Soil - Structure Interaction in the Leaning Tower of Pisa

Interaction du sol et de la structure de la Tour Penchée de Pise

Wechselwirkung Baugrund - Konstruktion beim schiefen Turm in Pisa

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The main aspects of the time-dependent behaviour of the leaning tower of Pisa are: creep and the non-linear response of the soil; second order effects due to the increase in leaning; the progressive reduction in the loaded area.

The model shown in Fig. 1 has been developed to take all these aspects adequately into account, by making the following assumptions: a) the structure of the tower is a rigid body; b) the foundation area is subdivided into 19 strips, arranged perpendicularly to the leaning plane; c) the soil under each strip is a Maxwell element, consisting of a non-linear spring and a non-linear dash-pot, applied to the strip's centre of gravity.

The response of the elements of the chain (for a unit of area) is illustrated in Figs. 2a and 2b (for non linear springs and non-linear dash-pots, respectively).

The displacement of a point of the foundation can then be expressed as:

$$y_i\,(t_K) = a\,(t_K) + b\,(t_K) \cdot x_i$$

where $a(t_K)$ and $b(t_K)$ denote the displacement of the foundation centre and the rotation at time t_K, respectively.

The equilibrium equations during the time step $\Delta t_K = t_K - t_{K-1}$ are:

$$\begin{aligned} \Delta a(t_K)\sum_{i=1}^{19} K_i A_i + \Delta b(t_K)\sum_{i=1}^{19} K_i A_i x_i - \sum_{i=1}^{19} \Delta y_i^c(t_K) K_i A_i &= \Delta N(t_K), \\ \Delta a(t_K)\sum_{i=1}^{19} K_i A_i x_i + \Delta b(t_K)\sum_{i=1}^{19} K_i A_i x_i^2 - \sum_{i=1}^{19} \Delta y_i^c(t_K) K_i A_i x_i &= \Delta M(t_K). \end{aligned} \tag{1}$$

where

$$\Delta y_i^c(t_K) = \sigma_i(t_{K-1})\beta_i\left[\log t_K - \log(t_{K-1})\right],$$

is the increase in displacement produced by the dash-pot.

The stiffness of the non-linear springs has been continuously updated according to the stress level, as shown in Fig. 2a, cutting off the elements in tension. Similarly, the external moment $\Delta M(t_K)$ on the right-hand side of eqs. (1) increases with increasing tilt (second order effect), while both the axial load and the external moment could vary if some stabilising measures were applied. Convergence has been reached, at each time step, through an iterative procedure.

The range of non-linear spring and dash-pot parameters has been deduced from the extensive geo-technical tests performed on the soil near the tower [1]; final calibration has been obtained by comparing the slope history provided by the model and the measurements taken on the tower since 1550 [2].

In Fig. 3, the tilt vs. time diagram obtained from the model is compared with the available measurements. Agreement seems to be very satisfactory; in particular the simultaneous effects of the reduction in the loaded area and second order effects produces a strong increase in the tilt rate, in keeping with the observations made since 1970.

The model makes it possible to predict the future behaviour of the tower and to assess its response to external actions. If no stabilising measures are taken, the tower will collapse between 2030 and 2040, due to divergence of equilibrium.

A simulation of a very soft stabilising intervention, consisting in the application of a vertical force of 2123 kN at a distance of + 6.83 m from the centroid of the foundation, is presented in Fig. 4. This would achieve a very small instantaneous reduction in tilt, but its delayed effect would be considerable, as a number of springs and dash-pots would return from the plastic to the elastic field, leading to a substantial stabilisation of the tower.

REFERENCES.

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[2] LANCELLOTTA R., PEPE M. C., "A Report on the movements of the Tower and the Surrounding Square", Rapporto di ricerca n. 4, Politecnico di Torino, (1.1991).

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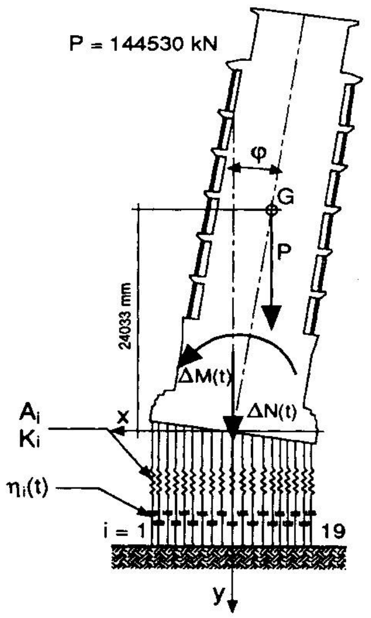

Fig. 1 - Soil-structure model.

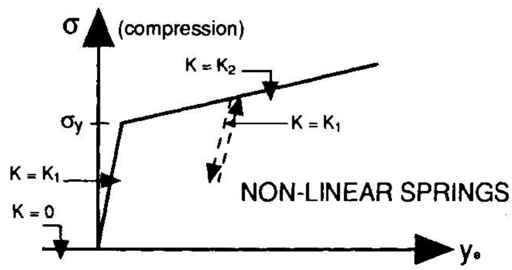

Fig. 2a - Stress/displacement relationship in non linear springs.

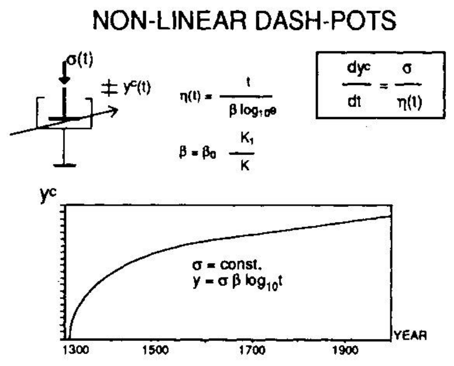

Fig. 2b - Stress/time/displacement relationship in non-linear dash-pots.

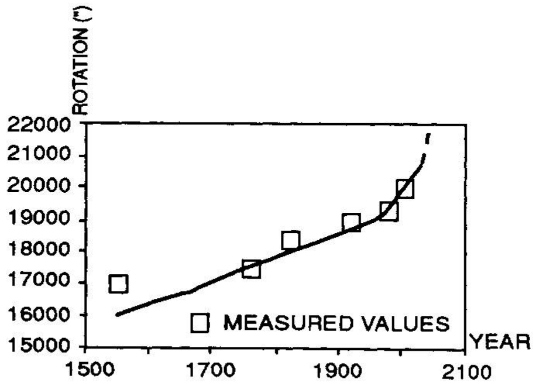

Fig. 3 - Rotation/time diagram and comparison with measured values.

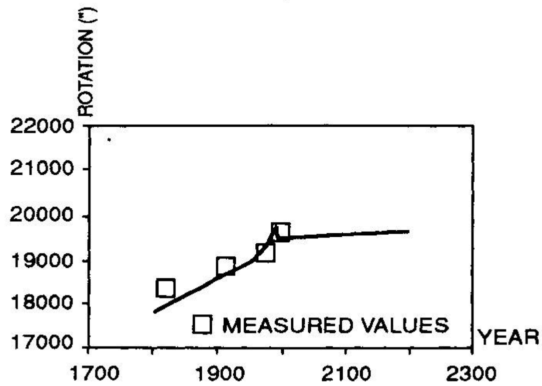

Fig. 4 - Instantaneous and delayed effects of the stabilisation measures.