3. How?

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because of progress in the other disciplines (for example, the study of such complex phenomena as polymers and imperfect crystals). Here are a few specific questions.

2.7.1. What is the essential basic algebra and analysis which we should like all students to know? What can be acquired at a school level? What must wait until university?

2.7.2. What are the 'traditional' subjects which have been given new life by the computer and today's applications? A typical example arises from differential equations. "Special functions" are now scarcely taught to mathematicians, yet one finds them in the syllabus for chemistry students at Jadavpur. Does the role of symmetry in Physics and Chemistry suggest a place for 'classical groups and special functions'?

2.7.3. What geometry should be included? (The geologists at Budapest still hold on to traditional elementary geometry and descriptive geometry. Solid-state physicists and chemists are interested in polyhedra. Everywhere there are demands for geometric interpretations. Is there a case for introducing fractals and the corresponding mathematics (Weierstrass, Cantor, von Koch, Hausdorff...)?).

2.7.4. What is the place of statistics and probability? Should these be introduced piecemeal as needs arise, or presented as a structured course? The response may differ in, say, Physics, Biology and Economics. There have also been interesting experiments over some years in medical education.

2.7.5. What is the appropriate mathematics for computer scientists and who should teach it? Wouldn't its algebra, algorithmics and finite mathematics be equally appropriate for other students?

2.7.6. Several institutions now list 'operational research' as part of the mathematics syllabus. How should this be interpreted? Is OR, in fact, a part of mathematics or rather an independent (as yet minor) discipline which should itself be seen as being served by mathematics.

2.7.7. Extreme positions are expressed on certain topics for engineers, for example, Schwartz distributions: useless? Indispensable?

2.7.8. Is the teaching of mathematical modelling — 'a necessity' (Jadavpur) or 'a beautiful dream' (Budapest)?

3. How?

In the best possible way. And it could be argued that once it has been decided what should be taught and who should teach it, then it is a matter to be determined solely by the individuals concerned. There are, however, many general points which merit particular consideration.

3.1. *Statements and Proofs*

There can be no justification for giving statements which are incorrect, for example, for stating — or suggesting — that the Fourier series of a continuous function converges uniformly to that function. Yet there will be times when the teacher wishes to make statements because they are simple and correct in a convenient frame. For example, an integrable function on R tends to zero at infinity (in the sense of distributions). Each function on R is Lebesgue-measurable (in a model of set-theory which excludes the axiom of choice). Each part of a probability space is an event (in the same model). An essential point is to make useful statements in the most primitive possible language.

The choice of good definitions and statements is the work of a mathematician, but one in which non-mathematicians can usefully participate. It must also be recognised that there is nothing sacrosanct about the order in which material is presented. For example, it is not forbidden to define the rotation (curl) of a vector field starting out from a physical interpretation of Stokes' Theorem (Berkeley Physics Course), rather than from the usual operator definition in terms of derivatives: the theorem can precede the definition or vice-versa.

In a course given to mathematicians the guarantee of exactitude and of cohesion is the chain of logical argument, proof. In a service course then sometimes one must replace proof (too long, non-illuminating) by other arguments, and develop, for example, what George Polyá termed 'plausible reasoning'. Good physical illustrations can be more enlightening and impressive than proofs: depressing the sustaining key on a piano, saying 'ohh' to the strings, and hearing the response 'ohhhh' is an excellent gateway to spectral analysis and synthesis (Berkeley, *Waves,* p. 91).

On the other hand, exploratory work and verification on a computer can give certain mathematical statements the status of 'experimental' truths. Mathematical rigour consists in distinguishing between mathematical proof and experimental verification — this distinction must not become blurred.

3.2. *Examples and concepts*

Must one begin with examples and from these derive the concepts, or should one start off with the concepts and flesh these out with examples? This is an old question. Should one restrict oneself to examples drawn from the major discipline? Advice varies and depends upon many external constraints, in particular, the time available and class size.

One possibility merits special attention: this is the introduction of exploratory data analysis at the beginning of university studies. Manipulation can be done without any great theoretical apparatus; in addition, important motivation can be provided for the study of linear algebra and probability.

In general, the relations between examples, concepts and intuition generate major pedagogical questions. The great unifying concepts (groups, measure) are not accessible, despite their apparent simplicity, unless they are supported with numerous illustrations and examples. This is true of all mathematics teaching, but unfortunately within service teaching students are not provided with the time in which such notions can become familiar and intuitive.

3.3. *Small or large classes?*

Generally, the response to this question depends almost entirely on local resources: large groups demand fewer teachers. There are clear administrative advantages in the tradition of teaching service mathematics to large groups, often drawn from several different departments: economies of preparation of both lectures and exercises, and the possibility of employing only such lecturers as have a direct interest in service teaching and who, over the years, amass experience concerning likely points of difficulty, general needs etc. The disadvantages include the lack of motivation for the students, the restrictions placed on the kind of learning activities which can be offered, and the impossibility of setting a common examination which matches the real needs and strengths of students drawn from a range of departments. (We note, however, that at Eindhoven, even though the 'large group' format has been retained, this has not prevented the introduction of a novel course which depends upon each student having his own programmable pocket computer).

The question is also bound to that of 'who teaches?' The case for having a large inhomogeneous class taught by a mathematician is very strong. Small groups, on the other hand, are better able to utilise exercises and examples which draw on their major disciplines.

Even with any one discipline, however, first-year students are likely to differ very greatly in their mathematical attainments and abilities. This creates difficulties for the lecturer and forces consideration of other methods of teaching and learning. For such reasons, we should like our study to pay particular attention to experiments which have been made to help resolve such pedagogical problems. We note, for example, that at Southampton first-year engineers follow an individualised, 'self-paced' course based on reading (with frequent testing) rather than lectures.

3.4. *The 'Ideal' situation*

Subject to the various constraints which have to be met, what patterns of service teaching are giving rise to local satisfaction? We have already referred

(Section 2.5.1. above), to the way in which mathematics is taught to physicists at Cardiff. Here we give other examples of situations considered 'ideal'.

At Southampton, the course for chemists is given by a mathematician but each student, together with three or four others, is seen fortnightly by a chemist who will give tutorial supervision using material and example sheets supplied by the mathematician.[1]) A similar system has operated for some years in the Physics department at Orsay to general satisfaction: the lectures to the whole class being given by a mathematician, directed work (to groups of 20 students) by physicists.

At Paris-Grignon, a 20-hour course for third-year students of Agriculture was mounted in the form of a dialogue between an economist and a mathematician, thus providing the framework for an effective investigation. Such 'team-teaching' is very motivating for students, but is very expensive in preparation time.

No doubt other 'ideal' situations having different characteristics can be found. Detailed descriptions of them would be extremely welcome.

3.5. *The use of computers*

As was written above, the impact of computers on the teaching of mathematics has already been the subject of an ICMI study. It is essential that we reflect on all the new possibilities offered by computers (rapid computation, graphics, experimentation) and on the changing needs caused by their introduction (changes both of curricular content and also of desirable qualities to be developed in students.

A feature of the reports we received was the limited use of computers in the teaching of those subjects which have traditionally made heavy use of mathematics.

3.6. *The use of books and papers*

Here there are two aspects. First, for service teaching it is good to use texts written collaboratively by mathematicians and specialists in the major disciplines. Such books do exist, but there are many gaps. It would be valuable to have the characteristics of the successful texts, and also the lacunae, described.

Secondly, as we have already stressed, students must learn how to read mathematics, both in order to learn more mathematics when there is no lecturer

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[1]) One chemist wrote of this arrangement: 'many of my colleagues agree with me that in Southampton Chemistry we have the ideal situation as far as academic considerations are concerned. In tutorials the chemists can relate the material covered to Chemistry, point out the relevance to the Chemistry course and (it is hoped) provide some motivation'.

to hand, and also to understand their professional literature. Descriptions of 'planned' reading tasks are not numerous, but appear of interest and potential value (e.g. readings of extracts from Laplace for students at the Paris Ecole des Ponts et Chaussées, a chapter of Volterra for biologists at Orsay).

3.7. *Examinations, assessment and control*

In many cases examinations supply the principal motivation for students (although, as we have indicated in Section 2.5.1., this need not necessarily be the case). If the examination is outside the lecturer's control (as in Florida, and even more in the preparatory classes for the 'grandes écoles' in France), then it also provides motivation for him. Therefore, the questions 'Why?' and 'How?' should not be asked of teaching alone, but must also be asked of evaluation and assessment. If the teaching of mathematical modelling is a primary goal, then this goal is unlikely to be attained, if all that is required to pass the examination is memory of a ragbag of techniques applied in stock, purely mathematical situations. On the whole examinations tend to freeze courses, and militate against such innovations as, for example, the introduction of computers, mathematical modelling, and 'planned' reading. On the other hand, all of these innovations can be effectively examined, and examples can be given. However, their assessment is extremely time-consuming and the large numbers of students involved in service courses present particular difficulties.

How, then are we to use examinations and assessment as a means for *improving* teaching and learning? What desirable changes can be made to entrance examinations or to national examinations? Are there forms of continuous assessment which enable teachers/students to monitor the assimilation of the mathematics they teach/learn? Can this be done within the short time allocated to service teaching? Are there still examinations which contribute little and might be better abandoned? Examples of good practice will be welcomed.

4. CALL FOR PAPERS

In this discussion document is has been possible only briefly to indicate some questions of great interest and concern. The next step is to take a selection of these and to delve into them more deeply, to flesh arguments out with examples taken from current practice, to examine philosophical and pedagogical points more critically, to report the results of relevant research. The planning committee for the study would very much welcome papers which so develop points made in this paper, and which, in their turn, could form the bases of discussions in Udine in April, 1987. Such papers would be welcomed from all concerned with service teaching, mathematicians, specialists in other disciplines, students, recent students and employers.