Subgroups with projective abelianization and trivial multiplicator.

Autor(en): **Dyer, Micheal**

Objekttyp: **Article**

Zeitschrift: **Commentarii Mathematici Helvetici**

Band (Jahr): **59 (1984)**

PDF erstellt am: **27.05.2024**

Persistenter Link: https://doi.org/10.5169/seals-45398

Comment. Math. Helvetici 59 (1984) 333–340 0010-2571/84/030333-08\$01.50 + 0.20/0

Subgroups with projective abelianization and trivial multiplicator

MICHEAL DYER

1. Introduction

In this note we study the exact sequence

$$L \rightarrowtail G \twoheadrightarrow H$$

of groups and homomorphisms where the first homology H_1L of L is a projective H-module and the second homology H_2L is trivial. We call such subgroups *projective* subgroups. They arise as examples of projective G-crossed modules. See [R] for more details.

The motivating topological setting is as follows: let X be a connected subcomplex of a connected two-dimensional aspherical CW-complex Y and let $i: X \to Y$ be the inclusion. Let $L = \ker\{i_\#: \pi_1 X \to \pi_1 Y\}$ be the normal subgroup of $G = \pi_1 X$ and $H = \operatorname{im} i_\#$. Then L is a projective subgroup ([D], [BD]).

Here are several examples of projective subgroups. Let X be a set and $F(X) = F$ be the free group on X. For any group H, consider $G = F * H$. Then setting elements of F equal to 1 gives an extension (which is split)

$$L \rightarrowtail G \twoheadrightarrow H$$

where L is the normal closure $\langle\langle F\rangle\rangle_G$ of F in G. It follows from the Kurosh theorem [Se, Theorem 14] that L is free on $\{hxh^{-1} \mid x \in X, h \in H\}$, that H_1L is a free H-module on X, and that $H_2L = 0$.

As a second example, consider a 1-relator presentation $\mathcal{P} = (X; r)$ of the group G. Let $F = F(X)$ and $R = \langle\langle r\rangle\rangle_F$. If the word r is *not* a proper power in F, then $H_1R \approx \mathbb{Z}G$ and $H_2R = 0$ (because R is free). This follows because the cellular 2-complex K modeled on $\mathcal{P}$ is aspherical [C].

Of course, if $L = G$ is projective, we simply mean that H_1L is free abelian and $H_2L = 0$. A projective group $L = G$ which is not a free G-crossed module would be one whose weight$>$rank H_1L. Here the weight of G is the minimal number of normal generators of G.

Another example is $H_1L = H_2L = 0$ (L is *superperfect*). Any such L is a projective subgroup.

Projective subgroups are *hereditary* in the following sense. Suppose L is a projective subgroup of $G = K * F$. Then $\bar{L} = L \cap K$ is a projective subgroup of K. See section 3.

In this note we study the lower central series of projective subgroups. The main theorem states that if H_1L is a submodule of a free H-module and $H_2L = 0$, then each quotient L_n/L_{n+1} $(n \geq 1)$ of the lower central series $\{L_n\}$ of L is the submodule of a free H-module. The proof is an extension of the proof devised by Ralph Strebel in [S]. Applications are given to the upper central series of G.

To fix notation, we let $[a, b] = aba^{-1}b^{-1}$ and $\mathbb{Z}G$ denote the integral group ring of the group G.

The author gratefully acknowledges the support of the Science Research Council of the United Kingdom during the summer of 1982, during which a portion of this research was done. Furthermore, I would like to thank Ronnie Brown and the University College of North Wales for their hospitality.

2. The lower central series of L

For any group L, define $L_1 = L$ and $L_{n+1} = [L_n, L]$ $(n \geq 1)$. This is called the *lower central series* of L. If L is a normal subgroup of G, then conjugation by elements of g ($l_n \to gl_ng^{-1}$) induces a left H ($H = G/L$)-module structure on each L_n/L_{n+1} $(n \geq 1)$. Note that $H_1L = L_1/L_2$. The graded object $\operatorname{gr} L = \{L_n/L_{n+1}\}_{n\geq 1}$ has the structure of a graded Lie-ring over $\mathbb{Z}H$ with the Lie bracket equal to [,].

If M is a left H-module, then the graded object

$$TM = \{\mathbb{Z}H, M, M \otimes M, M \otimes M \otimes M, \cdots\}$$

has the structure of a graded $\mathbb{Z}H$-algebra with multiplication given by tensoring: $m \cdot n = m \otimes n$. Here $M \otimes M$ means tensor product over $\mathbb{Z}$ with the diagonal H-action.

We now state and prove the main result of this paper.

MAIN THEOREM 2.1. *Suppose $L \rightarrowtail G \twoheadrightarrow H$ is an exact sequence of groups with H_1L a submodule of a free H-module and H_2L a torsion group. Then each successive quotient L_n/L_{n+1} of the lower central series L_n $(1 \leq n < \omega)$ is a submodule of a free H-module.*

Proof. We follow the proof of Theorem 1 of [S], p. 149, and check that at each stage the maps defined are isomorphisms of H-modules. This yields a graded Lie

$\mathbb{Z}H$-algebra isomorphism $\alpha: \operatorname{gr} L \to TH_1L$ from the graded Lie $\mathbb{Z}H$-algebra $\operatorname{gr} L$ associated with L *onto* the Lie $\mathbb{Z}H$-subalgebra of TH_1L generated by $H_1L = T^1H_1L$. It is clear that if H_1L is a submodule of a free H-module F, then TH_1L is a subalgebra of TF, with each $T^iF = F \otimes \cdots \otimes F$ (i times) a free H-module. Hence, each $\alpha(L_n/L_{n+1})$ is a submodule of T^nH_1L, which in turn is a submodule of T^nF.

Let $I = IL$ be the augmentation ideal of $\mathbb{Z}L$ and define a descending chain of normal subgroups of L by setting

$$D^i(L) = \{l \in L \mid l - 1 \in I^i\}$$

This series is a central series, and we can form the associated graded Lie $\mathbb{Z}H$-algebra $\operatorname{gr}\{D(L)\}$, because each $D^i(L)$ is *normal in* G.

In order to see that H acts on D^i/D^{i+1} via conjugation by elements of G, it is enough to show that if $l \in D^i(L)$, then, for any $\omega \in L$, $\omega l \omega^{-1} \equiv l \bmod D^{i+1}(L)$; i.e., $\omega l \omega^{-1} l^{-1}$ is a member of $D^{i+1}(L)$. But an easy calculation shows that $\omega l \omega^{-1} l^{-1} - 1 = (\omega - 1)(l-1)l^{-1} - \omega(l-1)(\omega^{-1}-1)l^{-1}$. So if $l - 1 \in I^i$, then $\omega l \omega^{-1} l^{-1} - 1 \in I^{i+1}$ and we have verified H acts on $\{D^i/D^{i+1}\}$ via conjugation.

The inclusion $L_j \subseteq D^j(L)$ allows one to define a Lie $\mathbb{Z}H$-algebra homomorphism $i: \operatorname{gr} L \to \operatorname{gr}(D(L))$.

Let $\operatorname{gr} \mathbb{Z}L$ denote the graded $\mathbb{Z}H$-algebra associated to the filtration $\{I^j\}_{0 \le j < \omega}$ of $\mathbb{Z}L$. Here H acts on I^j/I^{j+1} via conjugation by elements of G. This is well-defined because conjugation by elements of L is trivial $\bmod I^{j+1}$. The function $g \mapsto g - 1$ then defines an *injective* Lie $\mathbb{Z}H$-algebra homomorphism $\beta: \operatorname{gr}(D(L)) \to \operatorname{gr} \mathbb{Z}L$.

Finally, we use the isomorphism $\mu: H_1L \approx I/I^2$ $(l \cdot L' \mapsto (l-1) + I^2)$ to extend to a homomorphism $\mu: TH_1L \to \operatorname{gr} \mathbb{Z}L$ of graded associative $\mathbb{Z}H$-algebras, given in degree j by (H acts diagonally on TH_1L)

$$l_1L_2 \otimes l_2L_2 \otimes \cdots \otimes l_jL_2 \mapsto (l_1 - 1)(l_2 - 1) \cdots (l_j - 1) + I^{j+1}.$$

Clearly, μ is always surjective; it is also injective if H_1L is torsion free and H_2L is a torsion group (see [S], p. 150). The Lie $\mathbb{Z}H$-algebra homomorphism $\alpha: \operatorname{gr} L \to TH_1L$ is defined by $\alpha = \mu^{-1}\beta i$ in the following diagram:

$$\operatorname{gr} L \xrightarrow{i} \operatorname{gr}(D(L)) \xrightarrow{\beta} \operatorname{gr} \mathbb{Z}L \xleftarrow{\mu} TH_1L.$$

On page 151 of [S], Strebel shows that α is a monomorphism. ■

The following example shows that even if H_1L is a free H-module and

$L \rightarrowtail G \twoheadrightarrow H$ is *split*, the quotients L_n/L_{n+1} are not necessarily *projective*. Let $H = \mathbb{Z}_2 = \{e, h\}$ and $G = \mathbb{Z} * H$, where $\mathbb{Z}$ generated by x. Then $L = \langle\langle \mathbb{Z} \rangle\rangle_G$ is the free group of rank 2 with basis x and $y = hxh^{-1}$. We order $x < y$, as weight one basic commutators. The only basic commutator of weight 2 is $c_1 = [y, x]$. The action of H on $L_2/L_3 \cong \mathbb{Z}$, generated by $\bar{c}_1$, is non-trivial, because $h[y, x]h^{-1} = [hyh^{-1}, hxh^{-1}] = [x, y] = [y, x]^{-1}$. Hence L_2/L_3 is not projective, but is still a submodule of $\mathbb{Z}H$, as guaranteed by Theorem 2.1.

It is intriguing to ask just when the L_n/L_{n+1} might themselves be projective. The next theorem gives a partial result in this direction.

A group H is said to be *ordered* if there is a linear ordering $\leq$ on H such that if $a \leq b$ in H, then $ah \leq bh$ and $ha \leq hb$ for all h in H. Note that $1 \leq a$ in H iff $a^{-1} \leq 1$. For example, any torsion free nilpotent group is ordered [P, p. 581].

THEOREM 2.2. *Suppose $L \rightarrowtail G \twoheadrightarrow H$ is a split exact sequence of groups, with $H_2L = 0$, H_1L a free H-module, and H an ordered group. Then* gr *L is a free graded H-module.*

First, we prove the following.

LEMMA 2.3. *Suppose $F = F(X)$ is a free group with basis X and H is any ordered group. Form the group $G = F * H$ and the split exact sequence $L \overset{i}{\rightarrowtail} G \overset{\varphi}{\twoheadrightarrow} H$ where φ is obtained by setting elements of F equal to* 1 *and L is the normal closure $\langle\langle F \rangle\rangle_G$ of F in G. Then each free abelian group L_n/L_{n+1} $(n \geq 1)$ is a free H-module.*

Proof. If X is a basis for F, then $\bar{X} = \{hxh^{-1} \mid x \in X, h \in H\}$ is a basis for the free group $L = \langle\langle F \rangle\rangle_G$. We order X arbitrarily and $\bar{X}$ lexicographically according to the pairing (x, h). We consider the basic commutators in $\bar{X}$ (see [H], p. 166). Each basic commutator c_k of weight k is of the form (uniquely, as L is free)

$$c_k = [c_i, c_j]$$

where $wt(c_i) + wt(c_j) = k$, c_i, c_j are basic commutators and $c_i > c_j$. If $c_i = [c_r, c_s]$, then $c_j \geq c_s$. We *order* the basic commutators of weight k *lexicographically* by using the pairing (c_i, c_j).

We now will prove inductively the following: if c_l, c_m are basic commutators of weight k and $h \in H$, then (1) hc_lh^{-1} is a basic commutator of weight k and (2) $c_l > c_m$ implies that $hc_lh^{-1} > hc_mh^{-1}$, using the lexicographic ordering given above. It is clearly true for $k = 1$, using the ordering on $\bar{X}$ and that H is an ordered group. If $c_l = [c_i, c_j]$ with $c_i > c_j$, then $hc_lh^{-1} = [hc_ih^{-1}, hc_jh^{-1}]$, with $hc_ih^{-1} > hc_jh^{-1}$. Also if $c_i = [c_r, c_s]$ and $c_j \geq c_s$, then $hc_jh^{-1} \geq hc_sh^{-1}$, by induction. Finally, we must show that if $c_l > c_m$, then $hc_lh^{-1} > hc_mh^{-1}$. Let $c_m = [c_a, c_b]$ with $c_a > c_b$. If $c_i > c_a$, then $hc_ih^{-1} > hc_ah^{-1}$, while if $c_i = c_a$ and $c_j > c_b$, then $hc_jh^{-1} > hc_bh^{-1}$ is true by induction. Thus (1) and (2) are true for all basic commutators.

Now it is easy to find basic commutators of wt k which form an H-basis for L_k/L_{k+1}. These will consist of basic commutators of wt k whose first occurrence of an element of $\bar{X}$ is actually an element of X. For example, basic commutators of weight 2 look like $[hxh^{-1}, gyg^{-1}]$ where $x, y \in X$, $g, h \in H$ and $(x, h) > (y, g)$. Then the set $\{[x, hyh^{-1}] \mid x, y \in X,\ h \in H \text{ and } (x, 1) > (y, h)\}$ is a $\mathbb{Z}H$-basis because

$$\{h_1[x, hyh^{-1}]h_1^{-1} \mid h_1, h \in H, x, y \in X, (x, 1) > (y, h)\}$$

yields all weight 2 basic commutators with no repetitions. ∎

Proof of Theorem 2.2. Let $s: H \to G$ denote a splitting of the sequence $L \rightarrowtail G \twoheadrightarrow H$. Because H_1L is a free H-module and $H_2L = 0$, we may choose a subset X of L so that the image of X in H_1L is a $\mathbb{Z}H$-basis for H_1L and so that the corresponding $\bar{X} = \{(sh)x(sh^{-1}) \mid L \in H, x \in X\}$ (see [HS], p. 204) is the basis for a free subgroup $\bar{L} < L$. Let $F = F(X)$ be the free subgroup of L generated by X. Let $\bar{G} = F * H$ and consider the split exact sequence and commutative diagram:

$$\begin{tikzcd}
\bar{L} = \langle\langle F \rangle\rangle_{\bar{G}} \arrow[r, tail] \arrow[d, tail, "i"] & \bar{G} \arrow[r, two heads, shift left] \arrow[d, "\omega"] & H \arrow[l, shift left] \arrow[d, equal] \\
L \arrow[r, tail] & G \arrow[r, two heads, shift right] & H \arrow[l, shift right, "s"']
\end{tikzcd}$$

The map ω is defined by the inclusion $i: F \to L$ and the splitting s. Because $H_1(i)$ is an isomorphism and $H_2(i)$ is zero we have that the map

$$\bar{L}_n/\bar{L}_{n+1} \to L_n/L_{n+1}$$

is an isomorphism of $\mathbb{Z}H$-modules, which are free by the lemma. ∎

3. Applications to groups

In this section we apply the main theorem about the structure of gr L to show that often the center $\mathscr{Z}G$ of G must be "buried" inside L; i.e., $\mathscr{Z}G \subset \bigcap_{n<\omega} L_n = L_\omega$.

First we state a simple lemma about certain elements in group rings which are not zero divisors.

LEMMA 3.1. *Let H be a group and h be an element in H. Then $(h-1) \in \mathbb{Z}H$ is a zero divisor iff the order of h is finite: $(h+1)$ is a zero divisor in $\mathbb{Z}H$ iff the order of h is even. If $|n| \neq 1$, then $(h-n)$ is never a zero divisor in $\mathbb{Z}H$.* ∎

THEOREM 3.2. *Let* $\rightarrowtail G \overset{\varphi}{\twoheadrightarrow} H$ *be an exact sequence of groups with* H *torsion free,* H_1L *isomorphic to a submodule of a free* H*-module and* $H_2L=0$. *Let* $g \in G-L$. *Then any* $l \in L$ *which commutes with* g *must live in* $L_\omega = \bigcap L_n$.

Proof. Let the abelianization $L \to H_1L$ be denoted by $l \mapsto \bar{l}$ and $\varphi(g)=\hat{g}$ $(l \in L, g \in G)$. Then $glg^{-1}l^{-1}=1 \Rightarrow \hat{g}\bar{l}-\bar{l}=0$ in $H_1L \Rightarrow (\hat{g}-1)\bar{l}=0$ in $H_1L \subset \bigoplus_{i\in I} \mathbb{Z}H_i$. Write $\bar{l}=(\bar{l}_i)_{i\in I}$, where each $\bar{l}_i \in \mathbb{Z}H$. Thus $((\hat{g}-1)\bar{l}_i)=0$ and it follows from the lemma that each $\bar{l}_i=0$; hence $\bar{l}=0$. So $l \in L_2$. We use Theorem 2.1 and a similar argument to show that $l \in L_n$ for all $n \geq 2$. ■

Note 3.3. (a) A similar argument will show that if $g \in G-L$ and $l \in L$ satisfies $glg^{-1}=l^n$ $(n \in \mathbb{Z})$, then $l \in L_\omega$.

(b) Other identities may be used. For example, if $g_1, g_2 \in G-L$ and $l \in L$ with $[[g_1, l], g_2]=1$, then $l \in L_\omega$. This follows because $\overline{[[g_1, l], g_2]}=(1-\hat{g}_2)\overline{[g_1, l]}=(1-\hat{g}_2)(\hat{g}_1-1)\bar{l}=0$. Then apply the argument twice.

Recall that, if G is a group, then the nth order center of G, $\mathfrak{X}^nG$ $(n \geq 1)$, is inductively defined as $\mathfrak{X}^1G=$ center of G, $\mathfrak{X}^{n+1}G=\{g \in G \mid \varphi_n(g) \in \mathfrak{X}^1(G/\mathfrak{X}^nG)$, where $\varphi: G \to G/\mathfrak{X}^nG$ is the natural map$\}$.

COROLLARY 3.4. *Suppose that the exact sequence of groups is as in Theorem* 3.2 *with* $G-L \neq \varnothing$ *and* $H_1L \neq 0$. *Then all the centers* $\mathfrak{X}^nG$ $(n \geq 1)$ *are contained in* L_ω.

Proof. (a) We show that $\mathfrak{X}^1G \subset L_\omega$. Suppose there is an element $g \in (G-L) \cap \mathfrak{X}^1G$. Choose any $l \in L$ and observe that $[g, l]=1$. By Theorem 3.2, we see that $l \in L_\omega$. Hence $L=L_\omega$, which contradicts the hypothesis that $H_1L \neq 0$. Hence $\mathfrak{X}G \subset L$.

Now choose any $g \in G-L$ and $l \in \mathfrak{X}G \subset L$. Again, the proposition shows that $l \in L_\omega$. Hence $\mathfrak{X}^1G \subset L_\omega$.

(b) We observe that $g \in \mathfrak{X}^2G$ iff for all $g_1, g_2 \in G$ the commutator $[[g_1, g], g_2]=1$. Now suppose $g \in G-L \cap \mathfrak{X}^2G$. Choose any $l \in L$ and observe that $[[g, l], g_2]=1$ for all $g_2 \in G$. Choosing $g_2 \in G-L$, we see that $l \in L_\omega$ for all $l \in L$ by note (b). The rest of the argument is similar to (a).

(c) One can either prove $\mathfrak{X}^nG \subset L$ for $n \geq 2$ by induction or by studying the higher commutators $[\ldots, [[g_1, l], g_2] \ldots, g_n]$. ■

Note 3.5. Corollary 3.4 also follows without using 3.2 because L/L_ω is a non-abelian free group (under the assumption imposed above) and in that case $\mathfrak{X}^nG \subseteq \mathfrak{X}^nL \subseteq L_\omega$.

Note 3.6. The above corollary is false if $H_1L=0$ or if $G=L$. Let L be a superperfect group, let $G=\mathbb{Z}\times L$ and $G\to\mathbb{Z}$ be the projection with kernel L. Then $\mathbb{Z}\subset\mathscr{X}G$ is not contained in L. Also, if $G=L=\mathbb{Z}$, then $\mathscr{X}G=\mathbb{Z}$ is not contained in $L_\omega=0$.

In order to give the next application, we need the notion of a C-subgroup. We say that a subgroup $N<G$ is a *C-subgroup* if there is a G-projective resolution $P_*\twoheadrightarrow\mathbb{Z}$ of the trivial module $\mathbb{Z}$ for which the homomorphism $N\otimes\partial_3:\mathbb{Z}\otimes_N P_3\to\mathbb{Z}\otimes_N P_2$ is trivial. See [BDS] for properties and applications of this concept.

The next result shows that, in some sense, H is close to being a two-dimensional group, the "closeness" being measured by H_2L.

THEOREM 3.7. *Let $L\rightarrowtail G\twoheadrightarrow H$ be an exact sequence of groups with H_1L a projective H-module, $H_2L=0$ and L a C-subgroup. Then the cohomological dimension of H is ≤ 2.*

Proof. Let $P_*\to\mathbb{Z}$ be the resolution assured by L being a C-subgroup of G. By tensoring P_* with $\mathbb{Z}\otimes_L -$, using that H_1L is projective as an H-module and that L is a C-subgroup, we obtain an exact sequence of H-modules

$$H_2L\rightarrowtail\mathbb{Z}\otimes_L P_2\oplus H_1L\to\mathbb{Z}\otimes_L P_1\to\mathbb{Z}\otimes_L P_0\to\mathbb{Z}$$

with the inner three terms projective. The hypothesis that $H_2L=0$ does the rest. ■

The next result yields new information about the aspherical question of J. H. C. Whitehead ([BD] and [BDS]): Are subcomplexes of aspherical 2-complexes aspherical?

PROPOSITION 3.8. *Suppose $G=K*N$ is a free product of groups K and N and that L is a projective subgroup of G. Then $\bar{L}=K\cap L$ is a projective subgroup of K.*

Proof. We observe that $\bar{L}$ is a normal subgroup of K. It follows from the Kurosh structure theorem about subgroups of a free product [Se, Theorem 14] that $\bar{L}$ is a free summand of $L=\bar{L}*M$. It is easy to check that, if $\bar{H}=K/\bar{L}$ ($\bar{H}<G/L$), then H_1L is isomorphic to $H_1\bar{L}\oplus H_1M$ as $\bar{H}$-modules. Because H_1L is a projective G/L-module, it is a projective $\bar{H}$-module. We have $H_2\bar{L}=0$ because $H_2L\cong H_2\bar{L}\oplus H_2M=0$. Thus $\bar{L}$ is projective. ■

A $[G,2]$-complex X is a connected, 2-dimensional CW-complex with fundamental group isomorphic to G and a single zero cell. If X is a $[G,2]$-complex which is a subcomplex of an *aspherical* $[K,2]$-complex Y, we let $\bar{X}=X\cup Y^1$ be the union of X together with the 1-skeleton of Y and $i: X \hookrightarrow Y$, and $i_1: \bar{X} \hookrightarrow Y$ denote the inclusion maps. Furthermore, let $L = \ker \pi_1(i_1)$ and $\bar{L} = \ker \pi_1(i)$. Then the fundamental group $\pi_1(\bar{X}) \cong G * F$, where F is a free group whose rank corresponds to the number of 1-cells of Y outside of X. It is well known (see [BD]) that $L \rightarrowtail G * F \twoheadrightarrow \text{im}\ \pi_1(i_1) = H$ is a free $G * F$-crossed module. Thus $H_1 L$ is a free H-module and $H_2 L = 0$. Now $\bar{L} = \ker \pi_1(i) = L \cap G$ is a projective $\bar{H} = G/\bar{L}$-module by the previous proposition (see [D]). Furthermore (and this seems to be new), the sequence

$$\hat{L} = F \cap L \rightarrowtail F \twoheadrightarrow F/F \cap L = \hat{H} < H$$

is also a projective subgroup of the free group F, with $\hat{H}$ a 2-dimensional group.

BIBLIOGRAPHY

[BD] J. BRANDENBURG and M. DYER, *On J. H. C. Whitehead's aspherical question I*, Comm. Math. Helvetici, *56* (1981), 431–446.

[BDS] J. BRANDENBURG, M. DYER, and R. STREBEL, *On J. H. C. Whitehead's aspherical question II* appeared in Contemporary Mathematics, *20* (1983), 65–78.

[C] W. COCKCROFT, *On two-dimensional aspherical complexes*, Proc. London Math. Soc., *4* (1954), 375–384.

[D] M. DYER, *Subcomplexes of two-complexes and projective crossed modules*, preprint.

[H] M. HALL, *The Theory of Groups*, Macmillan, 1959, New York.

[HS] P. HILTON and U. STAMMBACH, *A Course in Homological Algebra*, Springer-Verlag, Heidelberg, 1970.

[P] D. PASSMAN, *The Algebraic Structure of Group Rings*, John Wiley and Sons, New York, 1977.

[R] J. RATCLIFFE, *Free and projective croseed modules*, J. London Math. Soc., *22* (1980), 66–74.

[Se] J.-P. SERRE, *Trees*, Springer-Verlag, Berlin–Heidelberg–New York, 1980.

[S] R. STREBEL, *A note on groups with torsion-free abelianization and trivial multiplicator*, Comm. Math. Helvetici, *54* (1979), 147–158.

University of Oregon
Eugene, Or. 97403 U.S.A.

Received July 27, 1983