

# **Addendum to the Paper Entitled: "Structural Theorems for Topological Actions of Z2-Torion Real, Complex and Quaternionic Projective Spaces".**

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**Addendum to the Paper Entitled:  
“Structural Theorems for Topological Actions of  
 $\mathbb{Z}_2$ -Torion Real, Complex and Quaternionic Projective Spaces”**

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In the statement of Theorem 3, there is a missing case that all connected components of  $F(G, X)$  are of *RP*-type. The condition for case 2 should be:  $Sq^2(\zeta)=\gamma \cdot \zeta \neq 0$  and  $Sq^1(\gamma)=0$ ; and there should be the following third case, namely,

(c) *Case 3,  $Sq^2(\zeta)=\gamma \cdot \zeta \neq 0$  and  $Sq^1(\gamma) \neq 0$ ; then all connected components of  $F(G, X)$  are of *RP*-type, i.e.,  $F^j \sim RP^{(k_j-1)}$  and there exist  $w_j, v_1, v_2 \in H^1(B_G)$  such that*

$$\gamma = v_1^2 + v_1 v_2 + v_2^2, \quad \alpha_j = w_j (w_j + v_1) (w_j + v_1 + v_2)$$

and

$$\iota_j^*(\zeta) = \xi_j^4 + \gamma \xi_j^2 + (Sq^1 \gamma) \xi_j + \alpha_j, \quad \iota_j^*: H_G^*(X) \rightarrow H_G^*(F^j).$$

And furthermore, the system of local weights at  $F^j, \Omega_j$ , is given by

$$\begin{aligned} \Omega_j = & \{(w_i + w_j), (v_1 + w_i + w_j), (v_2 + w_i + w_j), (v_1 + v_2 + w_i + w_j) \text{ multi. } k_i \ (i \neq j); \\ & v_1, v_2, (v_1 + v_2), 0 \text{ multi. } (k_j - 1)\}, \end{aligned}$$

and

$$F(x) \text{ is of } \begin{cases} \text{RP-type} & \text{if } v_1 \mid G_x \text{ and } v_2 \mid G_x \text{ are linearly independent,} \\ \text{CP-type} & \text{if } v_1 \mid G_x \text{ and } v_2 \mid G_x \text{ are linearly dependent but not all zero,} \\ \text{QP-type} & \text{if } v_1 \mid G_x = v_2 \mid G_x = 0. \end{cases}$$

Correspondingly, one should add the following proof to the proof of Theorem 3 for the above case 3:

In the case  $Sq^2(\zeta)=\gamma \cdot \zeta \neq 0$  and  $Sq^1(\gamma)=\delta \neq 0$ , it is not difficult to show that all connected components of  $F(G, X)$  are of *RP*-type. Let  $\xi_j$  be the generator of  $H^*(F^j)$ ,  $\iota_j^*$  be the restriction homomorphism of  $H_G^*(X)$  to  $H_G^*(F^j)$ . Then, it follows from the following equations:

$$Sq^1(\iota_j^*\zeta) = \iota_j^*(Sq^1\zeta) = 0 \quad \text{and} \quad Sq^2(\iota_j^*\zeta) = \iota_j^*(Sq^2\zeta) = \gamma \cdot (\iota_j^*\zeta)$$

that

$$\begin{aligned} i_j^* \zeta &= \xi_j^4 + \gamma \cdot \xi_j^2 + \delta \xi_j + \alpha_j; \\ Sq^1 \delta &= Sq^1 \alpha_j = 0, \quad Sq^2 \delta = \gamma \delta, \quad Sq^2 \alpha_j = \gamma \alpha_j. \end{aligned}$$

Let  $f_j = (0, \dots, 0, \xi_j^{(k_j-1)}, 0, \dots, 0)$  be the fundamental cohomology class of  $F_j$ . Then, simple computation will show that the ideal  $I_X(f_j)$  is generated by  $a(f_j) = \delta^{(k_j-1)} \cdot \prod_{i \neq j} (\alpha_i + \alpha_j)$ , (cf. the proof of Theorem 1). For simplicity, we may assume without loss of generality that  $\alpha_1 = 0$ .

(i) Suppose that at least one  $k_j > 1$ : Then it follows from Theorem B that  $\delta$  splits into product of linear factors, say  $\delta = v_1 \cdot v_2 \cdot v_3$ . Notice that

$$Sq^1 \delta = (v_1 + v_2 + v_3) \cdot \delta = 0 \quad \text{implies} \quad v_1 + v_2 + v_3 = 0,$$

i.e.,

$$\delta = v_1 v_2 (v_1 + v_2),$$

and

$$Sq^2 \delta = \gamma \cdot \delta \quad \text{implies} \quad \gamma = v_1^2 + v_1 v_2 + v_2^2.$$

Again, it follows from Theorem B that  $a(f_1) = \delta^{(k_1-1)} \cdot \prod_{j \neq 1} \alpha_j^{k_j}$  splits, and hence all  $\alpha_j, j \neq 1$ , split into product of linear factors, say

$$\alpha_j = w_j \cdot (w_j + \mu_{j,1}) (w_j + \mu_{j,2}) \cdot (w_j + \mu_{j,3}).$$

On the other hand,

$$\begin{aligned} Sq^1 \alpha_j &= (\mu_{j,1} + \mu_{j,2} + \mu_{j,3}) \cdot \alpha_j = 0 \quad \text{implies that} \quad \mu_{j1} + \mu_{j2} + \mu_{j3} = 0 \\ Sq^2 \alpha_j &= (\mu_{j1} \mu_{j2} + \mu_{j2} \mu_{j3} + \mu_{j3} \mu_{j1}) \cdot \alpha_j = \gamma \cdot \alpha_j \end{aligned}$$

implies that

$$\{\mu_{j1}, \mu_{j2}, \mu_{j3}\} = \{v_1, v_2, v_1 + v_2\}$$

as a set, namely

$$\alpha_j = w_j (w_j + v_1) (w_j + v_2) (w_j + v_1 + v_2).$$

(ii) Suppose all  $k_j = 1$ : Then  $s = (n+1) > 1$ ,  $a(f_1) = \prod_{j \neq 1} \alpha_j$ . Hence,  $\alpha_j$  again splits and the above proof will show that

$$\alpha_j = w_j (w_j + \mu_{j1}) (w_j + \mu_{j2}) (w_j + \mu_{j3}), \quad \mu_{j1} + \mu_{j2} + \mu_{j3} = 0,$$

and

$$(\mu_{j1}\mu_{j2} + \mu_{j2}\mu_{j3} + \mu_{j3}\mu_{j1}) = \gamma \quad \text{for all } j > 1,$$

and the assertion follows easily.

*Remark.* All actions of Case 3 are cohomologically modelled after the following type of linear actions on  $QP^n = Sp(n+1)/Sp(1) \times Sp(n)$ .

EXAMPLE 3c. Notice that the effective group of isometry on the symmetric space  $QP^n$  is  $Sp(n+1)/\{\pm 1\}$  and  $SO(3) = Sp(1)/\{\pm 1\}$  sits in  $Sp(n+1)/\{\pm 1\}$  as diagonal unit quaternions. Let  $G$  be a  $\mathbb{Z}_2$ -tori of  $Sp(n+1)/\{\pm 1\}$  containing a maximal  $\mathbb{Z}_2$ -tori of  $SO(3)$ . Then the restricted  $G$ -action on  $QP^n$  is of the type of Case 3.

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