The fracture theory of composite at bearing strain in end faces

The Fracture Theory of Composite at Bearing Strain in End Faces

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Summary

In continual approximation for construction of the theory of fracture the material is modelled by orthotropic elastic (in case of brittle fracture) or by elastic–plastic (in case of plastic failure) body. For modelling of the fracture mechanism considered the phenomenon of surface instability near the end face is used. The exact solution was used for computation of theoretical strength limits, corresponding to fracture at bearing strains on end faces. The comparison with experimental results was made for unidirectional fibrous materials.

1. Introduction

The article is devoted to investigation of mechanism of fracture of composite materials and of structures elements fabricated from these materials in compression, when in end faces bearing strains occur. This phenomenon occurs, for example, in uniaxial compression when fracture of material initiates near end faces. This fracture is not propagated far from end faces. For this reason, the ultimate strength of material, corresponding to fracture at bearing strains in end faces, is some what lower than the ultimate strength of the material at the fracture of the whole material (far from end faces). For description of this phenomenon the continual theory of fracture is proposed with modelling of material (in continual approximation) by orthotropic elastic (in case of brittle fracture) or by elastic–plastic (in case of plastic failure) body. The phenomenon of surface instability is applied for modelling the fracture mechanism considered. For description of phenomenon of surface instability under applied normal load the three–dimensional linearized theory of deformable bodies stability, which is presented, for an example, in [2] is used. For solution of formulated problems (within the framework of linearized theory of deformable bodies stability) the system of integral representations is used. As a result of exact solution equations are obtained for determination of theoretical values of ultimate strength related to fracture at bearing strains in end faces. Values of theoretical ultimate strength are computed related to fracture at bearing strains in end faces with application of composite materials with polymer (brittle fracture) and metal (plastic failure) matrix. Comparison was made with results of experimental studies for unidirectional fibrous composites.

2. Main relations and statements

Fracture at bearing strains in end faces occurs under compression. It consists of local fracture near end faces. When the increase of compressive load is insignificant no propagation of fracture far from end faces occurs. This fracture mechanism is realized also in structural elements fabricated from composite materials in places where they are joined with metal structural elements. The fracture at bearing strains is very marked in case of unidirectional fibrous composite under compression along the fibers and the layers, when the end faces are not fixed by special procedures. Consequently, following special features are characteristic of fracture mechanism at bearing strains in end faces.

1. It occurs mainly under compression of unidirectional fibrous composites and of laminated composites (under compression along fibers and layers).
2. It occurs mainly in cases with unfixed end faces.
3. It occurs near end faces and does not propagate far from end faces.

For an example we consider the unidirectional fibrous boron–aluminium in the case of 50% content of the fibers under uniaxial compression along the fibers. Nature of fracture at bearing strain in end face of this composite is shown on Fig.1. Since the compression along fibers or layers and end faces is analysed, it is logical to assume that at the initial stage a local stability loss occurs (the surface instability near the loaded end face). The foregoing considerations lead to following main statements of the present continual theory.

1. In the analysis of the phenomenon of bearing strains in end faces the influence of lateral surface of the specimen or of the structural element will be not accounted for. This statement allows to analyse the lower half–space ($x_3 = 0$).
2. The phenomenon of bearing strain in end faces at the initial stage will be assumed to occur as a result of surface stability loss near the loaded end face. Only surface instability will be analysed, when stresses and displacements attenuate at increasing distance from the end face.

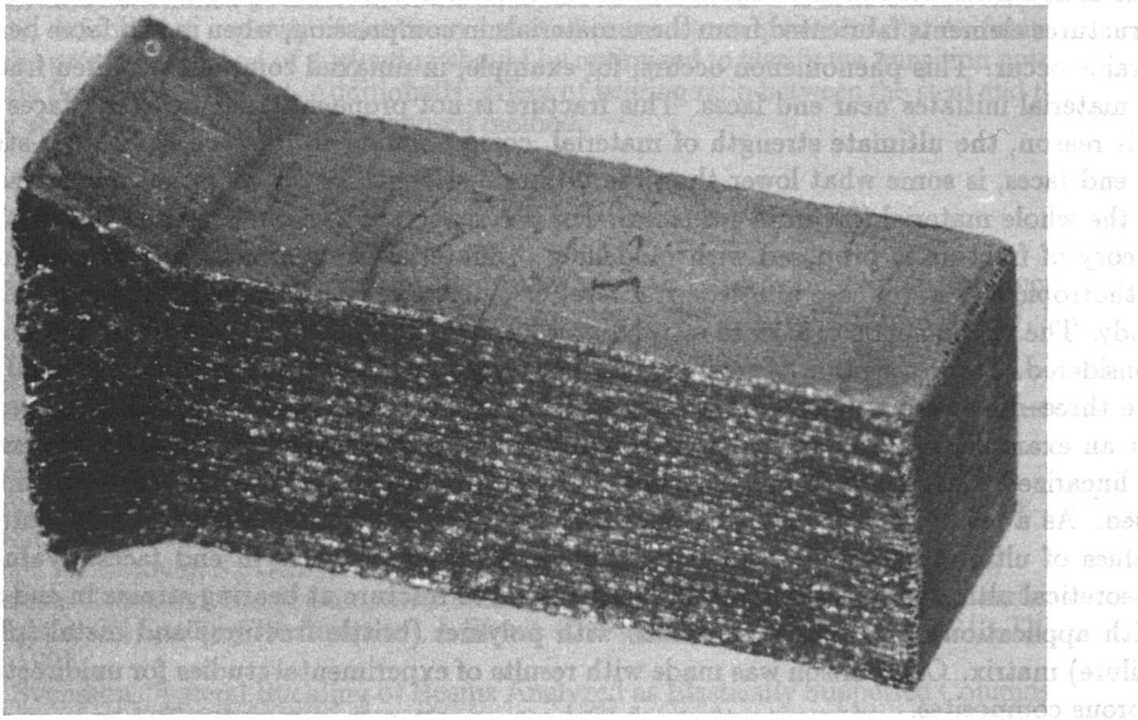

Fig.1. Fracture of unidirectional composite at bearing strains in end face in axial compression (boron–fibers, aluminium–matrix). The view after fracture

3. The analysis of surface instability near the loaded end face will be carried out within the framework of the three–dimensional linearized theory of deformable bodies stability [1,2]. The second variant of the small practical deformations theory will be used [2] (precritical state is determined with the use of geometrically linear theory).
4. In the analysis of plastic failure (materials with metal matrix) the generalized conception of continuing loading will be used [1,2]. Consequently, we will not take account of the change of unloading zones in the process of stability loss. This statement allows to analyse in the general form the brittle and plastic failure.
5. The precritical state will be assumed to be homogeneous in the analysis of surface instability near the loaded end face.
6. The external load at $x_3 = 0$ will be assumed "dead" load, and this validates the use of static method of analysis [2].

7. Laminated and fibrous composites will be considered. For laminated composites it will be assumed that layers are directed perpendicularly to the end face surface $x_3 = 0$. With application to fibrous composites the unidirectional or orthogonally reinforced materials will be analysed under the condition that the direction of main reinforcement is perpendicular to the end face $x_3 = 0$.
8. In continual approximation these composites will be modelled by compressible homogeneous orthotropic body. The model will be used of elastic linear body at brittle fracture (in case of polymer matrix) and the model of elastic–plastic body at plastic failure (in case of metal matrix). The assumption will be made that the axes of symmetry of material properties (in continual approximation) coincide with axes of the chosen coordinate system. In case of the model of transversely isotropic body the planes $x_3 = const$ will be assumed as isotropy planes.
Taking into account the foregoing main statements we will consider main relations. For three–dimensional precritical state main equations of the three–dimensional linearized theory of deformable bodies stability [2] with application to compressible bodies have the form

$$L_{m\alpha} u_\alpha = 0; \qquad n, m, \alpha, \beta = 1, 2, 3;$$

$$L_{m\alpha} = \omega_{nm\alpha\beta} \frac{\partial^2}{\partial x_n \partial x_\beta}; \qquad \omega_{nm\alpha\beta} = \omega_{nm\alpha\beta}(\sigma_{11}^0, \sigma_{22}^0, \sigma_{33}^0) \tag{2.1}$$

Components of the asymmetric tensor of stresses of Kirchhoff are determined [2] from the expression

$$t_{nm} = \omega_{nm\alpha\beta} \frac{\partial u_\alpha}{\partial x_\beta} \tag{2.2}$$

For homogeneous precriticall state (with application to the second variant of the small precritical deformations theory [2]) following relations hold

$$\omega_{nm\alpha\beta} = \delta_{nm}\delta_{\alpha\beta}A_{n\beta} + (1 - \delta_{nm})(\delta_{n\alpha}\delta_{m\beta} + \delta_{n\beta}\delta_{m\alpha})\mu_{nm} + \delta_{n\beta}\delta_{m\alpha}\sigma_{\beta\beta}^0 \tag{2.3}$$

In [2] expressions are presented for determination of values of $A_{n\beta}$ and μ_{nm} for various models. In case of brittle fracture (polymer matrix) it may be assumed

$$A_{n\beta}, \mu_{nm} = const; \qquad \mu_{nm} = G_{nm} \tag{2.4}$$

In case of plastic failure (metal matrix) it may also be assumed

$$A_{n\beta} = A_{n\beta}(\sigma_{11}^0, \sigma_{22}^0, \sigma_{33}^0) \qquad \mu_{nm} = \mu_{nm}(\sigma_{11}^0, \sigma_{22}^0, \sigma_{33}^0) \tag{2.5}$$

The existence of relations (2.5) complicates the analysis significantly.
On the end face ($x_3 = 0$) the following boundary conditions hold

$$t_{3m} = 0 \quad \text{at} \quad x_3 = 0; \qquad m = 1, 2, 3 \tag{2.6}$$

In view of the local character of the surface instability, the conditions of attenuation "at infinity" (at $x_3 \to -\infty$) should be put in the following form

$$u_m \to 0, \qquad t_{nm} \to 0 \quad \text{at} \quad x_3 \to -\infty \tag{2.7}$$

The formulation of the problems is complete with foregoing relations and with taking account of symmetry properties for components $\omega_{nm\alpha\beta}$ [2]. Consequently, the problem of eigenvalues is obtained (relative to loading parameter σ_{33}^0) in the form (2.1)–(2.7).

3. Theoretical results

Within the framework of the foregoing formulation we will consider now the computation of the theoretical strength limit corresponding to fracture at bearing strains in end faces. We will consider arbitrary form of stability loss in variables x_1 and x_2, including local (in x_1 and x_2) forms of stability loss. It should be remarked that in [2] surface instability was considered when the load was applied along the plane boundary ($\sigma_{33}^0 = 0$) and only periodical (in x_1 and x_2) forms of stability loss were investigated. In case of arbitrary forms of stability loss the solution will be represented in the form of Fourier integrals

$$u_n = \frac{1}{2\pi} \int \int_{-\infty}^{+\infty} v_n(x_3, \alpha_1, \alpha_2)\{\exp[-i(\alpha_1 x_1 + \alpha_2 x_2)]\} d\alpha_1 d\alpha_2;$$

$$t_{nm} = \frac{1}{2\pi} \int \int_{-\infty}^{+\infty} r_{nm}(x_3, \alpha_1, \alpha_2)\{\exp[-i(\alpha_1 x_1 + \alpha_2 x_2)]\} d\alpha_1 d\alpha_2; \qquad (3.1)$$

Introducing (3.1) into (2.1)–(2.7) we obtain the eigenvalues problem, formulated relative to functions v_n and r_{nm} (3.1). The solution of this eigenvalues problem will not be presented here as the extent of the article is restricted, additional information may be found in [3,4]. Here only final expressions will be presented, obtained by exact solution of this eigenvalues problem. In the exact solution characteristic equations are obtained corresponding to surface instability near the loaded end face. These expressions will be presented separately for the plane and the three–dimensional problems.

In the case of the plane problem (plane deformation in the plane x_1ox_3) with application to the model of orthotropic body characteristic equations are obtained in the following form

$$\omega_{1111} = 0; \quad \omega_{1331} = 0; \quad \omega_{1133} + \omega_{1313} = 0; \quad \Pi = 0 \qquad (3.2)$$

In the case of three–dimensional problem (with additional condition $\sigma_{11}^0 = \sigma_{22}^0$) with application to the model of transversely isotropic body (the axis ox_3 is the isotropy axis) characteristic equations have the form

$$\omega_{3333} = 0; \quad \omega_{3113} = 0; \quad (\omega_{1133} + \omega_{1313})^{-1} = 0; \quad \Pi = 0 \qquad (3.3)$$

In (3.2) and (3.3) the notation is introduced

$$\Pi \equiv \sqrt{\omega_{1111}\omega_{3333}}(\omega_{3113}\omega_{1331} - \omega_{1313}^2) + \sqrt{\omega_{1331}\omega_{3113}}(\omega_{1111}\omega_{3333} - \omega_{1133}^2) \qquad (3.4)$$

It should be pointed out that characteristic equations (3.2) and (3.3) taking into account the notation (3.4) are obtained in the unified general form for the finite precritical deformations theory [2] and for two variants of the small precritical deformations theory [2] in the case of elastic and elastic–plastic models. For derivation from (3.2)–(3.4) of results corresponding to the second variant of the amsll precritical deformations theory [2] (this problem is considered in this article) it is necessary to use for determination of components of the tensor ω the expressions (2.3).

In the following only the case of uniaxial compression along the axis ox_3 will be considered. In view of this the following condition should be assumed

$$\sigma_{11}^0 \equiv \sigma_{22}^0 = 0 \qquad (3.5)$$

We also introduce the following notations:

$(\Pi_3^-)_T$ – theoretical strength limit in compression along the axis ox_3, corresponding to the fracture of the whole specimen or structural element;

$(\Pi_3^-)_T^{SM}$ – theoretical strength limit in compression along the axis ox_3, corresponding to the

fracture of the specimen or structural element at bearing strain in end face;
$(\Pi_3^-)_{ex}$ – experimental value of the strength limit in compression along the axis ox_3, corresponding to the fracture of the whole specimen or structural element;
$(\Pi_3^-)_{ex}^{SM}$ – experimental value of the strength limit in compression along the axis ox_3, corresponding to the fracture of the specimen or structural element at bearing strain in end face;
$-(\sigma_{33}^0)_{cr}$ – critical value of the compressive load, corresponding to the internal [2] stability loss in the structure (for the whole specimen, corresponding to the infinite body);
$-(\sigma_{33}^0)_{cr}^{SM}$ – critical value of the compressive load, corresponding to the local stability loss near the loaded end face.
Taking into account the foregoing notations and the second main statement of the present theory the following equality may be written

$$(\Pi_3^-)_{T}^{SM} = -(\sigma_{33}^0)_{cr}^{SM} \tag{3.6}$$

In analogous manner [5] we may also write

$$(\Pi_3^-)_{T} = -(\sigma_{33}^0)_{cr} \tag{3.7}$$

Omitting all intermediate calculations, we will present only final results concerning the evaluation of the considered theoretical strength limits. We remark that for analysed composite materials (Fig.3 and 4), taking into account the notations (2.2) and (2.3) following inequalities are valid

$$A_{33} >> G_{13}; \qquad A_{33} >> \mu_{13} \tag{3.8}$$

with application to the brittle and plastic fracture.
Taking into account (3.8) with application to the case (3.5), from (3.2)–(3.4) after some cumbersome transformations the following result is obtained

$$[(\Pi_3^-)_T - (\Pi_3^-)_T^{SM}] \cdot [(\Pi_3^-)_T]^{-1} \approx \left(\frac{\mu_{13}}{A_{33}}\right)^2 \frac{A_{33}}{A_{11}} \left[1 - \frac{A_{13}^2}{A_{11}A_{33}}\right]^{-1} > 0 \tag{3.9}$$

For case of brittle fracture in (3.9) the value μ_{13} should be substituted for G_{13}. In view of inequalities (3.8) for structural materials the following result is obtained

$$(\Pi_3^-)_T^{SM} < (\Pi_3^-)_T \tag{3.10}$$

It means that the values of theoretical strength limits $(\Pi_3^-)_T$ and $(\Pi_3^-)_T^{SM}$ differ insignificantly. As an example, the laminated composite at brittle fracture will be considered. We introduce notations: E_a, ν_a and S_a – Young modulus, Poisson coefficient and concentration of the filler (or reinforcing elements); E_m, ν_m and S_m – Young modulus, Poisson coefficient and concentration of the binder (of matrix). For structural materials we assume also

$$S_a \approx S_m; \qquad E_a >> E_m \tag{3.11}$$

In this case, taking into account known results, from (2.9) for a laminated composite we obtain

$$[(\Pi_3^-)_T - (\Pi_3^-)_T^{SM}] \cdot [(\Pi_3^-)_T]^{-1} \approx \frac{1}{4(1+\nu_m)^2} \frac{1}{S_a S_m} \frac{E_m}{E_a} \tag{3.12}$$

From (3.12), which corresponds to the considered example, it follows that the difference between theoretical strength limits $(\Pi_3^-)_T$ and $(\Pi_3^-)_T^{SM}$ is insignificant.

4. Comparison with experimental results

Three results will be considered in this chapter which related with experiments. In our comparison of theoretical results with experimental results we will use the relation (3.10), since in experimental studies quite frequency the values of $(\Pi_3^-)_T$ and $(\Pi_3^-)_T^{SM}$ are not distingwished. Values of theoretical strength limits $(\Pi_3^-)_T$ are computed in [5] for some materials.
1. Nature of fracture of unidirectional composite at bearing strains in end face in axial compression (boron–fibers, aluminium–matrix) is shown on Fig.1. The photograph of Fig.1 correspond to the view of specimen after fracture. The above mentioned fracture arose near

first end of specimen. Design and technological technigues, excluding occurence of the above mentioned fracture near second end of specimen were utilized.

2. We consider unidirectional fibrous boron-reinforced plastic in case of brittle fracture at 50% content of fibers ($S_a = S_m = 0.5$). In this case following results were obtained

$$(\Pi_3^-)_{ex} = 3.10 GPa; \qquad (\Pi_3^-)_T = 2.00 - 3.00 GPa \tag{4.1}$$

In (4.1) in determination of $(\Pi_3^-)_T$ the scatter of properties of epoxy resin is taken account of.

3. We consider the unidirectional fibrous boron-aluminium at plastic failure in the case of 50% content of fibers ($S_a = S_m = 0.5$). Results were obtained [4,5] for annealed and non-annealed aluminium, we present the results in the Table

Material	$(\Pi_3^-)_{ex}^{SM}$, MPa	$(\Pi_3^-)_T^{SM}$, MPa
annealed	665	736
non-annealed	1282	1467

These results taking into account the inequality (3.10) demonstrate a good agreement between theoretical and experimental results. Additional information and analysis of related problems are presented in monograph [5] and article [6].

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