

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 23 (1977)  
**Heft:** 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** SOME CONTRIBUTIONS OF BENO ECKMANN TO THE DEVELOPMENT OF TOPOLOGY AND RELATED FIELDS  
**Autor:** Hilton, Peter  
**Kapitel:** Introduction  
**DOI:** <https://doi.org/10.5169/seals-48926>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 02.07.2025

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

# SOME CONTRIBUTIONS OF BENO ECKMANN TO THE DEVELOPMENT OF TOPOLOGY AND RELATED FIELDS<sup>1</sup>

by Peter HILTON

## INTRODUCTION

The work of Beno Eckmann in topology and related fields, extending over a period of 35 years and continuing today just as actively as in those early years, is marked by certain characteristic features which I will attempt to describe in this introduction and to illustrate by selections from his published work in the later sections of my talk.

If one is to look for the distinctive aspects of Eckmann's contributions to mathematics, one might attempt to summarize them as *unification*, *clarification* and *penetration*. Eckmann has shown quite unique discernment in identifying and developing the relationships between different parts of mathematics; in particular, between algebraic topology on the one hand and linear algebra, homological algebra, group theory and geometry on the other. Simply to say that Eckmann has developed links between algebraic topology and homological algebra is of course to understate the magnitude of his contribution in this area. As a founder of homological algebra, he has helped to forge the fundamental tools of the subject. However, Eckmann's contribution in this direction will be discussed by Saunders MacLane in his talk at this Congress, so that I will do no more than pay tribute to the decisive positive influence that Eckmann has had on the development of this important branch of algebra. I will myself be choosing some examples from Eckmann's publications to illustrate the relationships which he has observed and studied between algebraic topology and the other subjects which I have listed above.

I would like to add that Eckmann has always seen category theory as a means of unification within mathematics. He was one of the earliest contributors to the development of a *point of view* about mathematics which is now so commonplace that young mathematicians find it difficult to believe that it was certainly not obvious to the mathematicians of twenty years ago

---

<sup>1</sup>) Presented at the Colloquium on Topology and Algebra, Zurich, April 1977.

who had not come under the influence of Sammy Eilenberg or Saunders MacLane. Moreover, it is possible that those same young mathematicians perusing the literature may not have fully appreciated the significance of Eckmann's role in establishing the point of view to which I refer. Eckmann is not a professional categorist; on the other hand the unqualified benefit of a categorical point of view has been clear to him from his earliest work on group cohomology in 1945, and he has moreover encouraged the development, and the broadening and deepening, of this point of view by inviting to the research institute here in Zürich active exponents of it.

Eckmann achieves clarification primarily by the limpid style of his writing. In both his writing and his lecturing, he follows in the footsteps of his own great teacher Heinz Hopf and himself constitutes a model for his many students. It is difficult, if not impossible, by merely summarizing his work, to demonstrate the clarity of his presentation. There is surely no substitute for reading his own papers. However, it is possible, by taking examples from his published work, to illustrate how very often Eckmann shows what really lies behind an argument or a concept by stripping away much of the superfluous technical detail. It is often the case that an argument in mathematics compels acceptance without really enabling the reader to understand why the statement is true. (The "reader" may even be the author of the argument himself!) Eckmann's own arguments, expressed in his particularly felicitous style of writing, are never of this kind, and, frequently, his papers have been devoted precisely to the clarification of an existing theory and its establishment in an appropriate mathematical context.

It is also characteristic of Eckmann to return to the topics of earlier work in order to demonstrate progress made and the relevance of new tools and techniques to the solution of classical problems. Examples of this significant feature of his work will also be given below.

Of the penetrating nature of Eckmann's work, it is surely unnecessary for me to say a great deal. We would not be gathered here today at this congress to do him honor were it not clear to all of us that his contributions have had a profound effect on the development of our subject. But I would also wish to include under this rubric the penetration of Eckmann's understanding as evidenced by the facility he has to grasp, comprehensively, the significance of new ideas introduced into mathematics.

I find myself in some difficulty in addressing the very congenial task which the organizers of this congress have laid upon me. For, ever since the late 1950's, I have been very closely engaged in joint research with Beno Eckmann. Indeed, over the 15-year period from 1958 to 1973, Eckmann

published 37 papers of which 24 were joint papers with myself (and sometimes with a third collaborator, too, principally Urs Stammbach). This long and fruitful collaboration is of course my own adequate testimony to the high regard in which I have always held my good friend Beno Eckmann. But it would perhaps contradict certain canons of good taste if I were to cite our joint work in evidence of the depth of penetration of Eckmann's mathematical insights. Let me therefore only say of that work that I regard my collaboration with Eckmann, and my previous apprenticeship as a student of Henry Whitehead, as the two principal formative elements in my own mathematical growth and maturity. I would only wish to add a reference to the gratification which Eckmann and I felt that a *leit motif* of our joint research, the heuristic duality which we uncovered at the heart of homotopy theory and exploited, received recognition from Norman Steenrod in his listing of principal themes of algebraic topology.

I have said that Eckmann remains as active in mathematical research as ever. This is a source of great delight to us gathered here for this congress, as also for the many mathematicians, all over the world, who derive benefit from his contributions to the progress of our subject. For it is not enough to say that Eckmann remains active; he remains effective, discriminating and entirely contemporary. His most recent work, with Robert Bieri, on Poincaré duality groups and a certain natural generalization of such groups, of which you may hear from Bieri at this congress, exhibits all the qualities to which I have already referred. It is a remarkable tribute to Beno Eckmann that one may say of him that today, at the age of 60, he is still doing his best work.

But, as I feel sure Beno would himself agree, we have had enough of generalities—it is time to get down to some mathematics!

## 1. CONTINUOUS SOLUTIONS OF SYSTEMS OF LINEAR EQUATIONS

In [8; 1943] Eckmann considered the following problem. Suppose given a system of  $r$  linear equations in  $n$  unknowns,  $r < n$ ,

$$(1.1) \quad \sum_{k=1}^n a_{ik} x_k = 0, \quad i = 1, 2, \dots, r < n,$$

where the coefficients  $a_{ik}$  are continuous real-valued <sup>1)</sup> functions of a variable  $u$  which describes some topological space  $U$ , which will usually

<sup>1)</sup> Eckmann had considered the corresponding problem in the complex case in [3; 1942].