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Objekttyp: **Article**

Zeitschrift: **IABSE reports = Rapports AIPC = IVBH Berichte**

Band (Jahr): **74 (1996)**

PDF erstellt am: **24.05.2024**

Persistenter Link: https://doi.org/10.5169/seals-56082

Statistical procedures for design assisted by testing

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Summary

Three parameter asymmetric distributions, characterised by the mean μ, standard deviation σ and independent coefficient of skewness α, are considered to present necessary statistical techniques for estimating characteristic and design values of basic variables from test data of limited size. It is shown that the resulting estimates for characteristic strength may considerably depend on the applied method and on available prior knowledge; possible asymmetry of the distribution should be considered whenever the coefficient of skewness exceeds $\pm$ 0,5.

1. Introduction

The Eurocode 1 [1] provides in Section 8 "Design assisted by testing" application rules for design procedures performed on the basis of tests. Design values for a material property, a model parameter or a resistance value may be determined from tests in either of the following two ways:

a) by assessing a characteristic value, which is divided by a partial factor and possibly by an explicit conversion factor,
b) by direct determination of the design value, implicitly or explicitly accounting for the conversion aspects and the total reliability required.

A simple statistical technique for assessment of material quality from tests is described in the informative annexes A and D of the Eurocode 1 [1], further information is available in ISO/CD 12491 [2] and revised ISO 2394 [3]. The methods included in [1], [2] and [3] are based on Bayesian approach assuming symmetrical normal distribution and vague prior information. It is, however, noted in the above mentioned Annex D that in practice there may be prior knowledge available indicating that the distribution type is of more favourable nature (for instance lognormal distribution with zero origin). There may be also partial prior knowledge about the mean and standard deviation based on previous experience which may lead to more accurate design values.

The aim of this contribution is to suggest possible extension of basic statistical methods recommended in [1], [2] and [3], particularly to show effect of population asymmetry and to propose operational procedures and appropriate provisions which could be included in an expected revision of the Eurocode 1 [1]. Presented procedures follow from previous studies concerning estimation of fractiles assuming general lognormal distribution [4], [5] and effects of distribution asymmetry in structural reliability and statistical quality control [6], [7], [8] and [9].

2. Statistical techniques

2.1 Basic probabilistic concepts

From the probabilistic point of view the characteristic or the design value of a resistance variable like the strength of concrete can be defined as a specified fractile of appropriate probability distribution. Fractile x_p is generally defined as a value of a random variable X satisfying the following relation

$$\mathrm{P}\{X < x_p\} = p \tag{1}$$

where capital X denotes a random variable and small x its particular realisation, p denotes specified probability. For the characteristic strength often the probability $p = 0{,}05$ is assumed. However, for the design strength lower probabilities, say $p \cong 0{,}001$, are to be considered. On the other hand the design value of non-dominating variables may correspond to greater probabilities, say $p \cong 0{,}10$.

When assessing strength of building materials, usually a limited number of observations is available only. Moreover, relatively high variability (coefficient of variation up to 0,25) and mostly a positive distribution asymmetry should be expected. That is why applied statistical techniques should be chosen cautiously, particularly when design strength corresponding to small probability is investigated.

In the following a lower fractile x_p ($p < 0{,}5$) of a random variable X is considered only. It is assumed that the population mean μ is unknown and sample mean m is available. The standard deviation σ is assumed to be either known or unknown. In the later case the sample standard deviation s is used. The coefficient of skewness α is always assumed to be known from previous experience. Two basic statistical methods to estimate fractiles are used most frequently: the coverage method and prediction method. When previous observations of a continuous production is available Bayesian approach can be used.

2.2 Coverage method

The classical coverage method is based on the key notion of the confidence level γ (often assumed 0,75, 0,90 or 0,95) for which the one-sided estimate $x_{p,\mathrm{cover}}$ of a lower p-fractile is determined in such a way that

$$\mathrm{P}\{x_{p,\,\mathrm{cover}} < x_p\} = \gamma \tag{2}$$

If the population standard deviation σ is known, the lower p-fractile estimate $x_{p,\mathrm{cover}}$ is given as

$$x_{p,\mathrm{cover}} = m - \kappa_p\ \sigma \tag{3}$$

if the population standard deviation σ is unknown and the sample standard deviation s is used then

$$x_{p,\mathrm{cover}} = m - k_p\ s \tag{4}$$

The estimation coefficients $\kappa_p = \kappa(\alpha, p, \gamma, n)$ and $k_p = k(\alpha, p, \gamma, n)$ depend on the coefficient of skewness α, on the probability p corresponding to the desired fractile x_p, on the confidence level γ and on sample size n. Explicit knowledge of the probability γ, that the estimate $x_{p,cover}$ shall lay on the safe side from the actual value x_p, is the most important advantage of the method. To take account statistical uncertainty the value γ = 0,75 is recommended in [3]. However, when unusual reliability consideration is required, higher confidence level 0,95 seems to be appropriate [5], [6]. In the documents [1] and [3] only the normal distribution is considered without taking into account possible asymmetry of the population distribution.

It may be shown [4] that if the population standard deviation σ is known, then the estimation coefficient $\kappa(\alpha, p, \gamma, n)$ may be well approximated using formula:

$$\kappa(\alpha, p, \gamma, n) = - u_p + u_\gamma / \sqrt{n} \tag{5}$$

where u_p is p-fractile of standardised lognormal distribution having the coefficient of skewness α, and u_γ is γ-fractile of standardised lognormal distribution having the coefficient of skewness $\alpha / \sqrt{n}$. If the population standard deviation σ is unknown, then the coefficient $k(\alpha, p, \gamma, n)$ may be expressed as

$$k(\alpha, p, \gamma, n) = - t(\alpha, p, \gamma, \nu) / \sqrt{n} \tag{6}$$

where $t(\alpha, p, \gamma, \nu)$ is γ-fractile of the generalised noncentral t-distribution having the coefficient of skewness α, corresponding to the probability p and with $\nu = n$-1 degree of freedom. The noncentral t-distribution, describing distribution of the p-fractile of lognormal distribution with the coefficient of skewness α, is a modification [4] of well known noncentral t-distribution derived from normal distribution. Extensive numerical tables for both estimation coefficients (σ is either known or unknown) are available in the Klokner Institute of CTU Prague.

2.3 Prediction method

According to the prediction method [10] the lower p-fractile x_p is assessed by the prediction limit $x_{p,pred}$, determined in such a way that a new value x_{n+1} randomly taken from the population would be expected to occur below $x_{p,pred}$ with the probability p, thus

$$\mathrm{P}\{x_{n+1} < x_{p,\,pred}\} = p \tag{7}$$

The prediction estimate $x_{p,pred}$, defined by equation (7), asymptotically approaches the unknown fractile x_p with increasing n, and from this point of view $x_{p,pred}$ can be considered as an assessment of x_p. It can be also shown that the prediction estimate $x_{p,pred}$ correspond approximately to the coverage method assuming the confidence level γ = 0,75 [8].

If the population standard deviation σ is known, the lower p-fractile estimate $x_{p,cover}$ is given in terms of the sample mean m as

$$x_{p,pred} = m + u_p\,(1/n + 1)^{1/2}\,\sigma \tag{8}$$

where $u_p = u\,(\alpha, p, \nu)$ is p-fractile of standardised lognormal distribution having the coefficient of skewness α. If the population standard deviation σ is unknown and the sample standard deviation s is used then

$$x_{p,pred} = m + t_p\,(1/n + 1)^{1/2}\,s \tag{9}$$

where $t_p = t(\alpha, p, v)$ is p-fractile of a generalised Student t-distribution having the coefficient of skewness α for $v = n - 1$ degrees of freedom.

2.4 Bayesian approach

When previous observations of a continuous production is available an alternative technique is provided by Bayesian approach [1], [2] and [3]. Let m is the sample mean, s the sample standard deviation determined from a sample of the size n. Besides from previous observations the sample mean m' and sample standard deviation s' determined from a sample, which values and the size n' are unknown, are available. Both samples are assumed to be taken from the same population having theoretical mean μ and standard deviation σ. Hence both samples can be considered jointly. Parameters of the combination of both samples are [2], [3]

$$n'' = n + n'$$

$$v'' = v + v' - 1, \text{ when } n' \geq 1, \; v'' = v + v' \text{ when } n' = 0$$

$$m'' = (mn + m'n')\,/\,n''$$

$$s''^2 = (v s^2 + v' s'^2 + n\,m^2 + n'\,m''^2 - n''\,m''^2)\,/\,v'' \tag{10}$$

Unknown values n' and v' may be estimated using formulae for the coefficients of variation $V(m')$ and $V(s')$, which may be written as

$$n' = [\sigma / (\mu \;\; V(m'))]^2, \; v' = 1\,/(2\;V(s')^2) \tag{11}$$

Obviously, both values n' and v' may be chosen individually (generally $v' \neq n'$-1) depending on previous experiences concerning degree of uncertainty in estimating the mean μ and standard deviation σ.

In accordance with [2] and [3] the Bayesian estimate $x_{p,\text{Bayes}}$ is given by a formula similar to equation (9) used by prediction method assuming that σ is unknown

$$x_{p,\text{Bayes}} = m'' + t_p\,(1/n'' + 1)^{1/2}\,s'' \tag{12}$$

where $t_p = t(\alpha, p, v'')$ is again p-fractile of the generalised Student t-distribution having the coefficient of skewness α for v'' (generally different from n'' - 1) degrees of freedom.

When applying the Bayesian technique for determining strength of building materials, an advantage may be taken of the fact, that long term variability of the strength is usually stable. Thus, uncertainty in determining σ is relatively small, the value $V(s')$ is also small and v' given by (11) and v'' given by (10) is high. This may lead to a favourable decrease of the resulting value t''_p and to an favourable increase of the estimate for the lower fractile x_p (see equation (12)). On the other hand uncertainty in determining μ and $V(m)$ is usually high and previous information may not significantly affect the resulting n'' and m''.

If no prior information is available, then $n' = v' = 0$ and the characteristics m'', n'', s'', v'' equal the sample characteristics m, n, s, v. Equation (12) reduces to the previous expression (9) . In this special case the Bayesian approach leads to the same procedure as prediction method and equation (9), in the case of known σ equation (8), are to be used. It should be noted that this special case of Bayesian technique with no prior information is considered in the informative annex D of the Eurocode 1 [1] and in ISO documents [2] and [3].

3. Comparison of coverage and prediction method

To estimate the characteristic and design strength the coverage and prediction method are applied most frequently. These methods are compared here (see also [8]) assuming normal distribution (lognormal distribution with $\alpha = 0$) of the population. Table 1 shows the coefficients κ_p and $u_p(1/n+1)^{1/2}$ used in equations (3) and (8) for selected values of n and γ. It follows from Table 1, that differences between both coefficients are dependent on number of observations n as well as on confidence level γ. For $\gamma = 0,95$ and small n the coefficient κ_p of the coverage method is by almost 40% higher than the corresponding coefficient $u_p(1/n+1)^{1/2}$ used in the prediction method. If $\gamma = 0,75$ is accepted (as recommended in [2] and [3]) than the differences are less than 10%. Generally, however, the prediction method would obviously lead to higher (less safe) characteristic values than the classical coverage method for the confidence level $\gamma \geq 0,75$ (see also [8]).

Coefficients		Number of observations n								
		3	4	5	6	8	10	20	30	∞
κ_p	$\gamma = 0,75$	2,03	1,98	1,95	1,92	1,88	1,86	1,79	1,77	1,64
	$\gamma = 0,90$	2,39	2,29	2,22	2,17	2,10	2,05	1,93	1,88	1,64
	$\gamma = 0,95$	2,60	2,47	2,38	2,32	2,23	2,17	2,01	1,95	1,64
$-u_p(1/n+1)^{1/2}$		1,89	1,83	1,80	1,77	1,74	1,72	1,68	1,67	1,64

Table 1. Coefficients κ_p and $u_p(1/n+1)^{1/2}$ for $p = 0,05$ and known σ.

If the standard deviation σ is unknown, equations (4) and (9) are to be compared. Table 2 shows the appropriate coefficients k_p and $t_p(1/n+1)^{1/2}$ for the same number of observations n and confidence levels γ as in table 1. Obviously, differences between the coefficients corresponding to different confidence levels γ are much more significant than in previous case of known σ. For $\gamma = 0,95$ and small n the coefficient k_p used by the coverage method is by almost 100% greater than the coefficient $t_p(1+1/n)^{1/2}$ used by the prediction method. For $\gamma =$ 0,75 both coefficients are nearly the same. The coefficient k_p is, however always slightly greater than $t_p(1/n+1)^{1/2}$ except for $n = 3$ (see also [8]). Like in the previous case of known σ, the prediction method would generally lead to greater (less safe) characteristic strengths than the classical coverage method. The difference increases with increasing confidence level.

Coefficients		Number of observations n								
		3	4	5	6	8	10	20	30	∞
k_p	$\gamma = 0,75$	3,15	2,68	2,46	2,34	2,19	2,10	1,93	1,87	1,64
	$\gamma = 0,90$	5,31	3,96	3,40	3,09	2,75	2,57	2,21	2,08	1,64
	$\gamma = 0,95$	7,66	5,14	4,20	3,71	3,19	2,91	2,40	2,22	1,64
$-t_p(1/n+1)^{1/2}$		3,37	2,63	2,33	2,18	2,00	1,92	1,76	1,73	1,64

Table 2. Coefficients k_p and $t_p(1/n+1)^{1/2}$ for $p = 0,05$ and unknown σ.

4. Effect of asymmetry

Actual asymmetry of population distribution may have significant effect on results of fractile estimation, particularly when small samples are taken from a population with high variability [6]. Assuming general three parameter lognormal distribution with independent coefficient of skewness α effect of population asymmetry on 0,05-fractile estimate is shown below for two confidence levels considering three coefficients of skewness α = -1,00, 0,00 and +1,00. Table 3 shows the coefficient k_p for selected numbers of observations n and confidence γ = 0,75. Table 4 shows the coefficient k_p for the same numbers of observations n as in table 3, but for the confidence level γ = 0,95.

Coefficients of skewness	Number of observations n								
	3	4	5	6	8	10	20	30	∞
α = -1,00	4,31	3,58	3,22	3,00	2,76	2,63	2,33	2,23	1,85
α = 0,00	3,15	2,68	2,46	2,34	2,19	2,10	1,93	1,87	1,64
α = 1,00	2,46	2,12	1,95	1,86	1,75	1,68	1,56	1,51	1,34

Table 3. Coefficients k_p for p = 0,05, γ = 0,75 and unknown σ.

Coefficients of skewness	Number of observations n								
	3	4	5	6	8	10	20	30	∞
α = -1,00	10,9	7,00	5,83	5,03	4,32	3,73	3,05	2,79	1,85
α = 0,00	7,66	5,14	4,20	3,71	3,19	2,91	2,40	2,22	1,64
α = 1,00	5,88	3,91	3,18	2,82	2,44	2,25	1,88	1,77	1,34

Table 4. Coefficients k_p for p = 0,05, γ = 0,95 and unknown σ.

Comparing data given in both tables 3 and 4 it follows that the effect of distribution asymmetry on the estimate $x_{p,cover}$ considerably increases with increasing confidence level γ. Generally the effect decreases with increasing n, nevertheless, it never vanishes even for $n \rightarrow \infty$. Detailed analysis [8] shows that when assessing characteristic strength of concrete corresponding to the 0,05-fractile, actual asymmetry of probability distribution should be considered whenever the coefficient of skewness is greater (in absolute value) than 0,5.

Differences between estimates obtained assuming general lognormal distribution with a given coefficient of skewness $\alpha \neq 0$ and corresponding estimates assuming normal distribution with α = 0, increases also with decreasing probability p associated with the estimated fractile x_p (see also [8]). This is one of the reasons why design value of strength, corresponding to a very small probability p (say 0,001), should not be generally determined directly from test data. Direct assessment could be applied only in those cases when sufficient number of observations and a convincing evidence on appropriate probabilistic model (including information on asymmetry) are available. When such an evidence is not accessible, the design value should be preferably determined by assessing a characteristic value, which is divided by a partial factor and possibly by an explicit conversion factor, as recommended in Eurocode 1 [1].

Effect of asymmetry on the coefficient t_p used in the prediction method is shown in table 5 for the same coefficients of skewness α = -1,00, 0,00 and +1,00 as before. However, in Table 5 values of the coefficient t_p are given for various degrees of freedom ν and not for the sample size n. The reason for this arrangement is possible use of indicated values in the method based on the Bayesian approach.

Coefficients	Coefficients - t_p for degrees of freedom ν								
of skewness	3	4	5	6	8	10	20	30	∞
α = -1,00	2,65	2,40	2,27	2,19	2,19	2,04	1,94	1,91	1,85
α = 0,00	2,35	2,13	2,02	1,94	1,86	1,81	1,72	1,70	1,64
α = 1,00	1,92	1,74	1,64	1,59	1,52	1,48	1,41	1,38	1,34

Table 5. Coefficients - t_p for p = 0,05 and unknown σ.

Similarly as in the case of classical coverage method the effect distribution asymmetry decreases with increasing n, here with increasing value of the degrees of freedom ν, nevertheless, it never vanishes even for $\nu \to \infty$ (see Table 5).

5. Example

A sample of n = 5 concrete strength measurements having the mean m = 29,2 MPa and standard deviation s = 4,6 MPa is to be used to assess the characteristic value of the concrete strength f_{ck} = x_p, where p = 0,05. Using coverage method it follows from equation (4) and table 2 that for the confidence level γ = 0,75

$$x_{p,\,cover} = 29{,}2 - 2{,}46 \times 4{,}6 = 17{,}9 \text{ Mpa} \tag{13}$$

and for the confidence level γ = 0,95 it holds

$$x_{p,cover} = 29{,}2 - 4{,}20 \times 4{,}6 = 9{,}9 \text{ Mpa} \tag{14}$$

If the prediction method is used, it follows from equation (9) and table 2

$$x_{p,pred} = 29{,}2 - 2{,}33 \times 4{,}6 = 18{,}5 \text{ Mpa} \tag{15}$$

Thus, using the prediction method (which is recommended in [1], [2] and [3]), the estimate for the characteristic strength is only slightly greater than the value obtained by the classical method assuming the confidence level γ = 0,75 given by equation (13). However, when the confidence level γ = 0,95 is required, then the prediction method lead to the estimate which is greater by almost 90% than the value given by equation (14).

When information from previous production is available Bayesian approach can be used. Assume the following prior information

$$m' = 30{,}1 \text{ MPa}, \; V(m') = 0{,}50, \; s' = 4{,}4 \text{ MPa}, \; V(s') = 0{,}28 \tag{16}$$

It follows from equations (11)

$$n' = \left(\frac{4{,}6}{30{,}1}\,\frac{1}{0{,}50}\right)^2 < 1 \;,\; \nu' = \frac{1}{2}\,\frac{1}{0{,}28^2} \approx 6 \tag{17}$$

The following characteristics are therefore considered : $n' = 0$ and $\nu' = 6$. Taking into account that $\nu = n - 1 = 4$, equations (10) yield

$$n'' = 5, \quad \nu'' = 10, \quad m'' = 29{,}2 \text{ MPa}, \quad s'' = 4{,}5 \text{ Mpa} \tag{18}$$

and finally it follows from equation (12)

$$x_{p,\,\text{Bayes}} = 29{,}2 - 1{,}81 \times \sqrt{\frac{1}{5} + 1} \times 4{,}5 = 20{,}3 \text{ MPa} \tag{19}$$

where the value $t_p = 1{,}81$ is taken from Table 5 for $\alpha = 0$ and $\nu = 10$. The resulting characteristic strength is therefore greater (by 10 %) than the value obtained by prediction method. Also other available information (see annex D in [3]) on application of Bayesian approach clearly indicates, that when previous experiences are available this technique can be effectively used. Particularly in the case of a high variability of strength or in the case of assessment of existing structures Bayesian approach may be valuable.

For commonly used (low strength) concrete a positive asymmetry of probability distribution (with the coefficient of skewness up to 1) is often observed. It is assumed that the sample of $n = 5$ concrete strength measurements, analysed above, is taken from a population with lognormal distribution having the coefficient of skewness $\alpha = 1$. Using the classical coverage method for the confidence level $\gamma = 0{,}75$, equation (4) and coefficients given in Table 3 yield

$$x_{p,\,\text{cover}} = 29{,}2 - 1{,}95 \times 4{,}6 = 20{,}2 \text{ Mpa} \tag{20}$$

For the confidence limit $\gamma = 0{,}95$ it holds

$$x_{p,\text{cover}} = 29{,}2 - 3{,}18 \times 4{,}6 = 14{,}6 \text{ Mpa} \tag{21}$$

These values are greater by 13% and 47% respectively, compared to the previous case (equations (13) and (14)) when asymmetry was disregarded; thus, due to positive asymmetry more favourable estimates are obtained. Similarly using equation (9) the prediction method would yield the estimate for the characteristic strength as

$$x_{p,\,\text{pred}} = 29{,}2 - 1{,}74 \times \sqrt{\frac{1}{5} + 1} \times 4{,}6 = 20{,}4 \text{ MPa} \tag{22}$$

where the value $t_p = 1{,}74$ is taken from Table 5 for $\alpha = 1{,}0$ and $\nu = 5 - 1 = 4$. The resulting strength is by 10% greater than the previous value obtained for the normal distribution ($\alpha = 0$) given by equation (15) and again approximately equal to the value obtained by the classical coverage method assuming the confidence level $\gamma = 0{,}75$ given by equation (20). However, when the confidence level $\gamma = 0{,}95$ is required, then the prediction method lead to the estimate which is greater by almost 40% than the value given by equation (21).

When Bayesian approach is used, then it follows from equations (12), (17), (18) and Table 5

$$x_{p,\,\text{Bayes}} = 29{,}2 - 1{,}48 \times \sqrt{\frac{1}{5} + 1} \times 4{,}5 = 21{,}9 \text{ MPa} \tag{23}$$

which is the value by 8% greater than the corresponding estimate obtained in equation (19) for the coefficient of skewness $\alpha = 0$.

It should be, however, noted that possible negative asymmetry, which may occur in the case of some high strength materials, would cause an unfavourable effect on resulting fractile estimates, particularly when design value corresponding to small probabilities ($p < 0,001$) are considered.

Thus, using different statistical techniques and the same sample data the resulting estimate for the 5% characteristic strength is within a broad range from 9,9 MPa up to 20,3 for the coefficient of skewness $\alpha = 0$ (normal symmetrical distribution) and, within a range from 14,6 up to 21,9 Mpa for the coefficient of skewness $\alpha = 1$. Generally, it follows from the above numerical example and from numerical values given for various coefficients of estimation that resulting estimates for both the characteristic and design strength considerably depend on the applied method and on available prior knowledge.

6. Conclusions

(a) Design values of strength should be preferably determined by assessing a characteristic value, which is divided by a partial factor and possibly by an explicit conversion factor; direct assessment from test results could be used only in those cases when convincing evidence on appropriate probabilistic model is available.

(b) Considerably different estimates for characteristic and design strength may be obtained depending on applied statistical technique, specified probability, population asymmetry, sample size and in the case of coverage method also on accepted confidence level.

(c) Classical coverage method of fractile estimation with a given confidence level is recommended; in common cases the confidence level 0,75 may be accepted (which yields almost the same results as the methods recommended in the latest version of Eurocode 1), in special cases when increased reliability is required, higher confidence level (0,95) should be considered.

(d) When previous observations of a continuous production are available an alternative technique provided by Bayesian approach can be effectively used.

(e) Possible asymmetry of the population distribution should be considered by any estimation method whenever the coefficient of skewness exceeds $\pm$ 0,5.

7. References

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