$$\tilde{F} - F = \sum_{\alpha;\,\beta_1, \ldots, \beta_\alpha} p_{\beta_1,\ldots,\beta_\alpha}(x_1, \ldots, x_n)$$

$$\times\ \varphi_{\beta_1,\ldots,\beta_\alpha}(q_{\beta_1,\ldots,\beta_\alpha,1}(x_1, \ldots, x_n), \ldots, q_{\beta_1,\ldots,\beta_\alpha,k}(x_1, \ldots, x_n)) + R(x_1, \ldots, x_n),$$

where $| R(x_1, \ldots, x_n) | \leqslant \gamma(\varepsilon)\,\varepsilon$, $\gamma(\varepsilon) \to 0$ as $\varepsilon \to 0$, and

$$\varepsilon = \max_{\alpha;\,\beta_1, \ldots, \beta_\alpha} \sup_t | \varphi_{\beta_1,\ldots,\beta_\alpha}(t_1, \ldots, t_k) |$$

$$\leqslant \lambda \sup_{x \in G_n} | \tilde{F}(x_1, \ldots, x_n) - F(x_1, \ldots, x_n) |.$$

That $\gamma(\varepsilon) \to 0$ as $\varepsilon \to 0$ follows from the fact that as $\varepsilon \to 0$ the quantity

$$\varepsilon' = \max_{\alpha;\,\beta_1, \ldots, \beta_\alpha} \sum_{i=1}^{k} \sup \left| \frac{\partial \varphi_{\beta_1,\ldots,\beta_\alpha}(t_1, \ldots, t_k)}{\partial t_i} \right| \to 0,$$

provided only that the modulus of continuity of the partial derivatives of the functions $\{ \varphi_{\beta_1,\ldots,\beta_\alpha}(t_1, \ldots, t_k) \}$ is fixed. By 5.1.10 it follows that $r(A, F) \leqslant k$ in some subregion $G_n \subset D_n$. So we have obtained a contradiction to the assumption that $r(A, F) > k$ in any subregion $G_n \subset D_n$ and this proves the theorem.

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