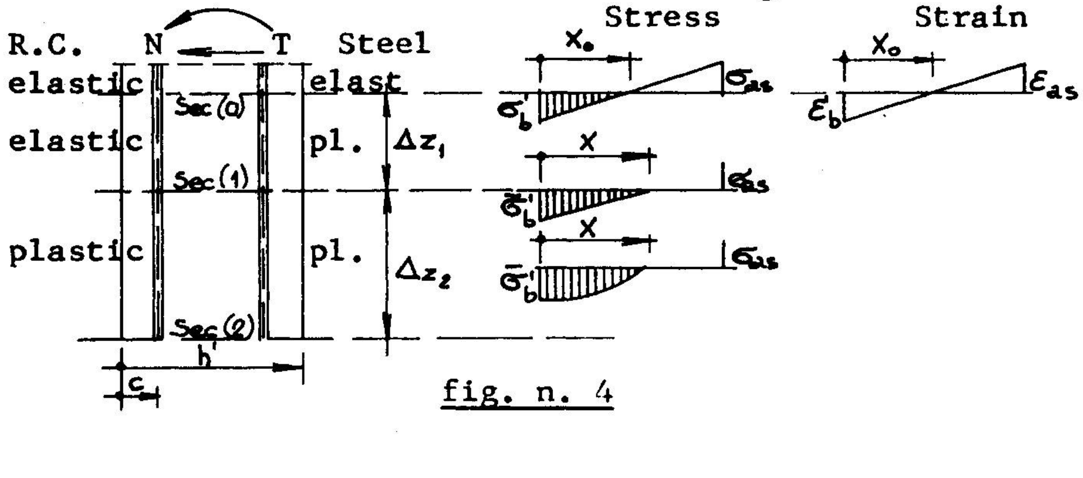

fig. n. 4

In the clamped-end cross-section, labelled (2) strain and stress distributions are assumed according to the usual assumptions of ultimate design. The compression region is supposed to be rectangular with effective depth $0.8\,x_{(2)}$.

Between cross-section (2) and (1) the compression stress of concrete is assigned a curvilinear distribution over the depth x, but the compression resultant is always taken at a distance of approximately 0.4x from the compressed edge. The total amount of compression may be expressed as

$$P_b = \bar{\sigma}'_b \cdot \alpha \cdot x \cdot b$$

α being a shape factor ranging between 0,5 (triangular elastic distribution) and 0,8 (ultimate distribution).

In cross-section (1) a triangular compression stress distribution is assumed with maximum value amounting to $\bar{\sigma}'_b$.

For cross-section (0) two alternative conditions are examined: both conditions (case a) stipulate that the steel is strained just at the yielding point with $\varepsilon_a = \varepsilon_{ae}$ ($\varepsilon_{ae} \simeq \sigma_{as}/E_a$) and $\sigma_a = \sigma_{as}$; in the first case (case a) compatibility of strain between steel and concrete is introduced; in the other (case b)

$$x_{(o)} = h'$$

is assumed, disregarding compatibility of deformations on account of bond slip being allowed to take place below the cross-section (0).

From the equilibrium of the forces acting at cross-section (0) expressed by the equation

$$0.5\,\sigma'_b\,x_{(o)}b - N + \sigma'_a F'_a = \sigma_{as} F_a \tag{1}$$

and from compatibility of deformations, written as:

$$x_{(o)} = h' \frac{\varepsilon'_b}{\varepsilon'_b + \varepsilon_{as}} \tag{2}$$

it is possible to obtain the position of the neutral axis ξ_o for case a, namely by solving the equation:

$$\xi_o^2 + 2n\,\xi_o\left[\mu_a(1+n_\alpha)+0.8\mu'_a\right] - 2\mu_a n(1+n_\alpha) = 0 \tag{3}$$

where $\varepsilon'_a \simeq 0.8\,\varepsilon'_b$ has been introduced for the sake of simplicity In the case b, we immediately have:

$$\xi_o = 1 \tag{4}$$

In the region between the cross-sections (0) and (1) let us take into consideration the variations, with respect to a parameter progressing along the column axis, of the equilibrium equations of both forces and moments. The quantities which may vary are, of course, σ'_b and the position of the neutral axis, x. The operator performing such variations is denoted by δ . Then we obtain:

$$0.5\, bx\, \delta\sigma'_b + 0.5\, \sigma'_b\, b\, \delta x = -0.8 n F'_a\, \delta\sigma'_b \tag{5}$$

$$\delta M = 0.5\, \sigma'_b\, b(h' - \tfrac{2}{3}x)\, \delta x + 0.5 bx(h' - \tfrac{1}{3}x)\delta\sigma'_b + 0.8 n F'_a (h'-c) . \delta\sigma'_b \tag{6}$$

By eliminating $\delta\sigma'_b$ between (5) and (6), the moment variation turns out to be:

$$\delta M = \frac{0.5}{3}\, \sigma'_b\, bx\, \delta x \tag{7}$$

hence, dividing by δ_z :

$$\frac{\delta M}{\delta z} = \frac{dM}{dz} = T = - \frac{0.5}{3}\, \sigma'_b\, bx \frac{dx}{dz} \tag{8}$$

This relation may be integrated between the cross-sections (0) ($x = x_{(o)}$) and (1) ($x = x_{(1)}$), after removal of σ'_b through the equilibrium of forces at a current coördinate z

$$\frac{\Delta z_1}{h'} = \frac{\sigma_{as} F_a}{T}\, k_1 \tag{9}$$

where

$$k_1 = \frac{(1+n_d)}{3} \left[\xi_o - 2\bar{\mu}_a(1+n_d) + 1{,}6 n \mu'_a \left(1 - \ln \frac{\xi_o + 1.6 n \mu'_a}{\xi_1 + 1.6 n \mu'_a}\right)\right. \tag{10}$$

In the region between the cross-sections (1) to (2) with the assumption that the compression reinforcement remains elastic over the whole length Δz_2, and $\sigma'_a \simeq \alpha / 0.8 \cdot \sigma_{as} = 1.25\, \alpha\, \sigma_{as}$, the variations of the equilibrium equations are expressed as:

$$\bar{\sigma}'_b\, bx\, \delta\alpha + \bar{\sigma}'_b\, b\alpha\, \delta x = -1{,}25\, \sigma_{as}\, F'_a\, \delta\alpha \tag{11}$$

$$\delta M = \bar{\sigma}'_b\, bh'^2 (0.4\, \xi^2 + \xi\bar{\mu}_a + 1.25 \frac{c}{h'}\, \bar{\mu}'_a)\, \delta\alpha \tag{12}$$

By eliminating ξ between (12) and the equation of equilibrium of forces at a current coor.z, δM as expressed by (12) may be integrated over the region between the cross-sections (1) (α =0.5) and (2) (α = 0.8), thus giving:

$$\frac{\Delta z_2}{h'} = \frac{\sigma_{as} F_a}{T}\, k_2 \tag{13}$$

where

$$k_2 = 0.3 \left[\bar{\mu}_a (1+n_d)^2 - 1.25 \frac{\bar{\mu}'_a}{\bar{\mu}_a} (0.5\, \bar{\mu}'_a - \frac{c}{h'})\right] \tag{14}$$

Equation (14) may undergo further simplification by recalling that $\mu_a = \mu'_a$.

The total length of the plastic region is therefore:

$$\frac{\Delta z}{h'} = \frac{\Delta z_1 + \Delta z_2}{h'} = \frac{\sigma_{as} F_a}{T} (k_1 + k_2) \tag{15}$$

4. EVALUATION OF THE PLASTIC-HINGE ROTATION

With the usual idealization of attributing the whole plastic rotation to a single point of the column axis, namely the idealized plastic hinge, the capacity of rotation φ_u may be obtained from an energy balance between the inner plastic work made in the whole plastic region of length Δz by both concrete and steel while strained beyond the elastic limits, and the external work $M_{ult} \cdot \varphi_u$.

In this manner, the problem is reduced to calculating the inner plastic work made both tension steel in the length Δz and by concrete in the inelastic range over the length Δz_2.

With the assumptions made in the foregoing the former is promptly calculated as:

$$\delta L_{pl(a)} = \frac{1}{2} \sigma_{as} F_a (\varepsilon_{au} - \varepsilon_{ae}) \Delta z \tag{16}$$

the stress in steel being constantly equal to σ_{as}.

The work of concrete in the region where this material is supposed to have abandoned the elastic range, may be espressed as:

$$\delta L_{pl(b)} = \int_0^{\Delta z_2} dz \int_{A'} \sigma'_b \, \delta\varepsilon'_b \, dA' \tag{17}$$

A' being the compression area as a function of z, namely $A' = bx(z)$ and $dA' = bdx$ .It is reasonable to assume a mean value for σ'_b over the compression area of each single cross-section, namely by putting

$$\sigma'_b \simeq \text{mean } \sigma'_b = \alpha(z)\, \bar{\sigma}'_b \tag{18}$$

in definition (17). Moreover, the strain $\varepsilon'_b(x')$ at any position in the compression area, may be expressed on account of compatibility of the deformation in the compression-concrete area, as

$$\varepsilon'_b(x') = \frac{x'}{x} \varepsilon'_b(z) \tag{19}$$

By introducing both (18) and (19) into the expression of plastic work, we have:

$$\delta L_{pl(b)} = \int_0^{\Delta z_2} \alpha^{-1} \bar{\sigma}'_b b \frac{\delta \varepsilon'_b}{x} dz \int_0^x x' dx' \qquad (20)$$

The inner integral may be calculated at once to yield $x^2/2$; so that (20) is reduced to an integral over coordinate z:

$$\delta L_{pl(b)} = \frac{1}{2} \bar{\sigma}'_b b \int_0^{\Delta z_2} \alpha x \, \delta \varepsilon'_b \, dz \qquad (21)$$

where α, x, $\delta\varepsilon'_b$, are all functions of z. On the basis of the equations of equilibrium both in finite and in varying form, and as the result of lengthy but plain calculations, we have succeded in eliminating x, and [for $\delta \varepsilon'_b(z)$ we may accept a linear expression: $\delta \varepsilon'_b = z/\Delta z_2 \, (\bar{\varepsilon}'_{br} - \varepsilon'_{be})$] and in transforming (21) into an integral of a known function of α over α itself. This can be solved through direct integration, yielding the final result:

$$\delta L_{pl(b)} = \frac{\sigma_{as}^2 F_a^2 h'}{T} (\bar{\varepsilon}'_{br} - \varepsilon'_{be}).k_3 \qquad (22)$$

being

$$k_3 = 0.25\, k_2 \left[1+n_1-0.389 \frac{\bar{\mu}'_a}{\bar{\mu}_a}\right] \qquad (23)$$

The plastic work of the compression concrete is however small (amounting to no more than a certain percentage of steel work) and sometimes it is not totally wrong to think of neglecting it in comparison with the work of steel. It may be useful to notice that, in any case, the amount of plastic work of concrete is not affected by the situation at cross-section (1), wether compatibility is assumed according to case a or free slipping of steel reinforcement is supposed (case b). In the latter case, the only modification to be brought into the foregoing results, involves, as previously pointed out, the length Δz_1 which in turn, enters into the expression of $\delta L_{pl(a)}$. As a consequence of this modification, $\delta L_{pl(a)}$ is expected, and has been actually calculated, to increase significantly (up to several times), probably overemphazizing the ductility of the column (*). Finally, the total

(*) A more general approach would be that of finding ξ_o from a condition of partial compatibility imposed on the deformations at cross-section (0). The concrete strain ε'_b would be compatible, in this case, not with the total steel strain ε_{ae}, but with a fraction of it obtained by subtracting from ε_{ae} the amount of slip rate $(du/dz)_o \simeq (f/\Delta_f)_o$ due to cracking, where f is the crack width and Δ_f the spacing

../..

ultimate rotation of the plastic hinge is calculated as:

$$\varphi_u = \frac{L_{pl(a)} + L_{pl(b)}}{M_{ult}} \tag{24}$$

M_{ult} being the ultimate moment of the column to be obtained from the equilibrium of cross-section (2). The deflection at the top of the column permitted by the rotation of the plastic hinge may be estimated as approximately

$$\eta_{pl} \cong \varphi_u (1 - \frac{\Delta z}{2}) \tag{25}$$

1 being the span of the column and supposing the plastic hinge reduced to a point located at the centre of the plastic zone.

In the diagrams of fig. 6 the ratios

$$\frac{\Delta z}{h'} \Big/ \frac{\sigma_{as} F_a}{T} \quad \text{and} \quad \varphi_u \Big/ \frac{\sigma_{as} F_a}{T}$$

are plotted versus the non-dimensional axial force n_d for three different reinforcement percentages. The capacity of rotation of the column decrease rapidly with increasing axial force, in spite of a not negligible spreading out of the plastic region.

In fig. 7 the effect of increasing the column depth can be followed, leading to decrements both of $\Delta z/h'$ and of φ_u, although with a clear tendency to asymptotes for very deep cross-sections. The effects of varying $\bar{\varepsilon}'_{br}$ and $\bar{\sigma}'_b$ have also been considered with the conclusions that φ_u increases nearly proportionally with $\bar{\varepsilon}'_{br}$ and even more decisively with the strength of concrete.

../..
of cracks, both calculated at cross section (0). The details of a procedure of this type at present are still to be worked out.

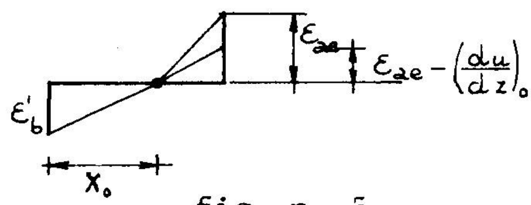

fig. n. 5

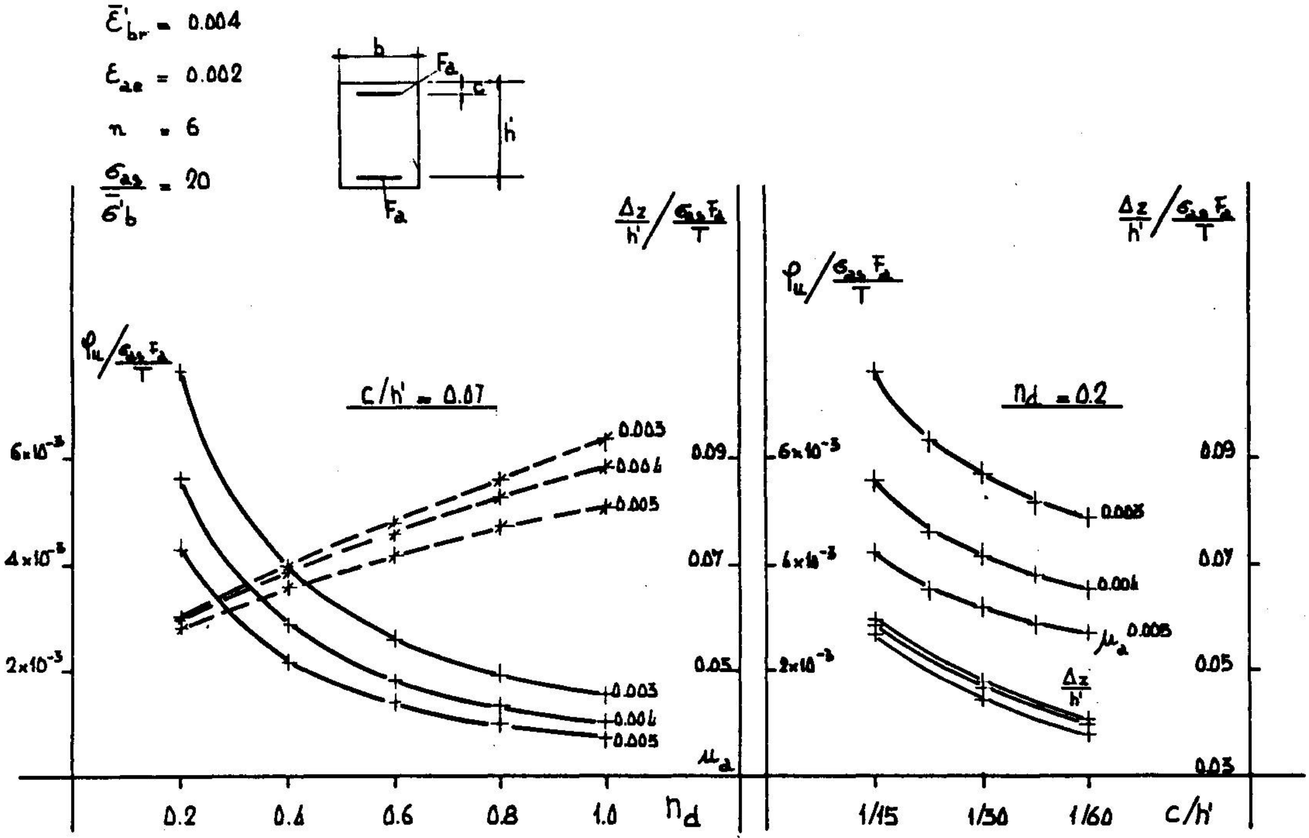

Fig. 6

Fig. 7

REFERENCES

[1] Macchi G. and E. Siviero
"Deformability of prismatic R.C. members with rectangular cross-section under combined bending and axial load"
Comité Européen du Béton - Bulletin d'Information n. 101 - Paris - 1974 - pagg. 191 and following.

[2] Corley G.W.
"Rotational capacity of reinforced concrete beams" - Journal of the Structural Division - ASCE - October 1966

[3] Jirsa J.O.
"Factors influencing the hinging behavior of reinforced concrete members under cyclic overloads"
Fifth World Congress of Earthquake Engineering - Roma - 1972 - pagg. 1198 and following

[4] Blume J.A., Newmark N.M. and L.H. Corning
"Design of Multistory Reinforced Concrete Buildings for Earthquake Motions"
Chicago - 1961 - Portland Cement Association - pagg. 113 and following

[5] Park R. and T. Paulay
"Reinforced Concrete Structures" - New York - 1975 - John Wiley and Sons - pagg. 248 and following