$$\{(1+|x|)^{-\alpha}\}_r(\lambda) \leqslant c(1+\lambda)^{-\alpha/n}, \ \lambda > 0 .$$

Combining estimates, we obtain

$$\int_{R^n} \phi^p w_1 dx \leqslant c \int_0^\infty \lambda^{-p}(1+\lambda)^{-\alpha/n} d\lambda < +\infty$$

if $1 - \frac{\alpha}{n} < p < 1$, as desired. This completes the proof of (ii).

To prove Theorem 3, let $f \in L^1_w$ and $w \in A_1$. Then (11) holds for F, p and w_1 as in the proof of Theorem 1 (ii). (The proof of (11) does not require $R_j f \in L^1_w$.) Hence, by Lemma 2 (see (8)),

$$N(F)(x) \leqslant c\left(|F(x,0)|^{\frac{n-1}{n}}\right)^{*\frac{n}{n-1}} .$$

Since $F(x,0) = \big(f(x), (R_1 f)(x), \ldots, (R_n f)(x)\big)$, the conclusion of Theorem 3 follows immediately with $\mu = (n-1)/n$.

To prove the fact stated at the end of the introduction, let

$$f, R_1 f, \ldots, R_n f \in L^1 .$$

Clearly,

$$P(R_j f)\hat{\ }(x,t) = \hat{P}(x,t)(R_j f)\hat{\ }(x) = e^{-2\pi t|x|}(R_j f)\hat{\ }(x),$$

$$(Q_j f)\hat{\ }(x,t) = \hat{Q}_j(x,t)\hat{f}(x) = i\frac{x_j}{|x|} e^{-2\pi t|x|}\hat{f}(x) \text{ a.e.},$$

where the Fourier transform is taken in the x variable with t fixed. (Note that for fixed t, $P(x,t)$ belongs to L^1 and $Q_j(x,t)$ belongs to L^2.) However, these expressions are all equal everywhere since $P(R_j f) = Q_j f$ by Theorem 1 and $P(R_j f) \in L^1$. Therefore, $(R_j f)\hat{\ }(x) = ix_j|x|^{-1}\hat{f}(x)$, as claimed.

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