

Erratum to 'Rational Lie Algebras and p-Isomorphisms of Nilpotent Groups and Homotopy Types'

Autor(en): **Roitberg, Joseph**

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Erratum to ‘Rational Lie Algebras and p -Isomorphisms of Nilpotent Groups and Homotopy Types’

JOSEPH ROITBERG

The author has noticed that the proof of Th. 3.2 of [R] contains a flaw. We shall indicate how to modify the argument in [R] so as to set things right.

The trouble occurs on p. 5, l. 12–13 where it is asserted that the homotopy equivalence $h_0: W \rightarrow W$ gives rise to a weak automorphism $\omega: L \rightarrow L$ of its associated Lie algebra. However, it may only be asserted that ω is an equivalence in the category in which it lies; it need not be an actual DGL map. To remedy this, we employ a theorem of Quillen ([Q1], [Q2]; esp. pp. 263–264] which provides an equivalence between the category of DGL algebras used in [R] and the category whose objects are reduced, rational DGL algebras which are free as graded Lie algebras and whose morphisms are homotopy classes (in an appropriate sense) of DGL maps. It is certainly the case that in this latter category, an equivalence always contains a representative ω which is a weak automorphism. To complete the proof of Th. 3.2 of [R], it is only necessary to replace Th. 3.1 of [R] (whose statement and proof are correct, except for a harmless misprint; in formula (3.5), one should have a plus sign rather than a minus sign since $(aa)ab = 2aaaab$, not $-2aaaab$) by the following variant.

THEOREM 3.1'. *There exists a rational, reduced DGL algebra L of finite type, free as a graded Lie algebra, such that any weak automorphism $\omega: L \rightarrow L$ is congruent, modulo L^2 , to the identity.*

L may be chosen so that $H(L)$ has totally finite dimension, but at the price of possibly foregoing the relation $\omega(x) \equiv x \pmod{L^2}$ for x of high degree.

Proof. Let L be the free, graded Lie algebra over \mathbf{Q} generated by elements a, b, c, e having degrees 1, 3, 2, 11 respectively and define a differential $d: L \rightarrow L$ by setting

$$da = 0, \quad db = aa, \quad dc = 0, \quad de = (ac)ccac + (ac)(ac)ab + (ab)cab.$$

Denote by F the (free) sub-Lie algebra generated by a, b, c . Writing for any weak

automorphism $\omega: L \rightarrow L$,

$$\omega(a) = ra, \quad \omega(b) = sb + u(ac), \quad \omega(c) = tc + v(aa), \quad \omega(e) = x \cdot e + f,$$

where $r, s, t, u, v, x \in \mathbf{Q}$, $f \in F$, we easily conclude, as in the proof of Th. 3.1 of [R], that $r \neq 0$, $t \neq 0$, $s = r^2$. Further, we obtain $\omega(de) = r^2t^4((ac)ccac) + r^5t^2((ac)(ac)ab) + r^6t((ab)cab) + g \pmod{L^7}$, where g lies in the ideal $I \subset F$ generated by aa . By a straightforward algebraic calculation, one shows that the elements $(ac)ccac$, $(ac)(ac)ab$, $(ab)cab$ are linearly independent modulo I . Then using $\omega(de) = d\omega(e)$, together with $df \in I$, we conclude that $r^2t^4 = r^5t^2 = r^6t = x$ so that $r = s = t = x = 1$.

To obtain the final assertion of Th. 3.1', we take any integer $n \geq 11$ and successively attach elements of degree $\geq n+1$ to L so as to kill all homology classes of degree $\geq n$.

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Hunter College
CUNY and
Graduate Center
CUNY

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