Addendum to the Paper Entitled: "Structural Theorems for Topological Actions of Z2-Torion Real, Complex and Quaternionic Projective Spaces".

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In the statement of Theorem 3, there is a missing case that all connected components of $F(G, X)$ are of RP-type. The condition for case 2 should be: $Sq^2(\zeta)=\gamma\cdot\zeta\neq 0$ *and* $Sq^1(\gamma)=0$; and there should be the following third case, namely,

(c) *Case* 3, $Sq^2(\zeta)=\gamma\cdot\zeta\neq 0$ *and* $Sq^1(\gamma)\neq 0$; *then all connected components of* $F(G, X)$ *are of RP-type, i.e.,* $F^j\sim RP^{(k_j-1)}$ *and there exist* $w_j, v_1, v_2\in H^1(B_G)$ *such that*

$$\gamma=v_1^2+v_1v_2+v_2^2,\qquad \alpha_j=w_j(w_j+v_1)(w_j+v_1)(w_j+v_1+v_2)$$

and

$$\iota_j^*(\zeta)=\xi_j^4+\gamma\xi_j^2+(Sq^1\gamma)\,\xi_j+\alpha_j,\qquad \iota_j^*:H_G^*(X)\to H_G^*(F^j).$$

And furthermore, the system of local weights at F^j, Ω_j, *is given by*

$$\Omega_j=\{(w_i+w_j), (v_1+w_i+w_j), (v_2+w_i+w_j), (v_1+v_2+w_i+w_j)\ \text{multi.}k_i\ (i\neq j);\\ v_1, v_2, (v_1+v_2), 0\ \text{multi}(k_j-1)\},$$

and

$$F(x)\ \textit{is of}\begin{cases} \textit{RP-type} & \textit{if}\ \ v_1\mid G_x\ \ \textit{and}\ \ v_2\mid G_x\ \ \textit{are linearly independent,}\\ \textit{CP-type} & \textit{if}\ \ v_1\mid G_x\ \ \textit{and}\ \ v_2\mid G_x\ \ \textit{are linearly dependent but not all zero,}\\ \textit{QP-type} & \textit{if}\ \ v_1\mid G_x=v_2\mid G_x=0.\end{cases}$$

Correspondingly, one should add the following proof to the proof of Theorem 3 for the above case 3:

In the case $Sq^2(\zeta)=\gamma\cdot\zeta\neq 0$ and $Sq^1(\gamma)=\delta\neq 0$, it is not difficult to show that all connected components of $F(G, X)$ are of RP-type. Let ξ_j be the generator of $H^*(F^j)$, ι_j^* be the restriction homomorphism of $H_G^*(X)$ to $H_G^*(F^j)$. Then, it follows from the following equations:

$$Sq^1(\iota_j^*\zeta)=\iota_j^*(Sq^1\zeta)=0\quad\text{and}\quad Sq^2(\iota_j^*\zeta)=\iota_j^*(Sq^2\zeta)=\gamma\cdot(\iota_j^*\zeta)$$

that

$$\iota_j^* \zeta = \xi_j^4 + \gamma \cdot \xi_j^2 + \delta \xi_j + \alpha_j;$$
$$Sq^1 \delta = Sq^1 \alpha_j = 0, \qquad Sq^2 \delta = \gamma \delta, \qquad Sq^2 \alpha_j = \gamma \alpha_j.$$

Let $f_j = (0, \ldots, 0, \xi_j^{(k_j-1)}, 0, \ldots, 0)$ be the fundamental cohomology class of F_j. Then, simple computation will show that the ideal $I_X(f_j)$ is generated by $a(f_j) = \delta^{(k_j-1)} \cdot \prod_{i \neq j} (\alpha_i + \alpha_j)$, (cf. the proof of Theorem 1). For simplicity, we may assume without loss of generality that $\alpha_1 = 0$.

(i) *Suppose that at least one* $k_j > 1$: Then it follows from Theorem B that δ splits into product of linear factors, say $\delta = v_1 \cdot v_2 \cdot v_3$. Notice that

$$Sq^1 \delta = (v_1 + v_2 + v_3) \cdot \delta = 0 \quad \text{implies} \quad v_1 + v_2 + v_3 = 0,$$

i.e.,

$$\delta = v_1 v_2 (v_1 + v_2),$$

and

$$Sq^2 \delta = \gamma \cdot \delta \quad \text{implies} \quad \gamma = v_1^2 + v_1 v_2 + v_2^2.$$

Again, it follows from Theorem B that $a(f_1) = \delta^{(k_1-1)} \cdot \prod_{j \neq 1} \alpha_j^{k_j}$ splits, and hence all α_j, $j \neq 1$, split into product of linear factors, say

$$\alpha_j = w_j \cdot (w_j + \mu_{j,1})(w_j + \mu_{j,2}) \cdot (w_j + \mu_{j,3}).$$

On the other hand,

$$Sq^1 \alpha_j = (\mu_{j,1} + \mu_{j,2} + \mu_{j,3}) \cdot \alpha_j = 0 \quad \text{implies that} \quad \mu_{j1} + \mu_{j2} + \mu_{j3} = 0$$
$$Sq^2 \alpha_j = (\mu_{j_1} \mu_{j_2} + \mu_{j_2} \mu_{j_3} + \mu_{j_3} \mu_{j_1}) \cdot \alpha_j = \gamma \cdot \alpha_j$$

implies that

$$\{\mu_{j1}, \mu_{j2}, \mu_{j3}\} = \{v_1, v_2, v_1 + v_2\}$$

as a set, namely

$$\alpha_j = w_j (w_j + v_1)(w_j + v_2)(w_j + v_1 + v_2).$$

(ii) *Suppose all* $k_j = 1$: Then $s = (n+1) > 1$, $a(f_1) = \prod_{j \neq 1} \alpha_j$. Hence, α_j again splits and the above proof will show that

$$\alpha_j = w_j (w_j + \mu_{j1})(w_j + \mu_{j2})(w_j + \mu_{j3}), \qquad \mu_{j1} + \mu_{j2} + \mu_{j3} = 0,$$

and

$$(\mu_{j1}\mu_{j2}+\mu_{j2}\mu_{j3}+\mu_{j3}\mu_{j1})=\gamma \quad \text{for all} \quad j>1,$$

and the assertion follows easily.

Remark. All actions of Case 3 are cohomologically modelled after the following type of linear actions on $QP^n = Sp(n+1)/Sp(1)\times Sp(n)$.

EXAMPLE 3c. Notice that the effective group of isometry on the symmetric space QP^n is $Sp(n+1)/\{\pm 1\}$ and $SO(3)=Sp(1)/\{\pm 1\}$ sits in $Sp(n+1)/\{\pm 1\}$ as diagonal unit quaternions. Let G be a $\mathbf{Z}_2$-tori of $Sp(n+1)/\{\pm 1\}$ containing a maximal $\mathbf{Z}_2$-tori of $SO(3)$. Then the restricted G-action on QP^n is of the type of Case 3.

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