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Comment. Math. Helv. 72 (1997) 466–480
0010-2571/97/030466-15 $ 1.50+0.20/0

Commentarii Mathematici Helvetici

Topology of complete intersections*

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Abstract. Let $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ be two n-dimensional complete intersections with the same total degree d. In this paper we prove that, if n is even and d has no prime factors less than $\frac{n+3}{2}$, then $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ are homotopy equivalent if and only if they have the same Euler characteristics and signatures. This confirms a conjecture of Libgober and Wood [16]. Furthermore, we prove that, if d has no prime factors less than $\frac{n+3}{2}$, then $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ are homeomorphic if and only if their Pontryagin classes and Euler characteristics agree.

Mathematics Subject Classification (1991). 13C40, 14M10, 57R50.

Keywords. Complete intersection, homotopy equivalence, homeomorphism.

§1. Introduction

A complete intersection is the transversal intersection of some complex hypersurfaces given by homogeneous polynomials in a complex projective space. In this paper we prove that the topology of a complete intersection is determined by several well-known invariants in most cases. It is a classical result of R. Thom that the topology of an n-dimensional complete intersection depends only on the degrees of the homogeneous polynomials. Let $X_n(d_1, d_2, ..., d_r)$ be a complete intersection defined by r homogeneous polynomials of $(n+r)$ variables and degrees $d_1, d_2, ..., d_r$, respectively. We call the unordered set $\mathbf{d} = (d_1, d_2, ..., d_r)$ the *multidegree* and the product $d_1 d_2 \cdots d_r := d$ the *total degree* of $X_n(\mathbf{d})$. It is known that the total degree d is a homotopy invariant of $X_n(\mathbf{d})$ when $n \geq 3$. By the Lefschetz hyperplane section Theorem, the inclusion

$$X_n(\mathbf{d}) \to \mathbb{C}P^{n+r}$$

is an $(n-1)$-equivalence.

In lower dimensions, the topology of a complete intersection is well understood by the general theory of differential topology. For example, $X_1(\mathbf{d})$ is a complex

*Supported in part by Max-Planck Institut für Mathematik

curve of genus $g = 1 - \frac{d}{2}(r + 2 - \sum_{i=1}^{r} d_i)$. $X_2(\mathbf{d})$ is a simply connected complex surface. By M.Freedman's celebrated work on the topology of 4-manifolds [8], the homeomorphism type of $X_2(\mathbf{d})$ is determined by its intersection form. In the smooth category, however, Ebeling [5] and Libgober-Wood [17] independently found examples of homeomorphic complete intersections but not diffeomorphic. $X_3(\mathbf{d})$ is a simply connected 3-dimensional complex manifold with torsion free homology groups. A complete classification of such manifolds was given by C.T.C.Wall [22] and Jupp [10].

$n = 4$ is the first nontrivial dimension in which we can not refer to any classical classification theory. In [7], S.Klaus and the author proved that two 4-dimensional complete intersections are homeomorphic if and only if their total degrees, Euler numbers and all Pontrjagin numbers agree. Even in this special dimension, the homotopy classification problem has not yet been solved.

On the other hand, some interesting partial results on the classification of complete intersections in high dimensions have been obtained under certain restriction on the total degree d. For example, under the condition that for all primes p with $p(p-1) \leq n+1$, the total degree d is divisible by $p^{[(2n+1)/(2p-1)]+1}$, Traving [21](c.f: [12]) proved that two complete intersections with the same total degree d are diffeomorphic if and only if their Euler numbers and all Pontrjagin classes agree. For the homotopy classification, Libgober and Wood [14] proved that, if the dimension n is odd and the total degree d has no prime factors less than $\frac{n+3}{2}$, then n-dimensional complete intersections with total degree d are homotopy equivalent if and only if their Euler numbers agree. They made a further conjecture [16] when n is even. In this situation, the topology becomes much more complicated. We refer to [15][16] for more details.

The following theorem and the work of Libgober and Wood [14] completes the homotopy classification problem of complete intersections for which, the total degree d has no prime factors less than $\frac{n+3}{2}$.

Theorem 1.1. *Let $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ be two complete intersections of even dimension with the same total degree d. Suppose that d has no prime factors less than $\frac{n+3}{2}$. If $n \neq 2$, then $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ are homotopy equivalent if and only if they have the same Euler characteristic and signatures.*

Remark. The conjecture of Libgober-Wood [16] is a corollary of Theorem 1.1.

Once the homotopy types of two complete intersections are the same, Sullivan's characteristic variety theory can be applied to handle the homeomorphic classification problem. Note that, for a complete intersection $X_n(\mathbf{d})$, the i-th Pontrjagin class p_i must be an integral multiple of x^{2i}, where $x \in H^2(X_n(\mathbf{d}), \mathbb{Z}) \cong \mathbb{Z}$ is a generator if $n \geq 3$. This multiple is independent of the choice of the generator x since $p_i \in H^{4i}(X_n(\mathbf{d}), \mathbb{Z})$. For convenience, throughout the rest of the paper, we view the Pontrjagin class p_i of $X_n(\mathbf{d})$ as the multiple of x^{2i}.

Theorem 1.2. *Let $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ be two homotopy equivalent complete intersections. If d is odd and $n \neq 2^i - 2$ for all $i \in \mathbb{Z}$, then $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ are homeomorphic to each other if and only if their Pontrjagin classes agree.*

Remark. Our proof of Theorem 1.2 can not be extended to the case when d is even. The reason is that we have to use Browder's result on the Kervaire invariants of framed manifolds [4].

Combining Theorem 1.1, Theorem 1.2 and [14] on the homotopy classification of complete intersections of odd dimensions, we solve the homeomorphism classification problem in the case when $n \neq 2^i - 2$, and d has no prime factors less than $\frac{n+3}{2}$. With some additional argument we will prove

Corollary 1.3. *Let $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ be two complete intersections of dimension $n \geq 3$ with the same total degree d. Suppose that d has no prime factors less than $\frac{n+3}{2}$. Then $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ are homeomorphic if and only if their Pontrjagin classes and Euler numbers agree.*

Another very natural question is as follows. If $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ are diffeomorphic/or homeomorphic/or homotopy equivalent, is $X_n(\mathbf{d}, a)$ diffeomorphic to $X_n(\mathbf{d}', a)$ for a natural number a? Here $X_n(\mathbf{d}, a)$ is the complete intersection with multidegree $(d_1, d_2, \cdots, d_r, a)$.

To give a partial answer to this question, we need a definition. M^{2n} and N^{2n} are said to be *S-diffeomorphic (homeomorphic, homotopy equivalent)* if there are integers r and s such that $M^{2n} \# rS^n \times S^n$ and $N^{2n} \# sS^n \times S^n$ are diffeomorphic(homeomorphic, homotopy equivalent).

Theorem 1.4. *Let $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ be two S-diffeomorphic(homeomorphic, homotopy equivalent) complete intersections. If $a_1, \cdots, a_k$ are positive integers such that*

$$max\{a_1, \cdots, a_k\} \leq min\{\mathbf{d}, \mathbf{d}'\},$$

then $X_n(\mathbf{d}, a_1, \cdots, a_k)$ and $X_n(\mathbf{d}', a_1, \cdots, a_k)$ are S-diffeomorphic(homeomorphic, homotopy equivalent).

Remark. Without loss of generality, we may assume that the multidegree $\mathbf{d}$ does not contain 1. By the above theorem, if $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ are S-diffeomorphic (homeomorphic, homotopy equivalent), then so are $X_n(\mathbf{d}, 2, \cdots, 2)$ and $X_n(\mathbf{d}', 2, \cdots, 2)$.

The organization of this paper is as follows. In §2 we study the homotopy types of complete intersections. The proof of Theorem 1.1 is given there. In §3 we first review Sullivan's characteristic variety theory. Using this potential theory as a tool, we prove Theorem 1.2 and corollary 1.3. In §4 we give a proof of Theorem 1.4.

§2. Homotopy type

In the topological category, every odd dimensional complete intersection is homeomorphic to the connected sum $K\#rS^n \times S^n\#N$, where K satisfies $H_n(K) = 0$, N is $(n-1)$-connected and

$$H_n(N) \cong \mathbb{Z} \oplus \mathbb{Z}$$

Following [16], we call K the topological core of the complete intersection. We define $K_n(\mathbf{d})$ to be the corresponding core of $X_n(\mathbf{d})$.

When $n = 1, 3$ or 7, the piece N is homeomorphic to $S^n \times S^n$. For other odd n, this is true if and only if either there is a homological trivial embedded n-sphere in $X_n(\mathbf{d})$ with nontrivial normal bundle, or the Kervaire invariant of a well-defined quadratic function on $H^n(X_n(\mathbf{d}), \mathbb{Z}_2)$ vanishes. The Kervaire invariant for hypersurface was calculated by Morita [18] and independently by Libgober (c.f: Proc. AMS, vol. 63 No.2, p.148). For general complete intersections, it was further calculated by J.Wood for d odd and W.Browder for all cases. The main results are:

Proposition 2.1. (J.Wood [23]) *There is no homological trivial n-sphere in $X_n(\mathbf{d})$ with nontrivial normal bundle if and only if*

• *The binomial coefficient* $\begin{pmatrix} m+l \\ m+1 \end{pmatrix}$ *is even, where $n = 2m+1 \neq 1, 3, 7$ and l is the number of even entries in $\mathbf{d}$.*

If • holds, for every element $x \in H^n(X_n(\mathbf{d}), \mathbb{Z}_2)$, its Poincarè dual can be represented by an embedded n-sphere. We know that the normal bundle of this sphere in $X_n(\mathbf{d})$ is stably trivial. In view of homotopy, this implies that the normal bundle corresponds to an element of the kernel of the stable homomorphism $\pi_{n-1}(SO(n)) \to \pi_{n-1}(SO)$. It is well known that this kernel is isomorphic to $\mathbb{Z}_2$. Let $q(x) \in \mathbb{Z}_2$ denote this element. This gives a well-defined quadratic function

$$q : H^n(X_n(\mathbf{d}), \mathbb{Z}_2) \to \mathbb{Z}_2$$

The Kervaire invariant is defined to be the Arf invariant of q. We denote by $k_{X_n(\mathbf{d})}$ the Kervaire invariant of $X_n(\mathbf{d})$. Clearly, if d is odd, $l = 0$. Thus • is satisfied and the Kervaire invariant is well-defined.

Theorem 2.2. (Browder[3], Morita[18], Wood[23]) *If d is odd, then*

$$k_{X_n(\mathbf{d})} = \begin{cases} 0, & \text{if } d = \pm 1 (mod 8); \\ 1, & \text{if } d = \pm 3 (mod 8). \end{cases}$$

Suppose • holds and d is even. Then $k_{X_n(\mathbf{d})} = 1$ if and only if $n = 1 (mod 8)$, $l = 2$ and d is not divisible by 8.

Note that, if $k_{X_n(\mathbf{d})} = 1$, the manifold N above is the Kervaire manifold.

For n even, the situation is quite different. One can not find a decomposition with a core K, for which $H_n(K) = 0$. This is because that there is an element in $H_n(X_n(\mathbf{d}))$ which is not spherical. Indeed, by [15] we have a decomposition with a core $K_n(\mathbf{d})$ such that $\mathrm{rank} H_n(K_n(\mathbf{d})) \leq 5$. The precise value of this minimum rank depends on the type of the intersection form as well as the total degree d. It is easy to see that, at least up to homotopy, the core is unique.

When n is odd, by [14] the cohomology ring

$$H^*(K_n(\mathbf{d})) \cong \mathbb{Z}[x, y]/\{x^{\frac{n+1}{2}} = dy, y^2 = 0\}.$$

If d has no prime factors less than $\frac{n+3}{2}$, it is proved [14] that $K_n(\mathbf{d})$ has the homotopy type of the $2n$-skeleton of E, where E is the homotopy fiber of

$$x^{\frac{n+1}{2}} : \mathbb{C}P^\infty \to K(\mathbb{Z}_d, n+1)$$

Here $x \in H^2(\mathbb{C}P^\infty, \mathbb{Z}_d)$ is a generator. Thus the homotopy type of $K_n(\mathbf{d})$ depends only on the total degree d. The similar method does not work when n is even. In [16], Libgober and Wood conjectured that the same conclusion holds for n even. This is essentially equivalent to Theorem 1.1. We will give a proof of this fact by using the surgery theory of F. Quinn.

Let M be a manifold of dimension $2n$ and N be a codimension 2 submanifold. Let $C = M - intU$, where U denotes a tubular neighborhood of N. We say N is *taut* if the pair $(C, \partial C)$ is $(n-1)$-connected. Let $f : M \to X$ be a map transversal to a CW subcomplex $Y \subset X$, where Y has a 2-dimensional normal bundle. Let $E(f, Y)$ and $E(f, X - Y)$ denote the fiber spaces over M

$$\begin{array}{ccc} E(f,Y) & \longrightarrow & Y \\ \big\downarrow & & \big\downarrow \\ M & \xrightarrow{f} & X \end{array}$$

and

$$\begin{array}{ccc} E(f,X-Y) & \longrightarrow & X-Y \\ \big\downarrow & & \big\downarrow \\ M & \xrightarrow{f} & X \end{array}$$

Following Quinn [19], f is called *almost canonical* with respect to Y if the natural maps

$$f^{-1}(Y) \to E(f, Y)$$

and

$$f^{-1}(X - Y) \to E(f, X - Y)$$

are $(n-1)$- and n-equivalences, respectively. When f is a homotopy equivalence and almost canonical, it is easy to see that the maps $f : f^{-1}(Y) \to Y$ and $f :$

$f^{-1}(X-Y) \to X-Y$ are $(n-1)$- and n-equivalences. The following theorem of F.Quinn plays an important role in this paper.

Theorem (F.Quinn[19]). *Let $Y \subset X$ have a dimension 2 normal bundle neighborhood. Then every map $f: M \to X$ is homotopic holding the boundary fixed to an almost canonical one with respect to Y.*

Proof of Theorem 1.1. The necessity is obvious. Assume that $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ have the same total degree d and the same signature, where n and d are as in Theorem 1.1. Since $n+1$ is odd, by [14] the cores $K_{n+1}(\mathbf{d})$ and $K_{n+1}(\mathbf{d}')$ are homotopy equivalent under the hypothesis on d. In our case, d is odd and so the Kervaire invariant of $X_{n+1}(\mathbf{d})$ and $X_{n+1}(\mathbf{d}')$ are well-defined and the same. Without loss of generality, we assume that

$$\mathrm{rank} H_{n+1}(X_{n+1}(\mathbf{d})) \geq \mathrm{rank} H_{n+1}(X_{n+1}(\mathbf{d}'))$$

There is an integer r such that $X_{n+1}(\mathbf{d}) \simeq X_{n+1}(\mathbf{d}')\# rS^{n+1} \times S^{n+1}$. Let $f: X_{n+1}(\mathbf{d}) \to X_{n+1}(\mathbf{d}')\# rS^{n+1} \times S^{n+1}$ be a homotopy equivalence.

Notice that $X_n(\mathbf{d}) \subset X_{n+1}(\mathbf{d})$ and $X_n(\mathbf{d}') \subset X_{n+1}(\mathbf{d}')\# rS^{n+1} \times S^{n+1}$ are taut submanifolds. They represent the generators of $2n$-dimensional homology groups of the ambient manifolds, respectively. By Quinn's theorem above, we can assume that f is almost canonical with respect to the submanifold $X_n(\mathbf{d}')$. Since f is a homotopy equivalence, this implies that $f^{-1}(X_n(\mathbf{d}'))$ is also a taut submanifold of $X_{n+1}(\mathbf{d})$ representing the same $2n$-dimensional homology generator. Freedman's uniqueness theorem on taut submanifolds [9] asserts that $X_n(\mathbf{d})$ and $f^{-1}(X_n(\mathbf{d}'))$ are stably diffeomorphic. By [9] we can add some copies of $S^n \times S^n$ to the taut submanifold $f^{-1}(X_n(\mathbf{d}')) \subset X_{n+1}(\mathbf{d})$ and keep f being an almost canonical map. Thus we can assume that the middle dimensional Betti number of $f^{-1}(X_n(\mathbf{d}'))$ is not less than that of $X_n(\mathbf{d})$. Therefore $f^{-1}(X_n(\mathbf{d}'))$ is diffeomorphic to $X_n(\mathbf{d})\# r' S^n \times S^n$ for some nonnegative integer r'.

Now we obtain a map

$$f: X_n(\mathbf{d})\# r' S^n \times S^n \to X_n(\mathbf{d}')$$

which is an n-equivalence. Moreover, f is a degree one map, since the two complete intersections have the same total degree. It follows that the sublattice $ker f_* \subset H_n(X_n(\mathbf{d})\# r' S^n \times S^n)$ is unimodular. From the commutative square of the Hurewicz homomorphisms it is easy to verify that $ker f_*$ consists of spherical elements. Moreover, $ker f_*$ is of even type. The signature of this sublattice is exactly the difference of the target and source manifolds, which is zero. Hence $ker f_*$ is isomorphic to the sum of some copies of the hyperbolic plane H, say mH. As in [16], from this algebraic decomposition, there is a topological decomposition $M\# mS^n \times S^n \cong X_n(\mathbf{d})\# r' S^n \times S^n$. Note that f is null homotopy when it is restricted to the factor $mS^n \times S^n - int D^{2n}$. By surgery on these $2m$ spheres

$S^n \times pt$ and $pt \times S^n$ we get a map $f' : M \to X_n(\mathbf{d}')$ with $kerf'_* = 0$. From this we conclude that f' is a homotopy equivalence. On the other hand, by assumption, $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ have the same Euler numbers, hence $m = r'$.

Therefore $X_n(\mathbf{d})\#mS^n \times S^n$ and $X_n(\mathbf{d}')\#mS^n \times S^n$ are homotopy equivalent. Using the same argument of [16] Proposition 3.3, one can check that $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ are homotopy equivalent. This completes the proof. □

Remark. The proof above also gives an affirmative answer to the conjecture in [16] (c.f: page 126).

It has been pointed out in [16] that the condition on d in Theorem 1.1 is sharp. By using K-theory one can get some stronger restrictions on the multidegrees of two homotopy equivalent complete intersections. To illustrate this, we give the following

Proposition 2.3. *Let $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ be two complete intersections with homotopy equivalent cores. Let l and l' denote the numbers of even entires in $\mathbf{d}$ and $\mathbf{d}'$, respectively. Then $l - l'$ is divisible by $2^{f(n)-1}$. Here $f(n)$, $n \in \mathbb{Z}_+$, is given by the following table*

1	2	3	4	5	6	7	8	$\cdots$	$m + 8$
1	2	2	3	3	3	3	4	$\cdots$	$f(m) + 4$

Proof. Suppose that the cores $K_n(\mathbf{d})$ and $K_n(\mathbf{d}')$ are homotopy equivalent, by definition, there are two $(n-1)$-connected almost parallelizable manifolds, say M and M', such that $X_n(\mathbf{d})\#M$ and $X_n(\mathbf{d}')\#M'$ are homotopy equivalent. We warn that M and M' are not necessarily smoothable.

By Atiyah [2], the stable normal spherical fibrations of $X_n(\mathbf{d})\#M$ and $X_n(\mathbf{d}')\#M'$ are fiber homotopy equivalent. By the Lefschetz hyperplane Theorem, there are natural $(n-1)$-equivalences $\mathbb{C}P^{[\frac{n}{2}]} \hookrightarrow X_n(\mathbf{d})$ and $\mathbb{C}P^{[\frac{n}{2}]} \hookrightarrow X_n(\mathbf{d})$. Therefore the restrictions of the stable normal bundles of $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ to $\mathbb{C}P^{[\frac{n}{2}]}$ are fiberwise homotopy equivalent. In other words, they present the same element in the J-group $J(\mathbb{C}P^{[\frac{n}{2}]})$. It is easy to check that the stable normal bundles of $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ are $H^{d_1} \oplus \cdots \oplus H^{d_r} - (n + r + 1)H$ and $H^{d'_1} \oplus \cdots \oplus H^{d'_{r'}} - (n + r' + 1)H$, respectively. Here H is the Hopf line bundle over $X_n(\mathbf{d})$. From the above argument we conclude that

$$H^{d_1} \oplus \cdots \oplus H^{d_r} - (n+r+1)H = H^{d'_1} \oplus \cdots \oplus H^{d'_{r'}} - (n+r'+1)H \in J(\mathbb{C}P^{[\frac{n}{2}]}) \qquad (2.4)$$

Consider the canonical S^1-fibration $\pi : \mathbb{R}P^{2[\frac{n}{2}]+1} \to \mathbb{C}P^{[\frac{n}{2}]}$. The complex line bundle $\pi^*(H^{d_i})$ has the trivial first Chern class if and only if d_i is even since $H^2(\mathbb{R}P^{2[\frac{n}{2}]+1}) \cong \mathbb{Z}_2$. Moreover, if d_i is odd, $\pi^*(H^{d_i}) \cong \pi^*(H) = 2\eta \in KO(\mathbb{R}P^{2[\frac{n}{2}]+1})$, where η is the Hopf real line bundle. Thus by (2.4) we conclude that

$$\pi^*\{(H^{d_1}\oplus\cdots\oplus H^{d_r}-(n+r+1)H)-(H^{d'_1}\oplus\cdots\oplus H^{d'_{r'}}-(n+r'+1)H)\}=\pi^*(l'-l)H=0$$

in $J(\mathbb{R}P^{2[\frac{n}{2}]+1})$. Therefore $2(l'-l)$ must be a multiple of the order of the J-group $J(\mathbb{R}P^{2[\frac{n}{2}]+1})$, which is equal to $2^{f(2[\frac{n}{2}]+1)}$ by [1]. This completes the proof. $\square$

§3. Sullivan's characteristic variety

This section is devoted to a proof of Theorem 1.2 and Corollary 1.3 by using Sullivan's characteristic variety theory [20]. Sullivan's characteristic variety theory is a very powerful approach to the problem when two homotopy equivalent manifolds are homeomorphic. For reader's convenience, we recall some main results in this theory with adaptations for our use in this paper.

Let M be an oriented PL m-manifold whose oriented boundary is the disjoint union of n-copies of closed oriented $(m-1)$-manifolds L(with the induced orientations). The polyhedron V obtained from M by identifying these copies of L to one another is called a *$\mathbb{Z}_n$-manifold*. Denote $L \subset V$ by δV and call it the *Bockstein* of V.

A finite disjoint union of $\mathbb{Z}_n$-manifolds is called a *variety*. If X is a polyhedron, a *singular variety* in X is a piecewise linear map $f : V \to X$, from a variety V to X. The $\mathbb{Z}_n$ manifold provides a nice model for $\mathbb{Z}_n$-homology class since every $\mathbb{Z}_n$-manifold V carries a well-defined fundamental class in $H_m(V;\mathbb{Z}_n)$. Clearly, every closed manifold is a $\mathbb{Z}_n$-manifold for each n with the Bockstein empty.

For a homotopy equivalence $f : L \to M$, where L, M are closed PL manifold, let $V \to M$ be an embedded connected singular $\mathbb{Z}_n$-manifold of dimension m. Assume that M, V and δV are all simply connected and $dim M \geq 3$. If $m = 2s$ is even, then f can be deformed to a map f' such that:
(i) f' is transversal regular to $(V, \delta V)$ with $U = {f'}^{-1}(V)$ and $\delta U = {f'}^{-1}(\delta V)$.
(ii) ${f'}^{-1} : \delta U \to \delta V$ is a homotopy equivalence.
(iii) $f' : U \to V$ is s-connected.

Let $K_s = ker f'_* \subset H_s(U, Z)$. This is a unimodular form. Moreover, when s is even, it is of even type and so its signature is divisible by 8. When s is odd, one has an Arf invariant in $\mathbb{Z}_2$.

By Sullivan, the *splitting obstruction* $\theta_f(V)$ of $f : L \to M$ along V is defined to be the Arf invariant of K_s if s is odd, $\frac{sigK_s}{8}(\bmod n)$ if $s \neq 2$ even and $\frac{sigK_s}{8}(\bmod 2n)$ if $s = 2$.

In general the splitting invariants $\theta_f(V)$ of a nonconnected singular variety V is defined as the collection of the corresponding invariants along its connected components.

Sullivan's characteristic variety Theorem ([20]). *Let $f : L \to M$ be a homotopy equivalence between two simply connected PL manifolds L and M of dimension $n \geq 6$. Then there is a (characteristic) singular variety in M, $V \to M$, such that f is homotopic to a PL homeomorphism if and only if the splitting invariants of f along V is identically zero.*

To apply this theorem, one needs to obtain a characteristic variety for a given manifold. But there is no natural way to define it in general. For the complex projective space $\mathbb{C}P^n$, as noted in [20], the characteristic variety is the union

$$\mathbb{C}P^2 \cup \mathbb{C}P^3 \cup \cdots \cup \mathbb{C}P^{n-1} \subset \mathbb{C}P^n$$

For a complete intersection $X_n(\mathbf{d})$, notice that there is an embedding $i : \mathbb{C}P^{[\frac{n}{2}]} \to X_n(\mathbf{d})$, which is a $(n-1)$-equivalence. We may identify $\mathbb{C}P^{[\frac{n}{2}]}$ and $\mathbb{C}P^k \subset \mathbb{C}P^{[\frac{n}{2}]}$ ($k \leq [\frac{n}{2}]$) with their images under i in $X_n(\mathbf{d})$.

Lemma 3.1. *If n and d are both odd and $n \geq 5$, then*

$$V = \cup_{i=1}^{[\frac{m}{2}]} X_{2i+1}(\mathbf{d}) \cup \cup_{i=2[\frac{m}{2}]+2}^{n-1} X_i(\mathbf{d}) \cup \cup_{i=1}^{[\frac{m}{2}]} \mathbb{C}P^{2i} \subset X_n(\mathbf{d})$$

is a characteristic variety, where $n = 2m+1$.

Proof. Note that the homology groups of $X_n(\mathbf{d})$ are all torsion free. Moreover, $H_n(X_n(\mathbf{d}))$ is the only nontrivial homology group in odd dimensions if $d \neq 1$. By [20] Theorem 5 the K-homology group

$$KO_{-1}(X_n(\mathbf{d})) \otimes \mathbb{Z}_{(odd)} \cong \Omega_{4*-1}(X_n(\mathbf{d})) \otimes_{\Omega_*} \mathbb{Z}_{(odd)} = 0.$$

By the Atiyah-Hirzebruch spectral sequence it is easy to see that $KO_{-1}(X_n(\mathbf{d})) \otimes \mathbb{Z}_{(odd)}$ has no odd torsion. For a generator $x \in H^2(X_n(\mathbf{d}), \mathbb{Z})$, $x^2 \in H^2(X_n(\mathbf{d}), \mathbb{Z}_2)$ is also a generator for $n \geq 5$ by the Lefschetz hyperplane section Theorem. Thus $Sq^2 : H^2(X_n(\mathbf{d}), \mathbb{Z}_2) \to H^4(X_n(\mathbf{d}), \mathbb{Z}_2)$ is an isomorphism. By the proof of Sullivan's characteristic variety Theorem(refer to [20] pages 33 and 34), we need only to show that

(i). A basis of $\oplus_{i\geq 1, \neq \frac{n-1}{2}} H_{4i+2}(X_n(\mathbf{d}), \mathbb{Z}_2)$ can be represented by the fundamental classes of V.

(ii). The image of the oriented bordism classes of V under the natural maps S_* and I_* below in the groups $\Omega^{so}_{4*}(X_n(\mathbf{d})) \otimes_{\Omega^{so}_*} \mathbb{Z}_{(odd)}$ and $\oplus_{i\geq 1} H_{4i}(X_n(\mathbf{d}))$ are basis, where

$$I_* : \Omega^{so}_{4*}(X_n(\mathbf{d})) \to \Omega^{so}_{4*}(X_n(\mathbf{d}) \otimes_{\Omega^{so}_*} \mathbb{Z}_{(odd)}/\text{torsion}$$

is the natural projection and

$$S_* : \Omega^{so}_{4*}(X_n(\mathbf{d})) \stackrel{\text{fundamental class}}{\longrightarrow} \oplus_{i\geq 1} H_{4i}(X_n(\mathbf{d}))/\text{torsion}.$$

(i) is clearly satisfied by our variety since d is odd.

To verify (ii), note that $\Omega_{4*}^{so}(X_n(\mathbf{d}))\otimes_{\Omega_*^{so}}\mathbb{Z}_{(odd)} \cong H_{4*}(X_n(\mathbf{d}),\Omega_*^{so})\otimes_{\Omega_*^{so}}\mathbb{Z}_{(odd)}$ is torsion free. Since all $4i$ dimensional homology generators of $X_n(\mathbf{d})$ are represented by some subvarieties of V, this completes the proof. □

The characteristic variety for n even is more complicated since we have to count the middle dimensional homology classes and represent them by some singular manifolds.

Let $x \in H^2(X_n(\mathbf{d})$ be a generator where n is even. We use h to denote the hyperplane class $x^{\frac{n}{2}} \cap [X_n(\mathbf{d})]$. By [15], the image of Hurewicz homomorphism $\pi_n(X_n(\mathbf{d})) \to H_n(X_n(\mathbf{d})) := \mathcal{H}$ is the orthogonal complement $h^\perp$. Let $\beta \in H_n(X_n(\mathbf{d}))$ satisfy $\beta \cdot h = 1$. Then $\mathcal{H} = h^\perp + Z\beta$. Notice that this is not an orthogonal decomposition. By [15] again, every element in $h^\perp$ can be represented by an embedded n-sphere with stably trivial normal bundle if $n \geq 2$ and β can be represented by an embedded $\mathbb{C}P^{\frac{n}{2}}$ with normal bundle $(\frac{n}{2}+r)H-\sum_1^r H^{d_i}$. Choose a basis for $h^\perp$ and represent them by embedded n spheres $\alpha_1, \cdots, \alpha_k$. Similar to Lemma 2.2 it is easy to check the following lemma. We omit the details.

Lemma 3.2. *Let $n = 2m \geq 6$ and d be odd. Then*

$$V = \cup_{i=1}^{[\frac{m}{2}]-1} X_{2i+1}(\mathbf{d}) \cup \cup_{i=2[\frac{m}{2}]+1}^{n-1} X_i(\mathbf{d}) \cup \cup_{i=1}^{[\frac{m-1}{2}]} \mathbb{C}P^{2i} \cup \beta(\mathbb{C}P^m) \cup \cup_{i=1}^{k} \alpha_i(S^{2m}) \subset X_n(\mathbf{d})$$

is a characteristic variety.

In general, we can also write down a characteristic variety for a complete intersection when d is even. But it is difficult to compute the splitting invariant of Arf type for the application of the characteristic variety Theorem.

Now we are ready to prove Theorem 1.2.

Proof of Theorem 1.2. We need only to show the sufficiency. Let $f : X_n(\mathbf{d}) \to X_n(\mathbf{d}')$ be a homotopy equivalence. By the characteristic variety Theorem we need only to show the splitting invariant $\theta_f(V) = 0$, where V denotes the variety defined above.

Let us consider first the case when n is odd. Notice that the splitting invariant along a $4i$-dimension subvariety, denoted by $X_{2i}(\mathbf{d}')$(or $\mathbb{C}P^{2i}$), is the difference $Sigf^{-1}(X_{2i}(\mathbf{d}')) - SigX_{2i}(\mathbf{d}')$. By assumption, $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ have the same Pontrjagin classes. Applying the Hirzebruch signature Theorem, it is easy to show that all splitting invariants along $4i$ $(1 \leq i \leq \frac{n}{2}$) dimensional subvarieties vanish.

The only difficulty is to show that the Arf type splitting invariants vanishes along V. Fortunately the main difficulty has been overcomed by Browder and Wood. When d is odd and $n \neq 1, 3, 7$, the Kervaire invariant of $X_n(\mathbf{d})$ is well-defined and independent of the framing (c.f. [3] or [23]) and its value depends only on the total degree d(mod8)(independent of the dimension). We prove now that the splitting invariant along $X_{n-2}(\mathbf{d}')$ vanishes.

By Quinn's theorem [19], we may assume that f is almost canonical with respect to $X_{n-1}(\mathbf{d}')$. Thus $f^{-1}(X_{n-1}(\mathbf{d}'))$ is a taut submanifold of $X_n(\mathbf{d})$ representing the dual of a generator $x \in H^2(X_n(\mathbf{d}))$. Freedman's Theorem [9] applies to assert that $f^{-1}(X_{n-1}(\mathbf{d}'))$ and $X_{n-1}(\mathbf{d})$ are stably diffeomorphic. Consider the restricted map $g : f^{-1}(X_{n-1}(\mathbf{d}')) \to X_{n-1}(\mathbf{d}')$, applying Quinn's and Freedman's Theorems again we can deform g to get a taut submanifold $g^{-1}(X_{n-2}(\mathbf{d}'))$ which is stably diffeomorphic to $X_{n-2}(\mathbf{d})$. When $n-2 \neq 1, 3$ or 7, the Kervaire invariant is a stably diffeomorphic invariant by the geometric definition. Therefore, with the exception of $n = 3, 7$ or 9, the Kervaire invariant of $g^{-1}(X_{n-2}(\mathbf{d}'))$ is the same as that of $X_{n-2}(\mathbf{d})$ and so as that of $X_{n-2}(\mathbf{d}')$. By the naturality of the splitting obstruction, the splitting invariants of g and f along $X_{n-2}(\mathbf{d}')$ are the same. Notice that the splitting invariant of g along $X_{n-2}(\mathbf{d}')$ is the difference of the Kervaire invariants of $g^{-1}(X_{n-2}(\mathbf{d}'))$ and $X_{n-2}(\mathbf{d}')$, which is identically zero. This implies that the splitting invariant along $X_{n-2}(\mathbf{d}')$ vanishes. Proceeding this we can show that the splitting invariants along $X_i(\mathbf{d}')(i$ odd) is zero if $i \geq 8$.

When $i = 7$, we have to deal with the framing. For a complete intersection, $X_n(\mathbf{d}) \subset \mathbb{C}P^{n+r}$, it can be endowed a natural framing $X_n(\mathbf{d}) \times \mathbb{R}^q \subset E$ where E is a vector bundle over $\mathbb{C}P^{n+r}$ representing $-(H^{d_1} + H^{d_2} + \cdots H^{d_r}) \in KO(\mathbb{C}P^{n+r})$. This framing is not determined by the smooth structure but by the complete intersection structure. For d odd, the Kervaire invariant is well-defined not only for n odd but also for n even [3]. Moreover, by Browder [3] page 100, the Kervaire invariant of a hyperplane section in $X_n(\mathbf{d})$ is the same as the Kervaire invariant of $X_n(\mathbf{d})$. If

$$f : X_8(\mathbf{d}) \to X_8(\mathbf{d}')$$

is an orientation preserving homotopy equivalence, the transversal preimage $f^{-1}(X_7(\mathbf{d}'))$ is a hyperplane section of the complex line bundle H over $X_8(\mathbf{d})$ and so it has the same Kervaire invariant as that of $X_7(\mathbf{d})$ and $X_7(\mathbf{d}')$. Therefore the splitting obstruction along $X_7(\mathbf{d}')$ is zero too.

The case of $i = 3$ is similar. One can also refer to [7] for this detail. This completes the proof in the case of n odd.

For $n \neq 2$ even, the argument is exactly the same but we have to count the splitting invariants along the subvarieties $\alpha_i(S^n)$ and $\mathbb{C}P^{\frac{n}{2}}$ when $n = 0(mod 4)$. When $n = 0(mod 4)$, these splitting invariants along α_i is the signature of its transversal preimage $f^{-1}(\alpha_i)$, which is zero since its Pontrjagin classes are zero. The splitting invariant along $\mathbb{C}P^{\frac{n}{2}}$ is $Sigf^{-1}(\beta) - 1$. Using the Hirzebruch signature Theorem one can check directly that $Sigf^{-1}(\beta) - 1 = 0$.

For $n = 2(mod 4)$ and $n \neq 2^i - 2$, the splitting invariants along α_i and β are exactly the Kervaire invariant of $f^{-1}(\alpha_i)$ and $f^{-1}(\beta)$ respectively, since both of $\alpha_i(S^n)$ and $\beta(\mathbb{C}P^{\frac{n}{2}})$ have no nontrivial middle dimensional homology class. Recall that a smooth framed manifold of dimension $n \neq 2^i - 2$ has trivial Kervaire invariant [4]. This concludes that the splitting invariants vanish identically along V.

Now Sullivan's Theorem applies to conclude our Theorem. □

Proof of Corollary 1.3. By [14] and Theorem 1.1 and 1.2, we need only to consider the case when n is even and to show the sufficiency. Note that the cores of $X_{n+1}(\mathbf{d})$ and $X_{n+1}(\mathbf{d}')$ are homotopy equivalent. We may assume that there is an integer r such that, $X_{n+1}(\mathbf{d})\#rS^{n+1}\times S^{n+1}$ and $X_{n+1}(\mathbf{d}')$ are homotopy equivalent. Consider the canonical hyperplane sections $X_n(\mathbf{d})\subset X_{n+1}(\mathbf{d})$, $X_n(\mathbf{d}')\subset X_{n+1}(\mathbf{d}')$. By assumption, one can easily check that all of the Pontrjagin classes of $X_{n+1}(\mathbf{d})$ and $X_{n+1}(\mathbf{d}')$ are the same. Applying Theorem 1.2(with a slight extension but identical proof) we conclude that $X_{n+1}(\mathbf{d})\#rS^{n+1}\times S^{n+1}$ and $X_{n+1}(\mathbf{d}')$ are homeomorphic.

Notice that $X_n(\mathbf{d})\subset X_{n+1}(\mathbf{d})\#rS^{n+1}\times S^{n+1}$ and $X_n(\mathbf{d}')\subset X_{n+1}(\mathbf{d}')$ are taut submanifolds. Freedman's Theorem [9] applies to conclude that $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ are stably homeomorphic. With the exception of $\mathbf{d}$ or $\mathbf{d}'=(1),(2),(2,2)$ or (3), the complete intersection $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ can split out a factor $S^n\times S^n$(c.f: [15]). Applying the cancellation Theorem [12] it follows that $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ are homeomorphic. This completes the proof. □

§4. Proof of Theorem 1.4.

This section is devoted to the proof of Theorem 1.4. The main idea is to use the branched covering $X_n(\mathbf{d},a)\to X_n(\mathbf{d})$ with branched set $X_{n-1}(\mathbf{d},a)$ constructed in [23].

Proof of Theorem 1.4. Without loss of generality, we consider only the case of $n\geq 4$, since lower dimensional complete intersections have been completely classified. By induction we may assume that $k=1$. By [23], $X_n(\mathbf{d}',a)$ is an a-fold branched cover over $X_n(\mathbf{d}')$ with branched set $X_{n-1}(\mathbf{d},a)$. Let $p':X_n(\mathbf{d}',a)\to X_n(\mathbf{d}')$ denote this covering map. Let $X_n(\mathbf{d})$ and $X_n(\mathbf{d}')$ be S-homotopy equivalent, we want to show that $X_n(\mathbf{d},a)$ and $X_n(\mathbf{d}',a)$ are S-homotopy equivalent under the assumption. The proofs for S-homeomorphism and S-diffeomorphsim are similar.

Suppose

$$\mathrm{rank} H_n(X_n(\mathbf{d}))\geq \mathrm{rank}\ H_n(X_n(\mathbf{d}'))$$

There is a degree one map $f:X_n(\mathbf{d})\to X_n(\mathbf{d}')$ which is an n-equivalence. By [19], we can assume that f is almost canonical with respect to $X_{n-1}(\mathbf{d}',a)\subset X_n(\mathbf{d}')$. Let $Y_{n-1}=f^{-1}(X_{n-1}(\mathbf{d}',a))$. Pulling back this covering to $X_n(\mathbf{d})$ we get a covering $\pi:Y_n\to X_n(\mathbf{d})$ with branched set Y_{n-1}. Also we get a map $g:Y_n\to X_n(\mathbf{d}',a)$ such that the following diagram commutes

$$\begin{array}{ccc}
Y_{n-1} & \xrightarrow{f} & X_{n-1}(\mathbf{d}', a) \\
\big\downarrow \cap & & \big\downarrow \cap \\
Y_n & \xrightarrow{g} & X_n(\mathbf{d}', a) \\
\big\downarrow \pi & & \big\downarrow p' \\
X_n(\mathbf{d}) & \xrightarrow{f} & X_n(\mathbf{d}')
\end{array}$$

One can easily verify that Y_n is a connected manifold and g is a degree one map. From this we conclude that $g : Y_n - Y_{n-1} \to X_n(\mathbf{d}', a) - X_{n-1}(\mathbf{d}', a)$ is n-connected and the latter space is $(n-1)$-connected. Moreover, $f : Y_{n-1} \to X_{n-1}(\mathbf{d}', a)$ is an $(n-1)$-equivalence. Therefore g is an $(n-1)$-equivalence.

On the other hand, by the Alexander duality, $H_q(Y_n, Y_{n-1}) \cong H^{2n-q}(Y_n - Y_{n-1}) = 0$ if $q \neq 0, n$ or $2n$. Note that the Euler class of the normal circle bundle of Y_{n-1} in Y_n is a generator of the 2-dimensional cohomology group. Applying the Gysin exact sequence it is easy to show that $H^{n-1}(Y_n) \cong \mathbb{Z}$ if $n-1$ is even and 0 if $n-1$ is odd. By the diagram above, one can easily verify that g is actually an n-equivalence. When n is even, we know that the signature of Y_n is the same as that of $X_n(\mathbf{d}', a)$. Thus Y_n and $X_n(\mathbf{d}', a)$ are S-homotopy equivalent(c.f: the proof of Theorem 1.1).

We claim that Y_n and $X_n(\mathbf{d}, a)$ are stably diffeomorphic. From this we conclude that $X_n(\mathbf{d}', a)$ and $X_n(\mathbf{d}, a)$ are S-homotopy equivalent.

To prove this claim, first notice that Y_{n-1} and $X_{n-1}(\mathbf{d}, a)$ are stably diffeomorphic [9], since both of them represent the dual of $ax \in H^2(X_n(\mathbf{d}))$, where x is a generator. Therefore there is a homotopy

$$h : X_n(\mathbf{d}) \times I \to \mathbb{C}P^N$$

(N large) such that $h_0^{-1}(X_{N-1}(a)) = X_{n-1}(\mathbf{d}, a)$ and $h_1^{-1}(X_{N-1}(a)) = Y_{n-1}$, where $X_{N-1}(a)$ is a hypersurface of degree a. (h_0 and h_1 denote the restriction of h at the two component of the boundary.) By Quinn [19], we can deform h relatively to the boundary to get an almost canonical map with respect to $X_{N-1}(a)$. Set $W = h^{-1}(X_{N-1}(a))$. W is a manifold with boundary $X_{n-1}(\mathbf{d}, a)$ and Y_{n-1}, and the map $h : W \to X_{N-1}(a) \times I$ is an $(n-1)$-equivalence. We conclude that W is an $(n-2)$-connected cobordism, i.e., $H_q(W, Y_{n-1}) = H_q(W, X_{n-1}(\mathbf{d}, a)) = 0$ if $q \leq n-2$. In particular, the first homology group of the complement of W in $X_n(\mathbf{d}) \times I$ is isomorphic to $\mathbb{Z}_a$. Consider an a-fold branched covering M over $X_n(\mathbf{d}) \times I$ with the branched set W. The boundary of M is the union of $X_n(\mathbf{d}, a)$ and Y_n with opposite orientations. It is easy to show that $H_q(M, Y_n) = H_q(M, X_n(\mathbf{d}, a)) = 0$ for $q \leq n-1$. Moreover, each embedded n-sphere in M has trivial normal bundle. Applying the handle subtraction technique [13] we know that there are two integers s and t such that $Y_n \# sS^n \times S^n$ and $X_n(\mathbf{d}, a) \# tS^n \times S^n$ are h-cobordant. Thus Y_n and $X_n(\mathbf{d}, a)$ are stably diffeomorphic. This completes the proof. $\square$

Acknowledgement

This work was initiated during author's visit to Universität Mainz and was written during his visit at Max-Planck Institut für Mathematik in Bonn and IHÉS. The author would like to thank these Institutes for the hospitality and Matthias Kreck for some valuable discussions. The author wishes to thank Profs. A.Ranicki and Z. Shen, the former for his interests on this work, and the latter for his efforts to correct some problems with the english in an earlier version. Finally, he wants to thank the referee for his/her comments and suggestions for improvements.

After the author had finished this paper, Matthias Kreck informed me that, under the assumption of Corollary 1.3, he and S.Stolz can prove the complete intersections are diffeomorphic to each other.

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(Received: September 6, 1996)