13. Use words correctly

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"one". It is in matters like this that common sense is most important. For what it's worth, I present here my recommendation.

Since the best expository style is the least obtrusive one, I tend nowadays to prefer the neutral approach. That does *not* mean using "one" often, or ever; sentences like "one has thus proved that ..." are awful. It does mean the complete avoidance of first person pronouns in either singular or plural. "Since p, it follows that q." "This implies p." "An application of p to q yields r." Most (all ?) mathematical writing is (should be ?) factual; simple declarative sentences are the best for communicating facts.

A frequently effective and time-saving device is the use of the imperative. "To find p, multiply q by r." "Given p, put q equal to r." (Two digressions about "given". (1) Do not use it when it means nothing. Example: "For any given p there is a q." (2) Remember that it comes from an active verb and resist the temptation to leave it dangling. Example: Not "Given p, there is a q", but "Given p, find q".)

There is nothing wrong with the editorial "we", but if you like it, do not misuse it. Let "we" mean "the author and the reader" (or "the lecturer and the audience"). Thus, it is fine to say "Using Lemma 2 we can generalize Theorem 1", or "Lemma 3 gives us a technique for proving Theorem 4". It is not good to say "Our work on this result was done in 1969" (unless the voice is that of two authors, or more, speaking in unison), and "We thank our wife for her help with the typing" is always bad.

The use of "I", and especially its overuse, sometimes has a repellent effect, as arrogance or ex-cathedra preaching, and, for that reason, I like to avoid it whenever possible. In short notes, obviously in personal historical remarks, and, perhaps, in essays such as this, it has its place.

13. Use words correctly

The next smallest units of communication, after the whole concept, the major chapters, the paragraphs, and the sentences are the words. The preceding section about pronouns was about words, in a sense, although, in a more legitimate sense, it was about global stylistic policy. What I am now going to say is not just "use words correctly"; that should go without saying. What I do mean to emphasize is the need to think about and use with care the small words of common sense and intuitive logic, and the specifically mathematical words (technical terms) that can have a profound effect on mathematical meaning.

The general rule is to use the words of logic and mathematics correctly. The emphasis, as in the case of sentence-writing, is not encouraging pedantry; I am not suggesting a proliferation of technical terms with hairline distinctions among them. Just the opposite; the emphasis is on craftsmanship so meticulous that it is not only correct, but unobtrusively so.

Here is a sample: "Prove that any complex number is the product of a non-negative number and a number of modulus 1." I have had students who would have offered the following proof: "$-4i$ is a complex number, and it is the product of 4, which is non-negative, and $-i$, which has modulus 1; q.e.d." The point is that in everyday English "any" is an ambiguous word; depending on context it may hint at an existential quantifier ("have you any wool?", "if anyone can do it, he can") or a universal one ("any number can play"). Conclusion: never use "any" in mathematical writing. Replace it by "each" or "every", or recast the whole sentence.

One way to recast the sample sentence of the preceding paragraph is to establish the convention that all "individual variables" range over the set of complex numbers and then write something like

$$\forall z \exists p \exists u\, [(p = |p|) \wedge (|u| = 1) \wedge (z = pu)].$$

I recommend against it. The symbolism of formal logic is indispensable in the discussion of the logic of mathematics, but used as a means of transmitting ideas from one mortal to another it becomes a cumbersome code. The author had to code his thoughts in it (I deny that anybody thinks in terms of $\exists$, $\forall$, $\wedge$, and the like), and the reader has to decode what the author wrote; both steps are a waste of time and an obstruction to understanding. Symbolic presentation, in the sense of either the modern logician or the classical epsilontist, is something that machines can write and few but machines can read.

So much for "any". Other offenders, charged with lesser crimes, are "where", and "equivalent", and "if ... then ... if ... then". "Where" is usually a sign of a lazy afterthought that should have been thought through before. "If n is sufficiently large, then $|a_n| < \varepsilon$, where ε is a preassigned positive number"; both disease and cure are clear. "Equivalent" *for theorems* is logical nonsense. (By "theorem" I mean a mathematical truth, something that has been proved. A meaningful statement can be false, but a theorem cannot; "a false theorem" is self-contradictory). What sense does it make to say that the completeness of L^2 is equivalent to the representation theorem for linear functionals on L^2? What is meant is that the proofs of both theorems are moderately hard, but once one of them has been proved, either one, the other can be proved with relatively much less work. The logically precise word "equivalent" is not a good word for *that*. As for "if ... then ... if ... then", that is just a frequent stylistic bobble committed by quick writers and rued by slow readers. "If p, then if q, then r." Logically all is well ($p \Rightarrow (q \Rightarrow r)$), but psychologically it is just another pebble to stumble over, unnecessarily. Usually all that is needed to avoid it is to recast the sentence, but no universally good recasting exists; what is best depends on what is important in the case at hand. It could be "If p and q, then r", or "In the presence of p, the hypothesis q implies the conclusion r", or many other versions.

14. Use technical terms correctly

The examples of mathematical diction mentioned so far were really logical matters. To illustrate the possibilities of the unobtrusive use of precise language in the everyday sense of the working mathematician, I briefly mention three examples: function, sequence, and contain.

I belong to the school that believes that functions and their values are sufficiently different that the distinction should be maintained. No fuss is necessary, or at least no visible, public fuss; just refrain from saying things like "the function $z^2 + 1$ is even". It takes a little longer to say "the function f defined by $f(z) = z^2 + 1$ is even", or, what is from many points of view preferable, "the function $z \to z^2 + 1$ is even", but it is a good habit that can sometimes save the reader (and the author) from serious blunder and that always makes for smoother reading.

"Sequence" means "function whose domain is the set of natural numbers". When an author writes "the union of a sequence of measurable sets is measurable" he is guiding the reader's attention to where it doesn't belong. The theorem has nothing to do with the firstness of the first set, the secondness of the second, and so on; the *sequence* is irrelevant. The correct statement is that "the union of a countable set of measurable sets is measurable" (or, if a different emphasis is wanted, "the union of a countably infinite set of measurable sets is measurable"). The theorem that "the limit of a sequence of measurable functions is measurable" is a very different thing; there "sequence" is correctly used. If a reader knows what a sequence is, if he feels the definition in his bones, then the misuse of the word will distract him and slow his reading down, if ever so slightly; if he doesn't really know, then the misuse will seriously postpone his ultimate understanding.