4. Ab Kmil's solution.

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tions. In reality, this classification is comparable with the standardized types which had long before been set up by the Babylonians [14]. It is interesting that Al-Khwārizmī shows no evidence of acquaintance with the work of the great Greek algebraist, Diophantus [15].

4. *Abū Kāmil's solution.*

Shūja' [16] also discusses the solution of the question $x^2 + 21 = 10x$, the problem treated by al-Khwārizmī. He solves the equation algebraically in the following steps. Modern symbols have been substituted for the sake of brevity.

$$x = \frac{10}{2} - \sqrt{\left(\frac{10^2}{2}\right) - 21} = 3$$

$$x = \frac{10}{2} + \sqrt{\left(\frac{10}{2}\right)^2 - 21} = 7 .$$

The equation is also solved directly for the two values of x^2:

$$x^2 = \frac{10^2}{2} - 21 - \sqrt{\left(\frac{10^2}{2}\right)^2 - 10^2 . 21} = 9$$

$$x^2 = \frac{10^2}{2} - 21 + \sqrt{\left(\frac{10^2}{2}\right)^2 - 10^2 . 21} = 49 .$$

Then he gives the following demonstration for the equation:

" I shall explain all this. I take the number, twenty one, which is together with the square and larger than the square. I construct the square as a square surface, ABGD, and add to it the twenty one which is the surface ABHL. This surface is greater than surface ABGD. Then, because of this, line BL is greater than line BD. The surface HD equals ten of the roots of ABGD. Then line LD is ten and the surface HB is twenty one, or equal to the product of LB and BD for BD equals BA. Line LD is then divided into two halves by the point X. It had already been divided into two unequal parts by point B. Therefore, the product of LB by BD added to the square on XB is equal to the square on XD. So says Euclid in his book, part two. But the product of line XD by itself is twenty five for its length is five. Line LB times BD is twenty one as has been shown.

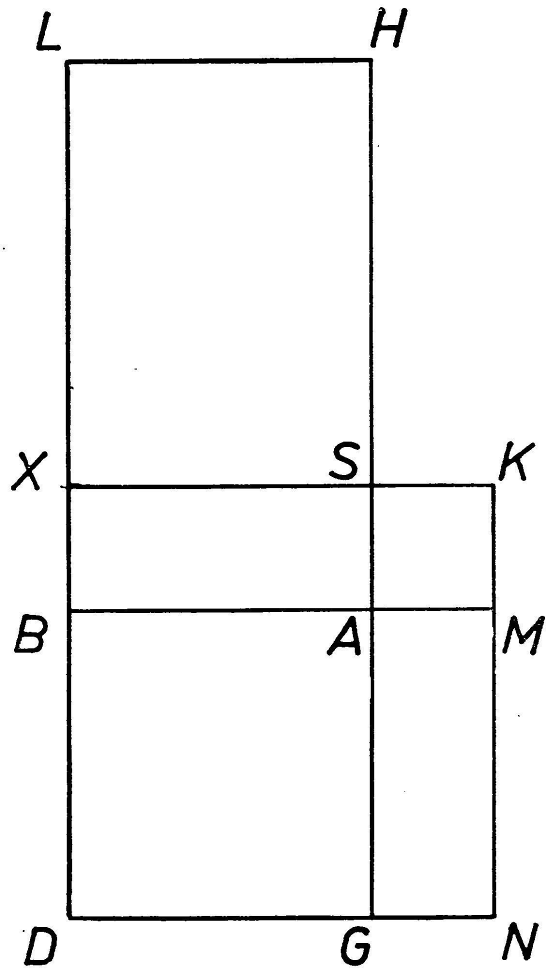

Fig. 4

Then the square on the line XB is four, and its side is two. But line XD is five. Then there remains line BD which equals three. This is the root of the square; the square is nine. If you wish that I prove what I have said, I construct a square on line XD, or surface KD equal to twenty five since line XD is five. Surface XG equals surface XH since line LX equals line XD. Surface AX equals surface AN. Therefore, the three surfaces AX, AD and AN together equal surface HB which is the product of LB by BD, or twenty one. So, surface KA remains equal to four. It is a square since line KN equals line KX, and line XS equals line NM. There remains line KS equal to line KM, equal to two. This is equal to line BX, also equal to two. Then line BD is three. This is the root of the square which is equal to nine.

" I shall explain this question. When you take half of the roots then the result of its multiplication by itself is more than the number, twenty one, that was placed with the square, and less than the square. I set the square as a square surface ABGD and add twenty one, which is the surface ABHW, to it. Surface AD is greater than surface AW as constructed, and line DB is longer than line BW.

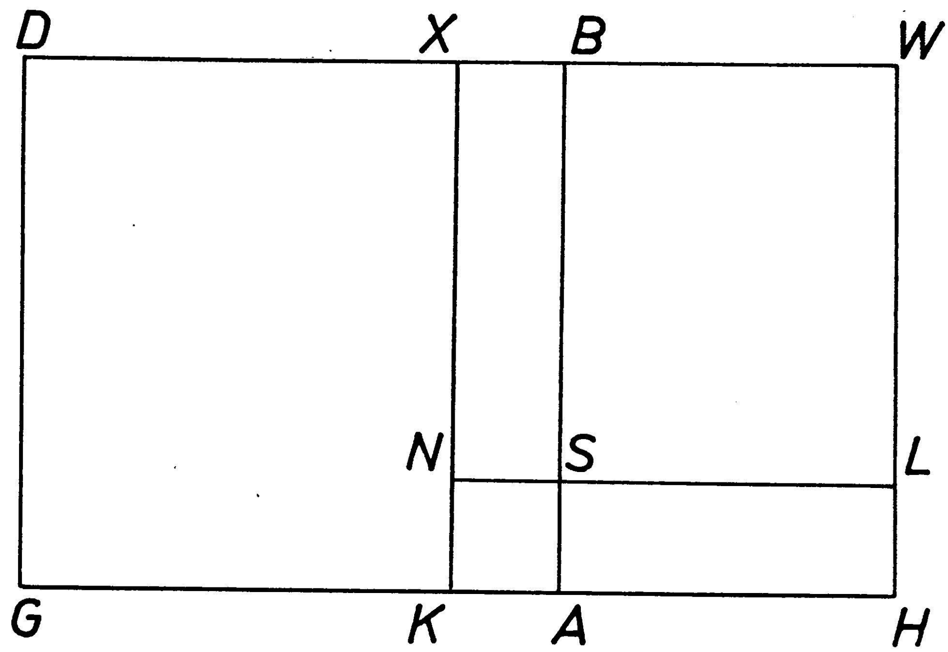

Fig. 5

The surface WG [17] equals ten of the roots of the surface AD. And so line DW is ten. The product of WB [18] by BD is twenty one. You divide line WD into two halves by the point X. Already it is divided into two unequal parts by point B. Thus, the product of WB by BD plus the product of BX by itself equals the product of XD by itself, as in Euclid's book, part two. The product of XD by itself is twenty five. The product of WB by BD is twenty one. There is left the product of line XB by itself, or four. The line XB is the root of four, or two. You add it to line XD which is five. The line BD is then seven, and it is the root of the square; the square is then forty nine. We have explained to you that when we set the square as less than the number, then we get the result by subtraction; when we set it greater than the number, then we get the result by addition.

"If you wish proof of all that we have said, you draw a square surface WN, on the line WX. Extend line XN out to point K. The surface (BN is equal to) surface NH since LN equals line NX, and line KN [19] equals line NS [20] since the surface AN is a square, as we said. Surface WN is twenty five and surface AW is twenty one. Surface AN remains as four and is a square figure since line AB equals line AG, and line KG equals BS since KG equals XD [21] and XD equals XW, and also XW equals XN, and XN equals BS. Therefore BS equals line KG, and line AK remains equal to AS. The surface AN is a square and line SN equals BX, equals two. You add it to line XD which is five to give line BD as seven. This is the root of the square which is forty nine."

Abū Kāmil then goes on with a geometric discussion of the equation $x^2 + 25 = 10x$, a special case where the square equals the number and the root of the square is equal to half of the root on the right side of the equation.

In Book II, Euclid has geometric demonstrations of algebraic formulas while, on the other hand, the works of the above mentioned Muslims are primarily algebraic with geometric explanations. It has been shown already that Greek geometry and algebra had no direct influence upon al-Khwārizmī [22]. The fact that geometric algebra is found in Euclid in such seemingly different form would tend to strengthen this idea. Moreover, on closer examination of Euclid, we find that " . . . the proofs of all the first ten propositions of Book II are practically independent of each other . . ." Heath then asks and answers the question, " what then was Euclid's intention, first in inserting some propositions not immediately required, and secondly in making the proofs of the first ten independent of each other ?

surely the object was to show the power of the method of geometrical algebra as much as to arrive at results" [23].

With the Babylonian accent on the algebraic form of geometry and the ensuing dependence of al-Khwārizmī upon this source, the latter's form of geometric algebra is fully expected. Thus, from the works of al-Khwārizmī and both Heron and Euclid, respectively representing the Babylonian and Greek forms of algebra, Abū Kāmil presented algebra on a unique level. This admits of theoretical explanation and demonstration, and provides the means of integrating Babylonian practice with Greek theory into a more virile approach [24].

C. Other examples of Abū Kāmil's methodology.

Abū Kāmil was the earliest algebraist to work out the solutions directly for the square of the unknown. In the problem quoted above he makes use of the following solutions:

$$x^2 = \frac{b^2}{2} - c - \sqrt{\left(\frac{c^2}{2}\right)^2 - b^2 c}$$

and for the second value

$$x^2 = \frac{b^2}{2} - c + \sqrt{\left(\frac{c^2}{2}\right)^2 - b^2 c}$$

The addition and substraction of [25] radicals was effected rhetorically by means of the relation now known as

$$\sqrt{a} \pm \sqrt{b} = \sqrt{a + b \pm 2\sqrt{a\,b}}$$

An example of this is given in $\sqrt{9} - \sqrt{4}$, whose solution is determined to be $\sqrt{9 + 4 - 2\sqrt{36}} = 1$ in the following [26]:

"On subtraction of roots from each other.

"When you wish to subtract (the root of) four from the root of nine so that the difference of the roots be another number, you add nine to four to give thirteen [27]. Then multiply nine by four to give thirty six [28]. Take two roots of it to give twelve. You subtract it from thirteen to get one. The root of one is the difference between the root of nine and