14. Use technical terms correctly

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either one, the other can be proved with relatively much less work. The logically precise word "equivalent" is not a good word for *that*. As for "if ... then ... if ... then", that is just a frequent stylistic bobble committed by quick writers and rued by slow readers. "If p, then if q, then r." Logically all is well ($p \Rightarrow (q \Rightarrow r)$), but psychologically it is just another pebble to stumble over, unnecessarily. Usually all that is needed to avoid it is to recast the sentence, but no universally good recasting exists; what is best depends on what is important in the case at hand. It could be "If p and q, then r", or "In the presence of p, the hypothesis q implies the conclusion r", or many other versions.

14. Use technical terms correctly

The examples of mathematical diction mentioned so far were really logical matters. To illustrate the possibilities of the unobtrusive use of precise language in the everyday sense of the working mathematician, I briefly mention three examples: function, sequence, and contain.

I belong to the school that believes that functions and their values are sufficiently different that the distinction should be maintained. No fuss is necessary, or at least no visible, public fuss; just refrain from saying things like "the function $z^2 + 1$ is even". It takes a little longer to say "the function f defined by $f(z) = z^2 + 1$ is even", or, what is from many points of view preferable, "the function $z \to z^2 + 1$ is even", but it is a good habit that can sometimes save the reader (and the author) from serious blunder and that always makes for smoother reading.

"Sequence" means "function whose domain is the set of natural numbers". When an author writes "the union of a sequence of measurable sets is measurable" he is guiding the reader's attention to where it doesn't belong. The theorem has nothing to do with the firstness of the first set, the secondness of the second, and so on; the *sequence* is irrelevant. The correct statement is that "the union of a countable set of measurable sets is measurable" (or, if a different emphasis is wanted, "the union of a countably infinite set of measurable sets is measurable"). The theorem that "the limit of a sequence of measurable functions is measurable" is a very different thing; there "sequence" is correctly used. If a reader knows what a sequence is, if he feels the definition in his bones, then the misuse of the word will distract him and slow his reading down, if ever so slightly; if he doesn't really know, then the misuse will seriously postpone his ultimate understanding.

"Contain" and "include" are almost always used as synonyms, often by the same people who carefully coach their students that $\in$ and $\subset$ are not the same thing at all. It is extremely unlikely that the interchangeable use of contain and include will lead to confusion. Still, some years ago I started an experiment, and I am still trying it: I have systematically and always, in spoken word and written, used "contain" for $\in$ and "include" for $\subset$. I don't say that I have proved anything by this, but I can report that (a) it is very easy to get used to, (b) it does no harm whatever, and (c) I don't think that anybody ever noticed it. I suspect, but that is not likely to be provable, that this kind of terminological consistency (with no fuss made about it) might nevertheless contribute to the reader's (and listener's) comfort.

Consistency, by the way, is a major virtue and its opposite is a cardinal sin in exposition. Consistency is important in language, in notation, in references, in typography—it is important everywhere, and its absence can cause anything from mild irritation to severe misinformation.

My advice about the use of words can be summed up as follows. (1) Avoid technical terms, and especially the creation of new ones, whenever possible. (2) Think hard about the new ones that you must create; consult Roget; and make them as appropriate as possible. (3) Use the old ones correctly and consistently, but with a minimum of obtrusive pedantry.

15. Resist symbols

Everything said about words applies, mutatis mutandis, to the even smaller units of mathematical writing, the mathematical symbols. The best notation is no notation; whenever it is possible to avoid the use of a complicated alphabetic apparatus, avoid it. A good attitude to the preparation of written mathematical exposition is to pretend that it is spoken. Pretend that you are explaining the subject to a friend on a long walk in the woods, with no paper available; fall back on symbolism only when it is really necessary.

A corollary to the principle that the less there is of notation the better it is, and in analogy with the principle of omitting irrelevant assumptions, avoid the use of irrelevant symbols. Example: "On a compact space every real-valued continuous function f is bounded." What does the symbol "f" contribute to the clarity of that statement ? Another example: "If $0 \leqq lim_n \, \alpha_n{}^{1/n} = \rho \leqq 1$, then $lim_n \, \alpha_n = 0$." What does "ρ" contribute