A Global Approach to Account for Time Effects in Composite Structures

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Summary

Based on the theory of linear viscoelasticity, an incremental relationship is proposed to account for the global behaviour of composite structure members. This formulation can be easily implemented in a finite element program. A specific procedure allows for construction process by successive phases. An example of application is given for a cable-stayed footbridge made of concrete, wood, ordinary and prestressing steel. The calculation provides detailed informations about the time behaviour of the structure.

1. Introduction

The long term behaviour of structures depends on time effects such as shrinkage, creep or relaxation in their constituting materials. In composite members, the rheological properties of the materials causes a progressive redistribution in the stresses throughout any cross-section. In hyperstatic structures, the internal forces will be longitudinally redistributed. As a consequence, the serviceability of the structure may be endangered because of excessive displacements or unexpected cracking of concrete for example.

It is generally assumed that the analysis of time effects during the service life of structures, should refer to the theory of linear viscoelasticity. Our work is based on an incremental formulation of the viscoelastic behaviour [1,2]. We show how to express the global behaviour of a composite member, taking the time dependent behaviour of its constituting materials into account. The formulation may be implemented in a finite element program. This approach is convenient for the analysis of time effects in composite structures during the service life, beginning with the period of construction. The efficiency of the method is illustrated by an application to a cable-stayed footbridge.

2. Time dependent behaviour of building materials

Building materials such as concrete, prestressing steel, wood... are usually assumed to behave as linear viscoelastic under service conditions. This behaviour may be represented by a relaxation function $R(t,t_o)$, expanded into a Dirichlet's series, where E is the longitudinal modulus of elasticity, α_μ and λ_μ are material parameters :

$$R(t,t_o) = E(t_o) \sum_{\mu=0}^{m} \alpha_\mu(t_o) e^{-\lambda_\mu(t-t_o)} \quad \text{where} \quad : \quad \lambda_o = 0 \ ; \ \sum_{\mu=0}^{m} \alpha_\mu = 1 \ \ \forall\, t_o \tag{1}$$

E and α_μ depends on the time t_o at loading for ageing materials such as concrete. For other materials, these parameters have constant values $\forall\, t_o$.

Based on the theory of linear viscoelasticity, it is possible to propose an incremental relationship to express the stress variation $\Delta\sigma$ induced by a strain variation $\Delta\epsilon$ during a finite interval of time $[t, t+\Delta t]$:

$$\forall\, t, \Delta t: \quad \Delta\sigma = \sum_{\mu=0}^{m} \Delta\sigma_\mu \quad \text{where} \quad \Delta\sigma_\mu = \kappa_\mu(t)\, E(t)\, (\Delta\epsilon - \Delta\epsilon^*) - \left(1 - e^{-\lambda_\mu \Delta t}\right) \sigma_\mu(t) \tag{2}$$

where $\Delta\epsilon^*$ denotes a free strain variation if any (shrinkage, thermal dilatation ...). $\sigma_\mu(t)$ represents a set of cumulative variables, whose actual values depend on the stress history since first loading. The $\kappa_\mu(t)$ parameters are defined as follows :

$$\kappa_o(t) = \alpha_o(t) \left(1 + \frac{\Psi_o}{2}\right) \ ; \ \kappa_\mu(t) = \frac{\alpha_\mu(t)}{\lambda_\mu \Delta t} \left[\left(1 - \frac{\Psi_\mu}{\lambda_\mu \Delta t}\right)\left(1 - e^{-\lambda_\mu \Delta t}\right) + \Psi_\mu\right] \quad \forall\, \mu \geq 1 \tag{3}$$

$$\text{where} \quad \Psi_\mu = \frac{\alpha_\mu(t+\Delta t)}{\alpha_\mu(t)} \frac{E(t+\Delta t)}{E(t)} - 1 \qquad [\Psi_\mu = 0 \ \text{ for non-ageing material}].$$

The α_μ parameters are calibrated by fitting Equation (1) to a reference curve (from tests or design standard). The m and λ_μ parameters being given fixed values (satisfying results are generally obtained for $m = 4$). In the case of an ageing material (concrete), the calibration procedure is performed for various initial times t_o. The $\alpha_\mu(t_o)$ functions are then approximated by suitable analytical functions.

3. Finite composite beam element

Specific finite beam elements have been developed to account for time effects in composite structures [3]. The case of a composite member made of several elastic or viscoelastic materials is presented below. The hypothesis are as follows :

— The cross-section of the beam is symmetrical. It is compound of $n \geq 1$ homogeneous areas A_i, each of them corresponding to a specific material (Eq.2 and Eq.3 apply for elastic material with $m = 0$).

— The distribution of the longitudinal strain is assumed linear throughout the cross-section of the beam (Bernoulli hypothesis). The shear strains are neglected. Therefore, the usual shape functions may be used to build the finite beam element corresponding to any longitudinal segment of the composite member.

— The change in the limit conditions or in the external loading (live loads, dead weight, prestressing force...), are supposed to occur within a very short interval of time (treated as instant).

The application of the principle of virtual work for any finite time interval $[t, t + \Delta t]$ yields the fundamental relationship of the finite element method :

$$\forall\, t, \Delta t : \qquad \left[\tilde{K}(t)\right]\{\Delta q\} = \{\Delta F^*\} - \{F^{his}(t)\} \tag{4}$$

In this equation, $\{\Delta q\}$ denotes the increase in the node displacements, due to creep and free strain development during the time interval Δt. $[\tilde{K}(t)]$ is a fictitious matrix of stiffness which accounts for the composition of the member and the rheological properties of its constituting materials. The nodal forces $\{\Delta F^*\}$ and $\{F^{his}(t)\}$ correspond to a fictitious loading. They account for the free strain and for the previous states of stress in the different parts of the member. The corresponding expressions are detailed in Table 1.

finite "composite beam" element		composite cross-section
$[\tilde{K}(t)] = \int_e [B]^t [\tilde{H}(t)] [B]\, dx$	stiffness matrix	$[\tilde{H}(t)] = \sum_{i=1}^{n} \tilde{E}_i(t) \begin{bmatrix} A_i & z_i A_i \\ z_i A_i & I_i \end{bmatrix}$
$\{F^{his}(t)\} = \int_e [B]^t \{S^{his}(t)\}\, dx$	term of history	$\{S^{his}(t)\} = \sum_{i=1}^{n} \int_{A_i} \begin{Bmatrix} 1 \\ z \end{Bmatrix} \sigma_i^{his}(t)\, dA$
$\{\Delta F^*\} = \int_e [B]^t \{\Delta S^*\}\, dx$	free strain	$\{\Delta S^*\} = \sum_{i=1}^{n} \int_{A_i} \begin{Bmatrix} 1 \\ z \end{Bmatrix} \Delta\sigma_i^*\, dA$

Table 1 : Expressions for the fictitious stiffness matrix and loading vectors in Equation (4)

$[B]$ is the matrix derivative of the shape functions. Every z_i and I_i denote the location of the centroid and the second moment of inertia of the corresponding area A_i, related to the middle-line of the finite element. The values of $\tilde{E}_i(t)$, $\sigma_i^{his}(t)$ and $\Delta\sigma_i^*$, related to any elementary area A_i of the composite member, are detailed in Table 2.

	$\tilde{E}_i(t)$	$\Delta\sigma_i^*$	$\sigma_i^{his}(t)$
elastic	E_i	$E_i\, \Delta\epsilon^*$	0
viscoelastic	$E_i(t) \sum_{\mu=0}^{m_i} \kappa_\mu^{(i)}(t)$	$\tilde{E}_i(t)\, \Delta\epsilon^*$	$-\sum_{\mu=1}^{m_i} \left(1 - e^{-\lambda_\mu^{(i)} \Delta t}\right) \sigma_\mu^{(i)}(t)$

Table 2 : Expressions for $\tilde{E}_i(t)$, $\Delta\sigma_i^*$ and $\sigma_i^{his}(t)$ in Table 1

According to this global approach, the contribution of every elementary parts of the composite beam segment are taken into account in one single finite "composite beam" element.

4. Implementation in a finite element program

According to Equation (4), the time analysis of a composite structure is divided in a number of elastic or viscoelastic calculation steps. The period of construction is divided into elementary stages, each of them corresponding to the erection of a new part of the structure. Specific internal limit conditions are introduced to account for the connection between parts. Each new stage induces an elastic step followed by one or several viscoelastic steps. Similarly, the service life of the structure is described by a number of viscoelastic steps of calculation. Further details may be found in reference [3].

In order to insure a satisfying accuracy, the size of the time intervals must be adjusted to the rate of creeping of the material. Taking account of the decreasing rate of creeping with time, we propose to fix the time intervals as follows :

$$\Delta t_i = t_i - t_{i-1} \quad \text{where} \quad t_i = t_o + i^n \Delta t_1 \quad [\, i = 1,2,\dots \; ; \; n \geq 1 \,] \tag{5}$$

where t_o is the time corresponding to the last elastic step. From our experience, a quite satisfactory accuracy is achieved for $n=2$ and $\Delta t_1 = 1$ day. This means that a number of 105 calculation steps is sufficient to cover a period of 30 years (11000 days).

5. Application to a cable-stayed footbridge

This example has been selected because of the composite constitution of the structure (Fig.1), and because of the way of construction. The first stage of the construction begins with the cast of the pylon and abutments. They are made of reinforced concrete. The second stage corresponds to the positioning of the glulam beams. Each beam rests on the centre pier and on the two abutments, and on six temporary supports beneath every cable anchorage. The connection is insured by steel bars embedded in the concrete slab and in the beams. The cable are tensioned once the concrete strength has gained its nominal value, in such a way to counterbalance the reaction of the beams on the temporary supports (forth stage). The final stage consists into setting the usual equipment (asphalt protection, parapets...) on the deck. The service life of the structure begins from this moment.

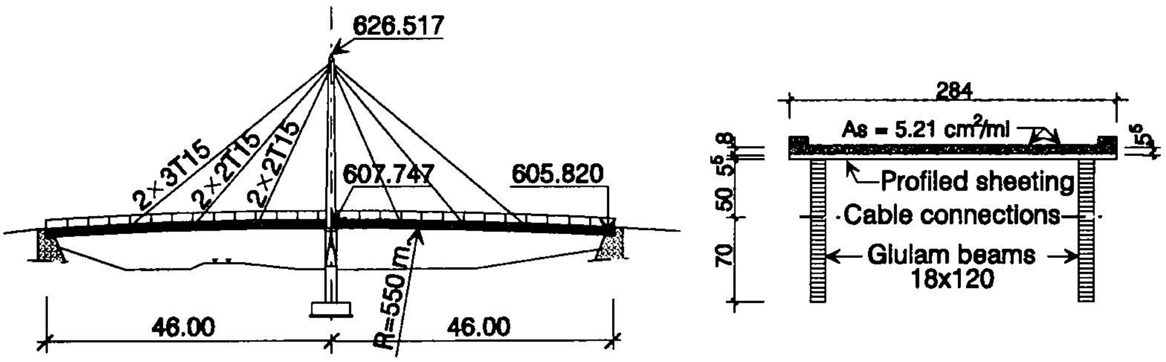

Fig.1 : Cable-stayed footbridge, general layout and cross-section of the deck

The characteristics of the materials are as follows :

— Concrete class is C25 for the pylon and the slab. Creep and shrinkage are assumed to develop according to Eurocode 2 provisions [4].

— The wood behaviour is assumed linear viscoelastic (longitudinal modulus : 11500 N/mm^2, creep ratio : 1.5, no longitudinal shrinkage).

— The behaviour of the concrete reinforcement and profiled steel sheeting is pure elastic (longitudinal modulus : 200000 N/mm^2).

— The cable are made of low relaxation T15 tendons (cross-area : 150 mm^2/tendon, longitudinal modulus : 190000 N/mm^2). The relaxation function is derived from Eurocode 2 provisions.

The time analysis of the structure has been performed according to the global approach detailed above. The pylon and the deck are represented by finite "composite beam" elements. For the pylon, each finite element accounts for the concrete and the reinforcing steel. Concerning the deck, the finite elements account for the glulam beams during the second stage of the construction. Once the concrete slab has been cast (third stage), the cross-section characteristics and the rheological properties of the concrete, reinforcement and steel sheeting are included in the same finite element. The cables are represented by linear finite elements. Their longitudinal modulus is adjusted to account for the sag caused by the cable self-weight.

The calculation covers the period of construction (20 calculation steps for 110 days), followed by the first ten years of service life of the structure (30 steps of calculation). The low number of calculation steps results from the choice of time adjusted steps (Eq.5). The loading accounts for the dead weight of the structure (live loads not taken into account). The main results are discussed below.

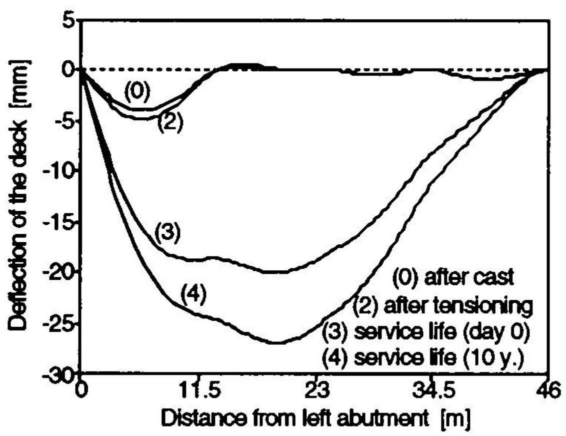

Fig.2 : Deflection of the deck (left span)

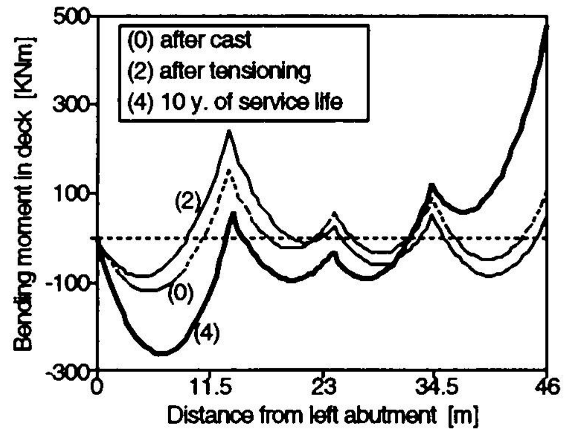

Fig.3 : Bending moment in the left span

On Fig.2, it is shown that the maximum deflection of the deck doesn't exceed 30 mm after 10 years. Anyway, despite the procedure adopted for the tensioning of the cables, a significant redistribution in the bending moment is observed in the deck (Fig.3). The cable tensions increase about 15% when the deck receives its final equipment (Fig.4), but they remain about constant during the service life of the structure. As a consequence, the tensile stress ranges about 1.7 Mpa to 1.9 Mpa at the uppermost fiber of the composite slab, in the centre cross-section of the deck (Fig.5).

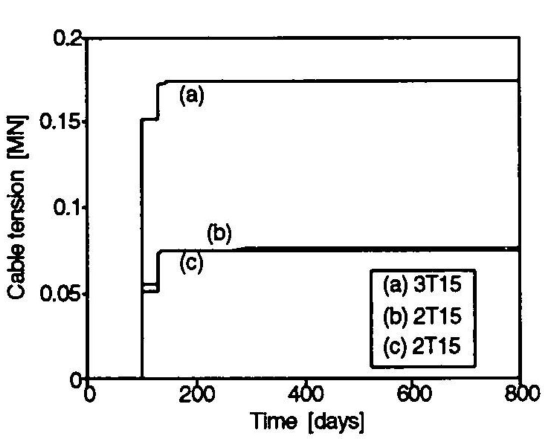

Fig.4 : Tensions in the cables

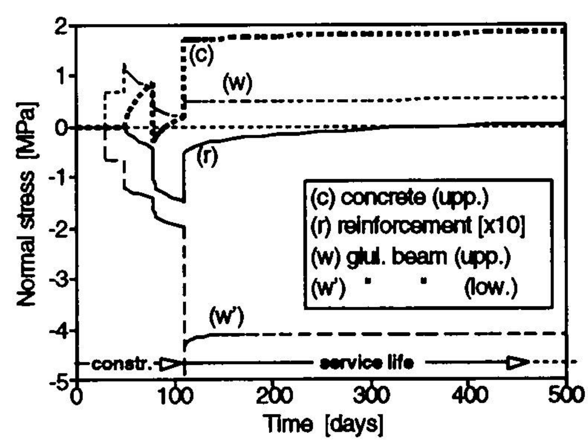

Fig.5 : Normal stresses in the mid-section

A better design of the structure could low this tensile stress in order to avoid the cracking of the concrete. This could be achieved by adjusting the initial tensions in the cables in order to decrease the bending moment in the mid-section of the deck, or by removing the support at the centre pier. The last figure (Fig.6) shows the distribution of the force of connection between the composite slab and the wooden glulam beams, along the left span of the deck. The connection force is increasing in time. The highest values are reached near the abutments and the centre pier.

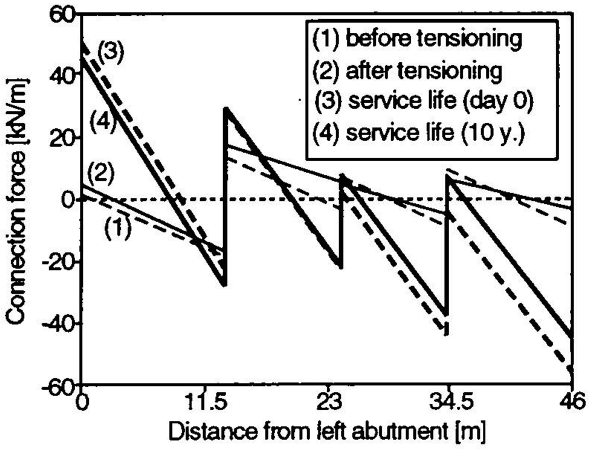

Fig.6 : Force of connection between glulam beams and composite slab

6. Conclusion

The combination of a global approach of the behaviour of structural members and an incremental formulation based on the theory of linear viscoelasticity, proves very efficient for the time analysis of composite structures. This formulation may be easily implemented in a finite element program. Each finite "composite beam" element accounts for the constitution of the composite member and for the time dependent behaviour of its constituting materials. The incremental formulation makes the computation process quite simple, specially while taking the process of construction into account. The choice of a time adjusted length for the calculation step, insures a good accuracy for low calculation cost. The efficiency of the method is illustrated by an application to the time analysis of a cable-stayed footbridge made of several materials.

7. References

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