PRINCIPLES IN TRAINING SCHOOL MATHEMATICS TEACHERS

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by Yasuo AKIZUKI

It is most urgent at present to reform mathematics teachers training for having success in the improving of school mathematics. So I would like to express my own opinion on the matter.

At first I wish to sketch my philosophy on qualification of a good mathematics teacher. He does not need be a well studied mathematician. However he should have a general, global view on mathematics and he should have a sound and fresh mind on mathematical thinking. Especially he should realize that mathematics is a science which will never have been closed but will develop for ever. But often we find on the contrary among the so-called well experienced teachers those, who are very well in teaching mathematical techniques as a pattern of behaviour and thus lead students to be able to solve some problems of certain definite types but would lead their students to the misunderstanding that mathematics had been already completed. Such teachers can not educate so that their students may apply mathematics actively for their own new problems in future, not speaking of any inventions in the new mathematical sciences.

School teachers should have a sound knowledge on the fundamentals of mathematics; they should firmly grasp the true meaning of the fundamental concepts and the elements of methods in mathematical treatments. In learning these they need to study at the beginning various examples, illustrations and they must also solve various problems of exercises. This course of learning should be called the step of " clothing ". As usually even this step is not so easy, in many cases it is misunderstood as if this procedure is a complete course of mathematical

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education. In this case, teachers thus trained will become to push their students to remember almost everything, such and such theorems are necessary for solving such problems. Moreover this attitude had been encouraged by the demand of the so-called applied mathematicians (of old type) who estimate formulae very highly, from which everything will follow in a certain routine way, and who want to teach many, many formulae. However the process of such " clothing " is a mere first step of learning of mathematics. In the teacher training, so it is necessary to let them understand how it is important to take off the clothes after furnishing up the course of " clothing ", and further if possible, to anatomize the body. Namely one should let them classify various theorems such that which theorems are essentially important and which theorems are their immediate consequences; thus we should let them reduce their knowledge in the essence to make spare space to remember the results in their further developments. In teachers training it is essentially important, I think, to recall a phrase of the ancient philosopher: " Forgetfulness is as precious as memory ". Teachers should recognize well the fact that one should forget all superficial trivials keeping only the essentials firmly. For example for memory of almost all formulae in trigonometry it may be reduced to the single one: $(\cos x+i \sin x)(\cos y+i \sin y)=\cos(x+y)+i \sin(x+y)$. So in teachers training we should train them to be used in doing such reductions.

Thus we might lead them to critify what parts are essentially important and what parts not so. Moreover we might guide them in this way to realize the true meaning of axiomatic attitude. And also we might educate teachers to have fresh and elastic mind so that they may apply the fundamentals freely to various problems.

Teachers should keep their attitude to study by themselves even after when he will become to teach. In this case there is a danger by the very reason that he is studying. He should not be fallen in one-sided! He cannot study mathematics over all sides, he can not do an all round play even if he wishes, as any mathematicians of today cannot do so. Even under such a condition, if one tries eagerly in his own specific field to enter in

the essence, then he might find out there the nature of mathematical thoughts which is common to all the others. However if one is not so keen to get the essence but hurries up to do something socially by his obtained knowledge, then sometimes he would be fallen into a superficial, one-sided conviction, which would spoil his students very much. The aim of the study of teachers by themselves should be laid mainly on improving their knowledges more wide and more sound and making their brain fresh and active. In education it is most important to offer to students the elementary fundamental preparations for their further developments in future as more effectively as possible. The most glorious delight of teachers might be the pleasure to find later the brilliant success of their students. The study of teachers by themselves should be an accompaniment of their duty. But this accompaniment is indispensably important for educations because otherwise teachers might neither encourage nor stimulate their good students.

Even when teachers wish eagerly to study by themselves, it would be very difficult to follow up the actual developments of science and mathematics, as they are so fast and so abundant. Teachers should be supported by some organizations. In my opinion, in each country, it should be established, that every teacher should study as compulsory every year in some institutes on the fundamentals and informations of modern developments for at least 3-4 weeks.

Concerning learning of mathematics I have already written about putting on and taking off the clothes. There remains one more important point. Many educators advise in instruction to proceed gradually and smoothly rejecting any jumps. It is, of course, desirable, if it is possible. And, I consider, this would be the basic attitude for mass-education. But this might not always be possible in mathematics, I believe. If it is limited to the ordinary pattern (or type) then it would be possible, but for a problems of a quite new type it needs some jumps called as invention. We need deep concentration of thinking through long durability. Also we need activity of association by looking for in various directions. To educate such activity may we do always so smoothly? Is'nt it similar with the following case?

Namely we do not, in usual case, become angry gradually, but we endure the feeling for a while, at last suddenly the feeling will be exploded. Even in the elementary course the process of mathematical discovery might be the same in psychology. It is important to let teachers recognize this fact and let them study about the psychology of this kind besides that of how to learn it easily.

Now I would like to write about the outline of the content of the curriculum of mathematics teachers training. I think, there should be two directions, the one horizontal, the other vertical.

As the horizontal I mean the understanding of the fundamental concepts and methods. To achieve this we should have the course on basic concepts of numbers and spaces before the ordinary courses (on algebra, geometry and analysis). We should clarify the true mathematics distinguishing it to the pre-mathematics, moreover the distinction of the categorical to the non-categorical.

Taking the axioms of Peano or the Euclidian axioms we may clarify the primitive step of the axiomatic approach and the first step of mathematical method in application to sciences. Moreover by comparing the axioms of Peano to those of an integral domain (or the Euclidian to the projective) we may clarify the categorical to the non-categorical. Thus we may realize the distinction of the existence and the assumption of the existence. Concerning the good point of the non-categorical we may make it clear by showing how negative integers and positive fractions may be introduced in a unified way as the construction of a group from a semi-group. Further by introducing polynomials we may show that to solve a equation $f(x)=0$ is nothing but to construct the rest-class field modulo $f(x)$. Thus we may introduce the axiomatic method how to apply it to the raw materials and how to analyse it.

Related to the order of numbers we may introduce various concepts of topology. By discussing real numbers we may cover almost all elementary concepts of the set-theoretical topology.

Such course is very important to teachers even though it has no direct relation to the contents which they will teach. Because otherwise teachers might not understand well the distinction of

the true-mathematics and the pre-mathematics and thus the unclearness on this point might cause some confusions, in their presenting fundamental notions to students. If they understand well, then they might judge appropriately how to present them in the sense of educations.

The modern abstract concepts and methods are of course very important. Nevertheless, I believe, in education for beginners these are to be considered as the very effective frames of mathematical thinking, the substance of mathematics something else, since there may be almost everytime some intuitional images as a background, as a model of the abstract being. So it is important even at present, rather more important than the past, to train the teachers on the substance of mathematics interacting them to the frames of modern abstract thinking. The education along this line should be done especially in analysis. It should be emphasized in education how to formulate the problems exactly in our languages and how to prove them rigorously. But at the same time the vague geometrical imagination should be accompanied with the above. The rigorous discussions (or the abstract grasp) should be lead or surveyed always by geometrical insights.

As the digital calculations are very important in applications, the training in this line should not be neglected. But even in calculations it should be accompanied by the qualitative consideration, thus we may lead to the modern numerical analysis.

As the vertical I mean the understanding of the nature of mathematical thoughts and of how it has growed up. One who enters in the mountain can not look at the global sight of the mountain. One who studies only along the horizontal might be compared with the one who learns how to climb up mountains step by step. So it is not sufficient for teachers, I think, to study only along the horizontal axis.

We should give them the understanding on the historical development of mathematical thoughts and relations of mathematical thoughts with those of other sciences. One may see how mathematical thoughts have been developed. (But this does not mean at all that the order of the curriculum should be obeyed after the order of the historical development). Also one

may see how the great thought has brought on the great influences on the human culture. For example take the case of Newton. Moreover one may realize what thoughts are deeper than the other, thus the activity to critify them might be breeded.

Even though the contents in the curriculum of school mathematics do not include the result of the modern mathematics, it is desirable to let teachers understand the meaning of the modern mathematics. So it is also desirable to have a course on the development from the 19th century till the present. Through Gauss, Riemann and Galois we may realize how the spirit in the ancient greek has become rivived against mechanics only. And through Cantor and Hilbert to the present how the new mode of thinking has been developed and how it has brought on various new applications.

But here it is to be noticed that the harmony of mathematics with theoretical sciences which we found in the age of Poincaré has disappeared today and that there exists a clear distinction of pure mathematics and theoretical physics. It seems that it depends on the fact that each progress of mathematics and theoretical physics has been done rather independently and each of them was so plentiful and rapid. Moreover the distance of them has been enlarged more and more as time has gone forward as mathematical education has remained always the same traditionally. (But in Russia we may find some different situation). To remedy of such a gap should be considered as one of the important reasons why we must reform mathematical education today. So also in the training of teachers we should remind this point in every step.

Moreover, I believe, it is desirable to have nice lectures on the survey of the development of sciences and technologies of today (without entering in details) for teachers.

Mathematics is something to do, not to look at. And teachers must teach how to do. So every student for teachers, of course, must learn how to walk by his own feet. All of them must train themselves by solving the problems actually. But on my belief it would be sufficient to do such manipulation in the newly coming course for them, no necessary the special drill on the material of the lower grade, on which they should teach in

future. The reason for my claim lies in the fact that if one studies mathematics of the higher grade then he may well master that of the lower grade.

Mathematics is not something to look at. But at present usually we must make a long trip and in such a long trip we take jet-planes. So it is necessary to us to know well about the geography previously. Today it is the same in mathematics, as it is the age of electronic computers. Thus it is essentially important for us to get the perspecitivity of our program. Therefore we should let students for teachers always reflect by themselves about what, and for what aim they are studying and what they have studied.

Concerning the exercises of the practices of teaching, I am convinced of, that it should be done at first after when students have finished their full course of mathematics. In the exercises the instructors should not push them the traditional, unified method but let them be free as possible so they may act originally even in teaching. Of course there should be some basic laws in mathematics teaching in general but also there may be many different ways depending on the character of each teacher. To make precise how our brain, differently for each person, grow up and how it will become to invent something, is a precious and an interesting task as well as to develop and create new discoveries in mathematical field. This should be considered as the very task of mathematics teachers of tomorrow. This might be done first only by practices of teachers who have understood well the meaning of the true mathematics of today, I believe.

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