1. Introduction

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INVOLUTIONS IN SURFACE MAPPING CLASS GROUPS

by John McCarthy and Athanase Papadopoulos

1. Introduction

Let F be a compact orientable surface with negative Euler characteristic. A *mapping class* is an isotopy class of orientation-preserving homeomorphisms of F. Mapping classes form a group under composition, the *mapping class group*, $M(F)$. An element of order two of the mapping class group will be called an involution. In this article, we prove two theorems about products of involutions.

The first theorem is group-theoretical. We assume that the surface F is closed and we study the subgroup of $M(F)$ which is generated by involutions. In particular, for closed surfaces of genus greater than or equal to three, we prove that the mapping class group is generated by involutions.

The second theorem is geometric. There is a classification, into 3 types, of mapping classes, which is due to Thurston. This theorem is about the type of the product of two involutions. As is natural in this setting, the theorem and its proof are in terms of the action of the mapping class group on Teichmüller space and its Thurston boundary.

The second theorem is analogous in nature to the following elementary facts about the product of order-two elements of a discrete group of isometries of hyperbolic 2 or 3-space:

(i) the product is an elliptic isometry if and only if the two order-2 isometries have a common fixed-point in hyperbolic space.

(ii) if the two order-2 isometries have no common fixed point in hyperbolic space and have a common fixed point on the boundary at infinity, their product is a parabolic isometry.

(iii) if the two isometries do not have any common fixed point, neither in hyperbolic space nor on the boundary at infinity, their product is of hyperbolic type.

In section 2, we prove the group theoretical result. Then, in section 3, we give an outline of some of the background material on Thurston's classification. For a complete exposition of Thurston's theory, we refer the reader to [4], and for more information on Teichmüller space, to [1]. Finally, in section 4, we prove the theorem on the type of the product of two involutions.

The problem of studying the types of products of involutions in the mapping class group was suggested to the second author by François Laudenbach. The theorem on the subgroup generated by involutions arose out of an attempt to obtain more precise information about the mapping classes which occur as products of two involutions.

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2. The subgroup generated by involutions

Let $M(F_g)$ denote the mapping class group of a closed orientable surface F_g of genus $g \geqslant 2$. Let $I(F_g)$ denote the subgroup of $M(F_g)$ which is generated by involutions. We wish to describe $I(F_g)$ as a subgroup of $M(F_g)$. Clearly $I(F_g)$ is a normal subgroup of $M(F_g)$. Hence, we shall give our description in terms of the quotient, $M(F_g)/I(F_g)$.

We begin by recalling some algebraic facts about $M(F_g)$. For a general introduction to the algebraic structure of $M(F_g)$, we refer the reader to [2].

It is a classical fact that $M(F_g)$ is generated by Dehn twists about nonseparating simple closed curves on M. In fact, $M(F_g)$ is generated by a finite number of such twists. The minimum number of twists which is required to generate $M(F_g)$ was given by Humphries [5].

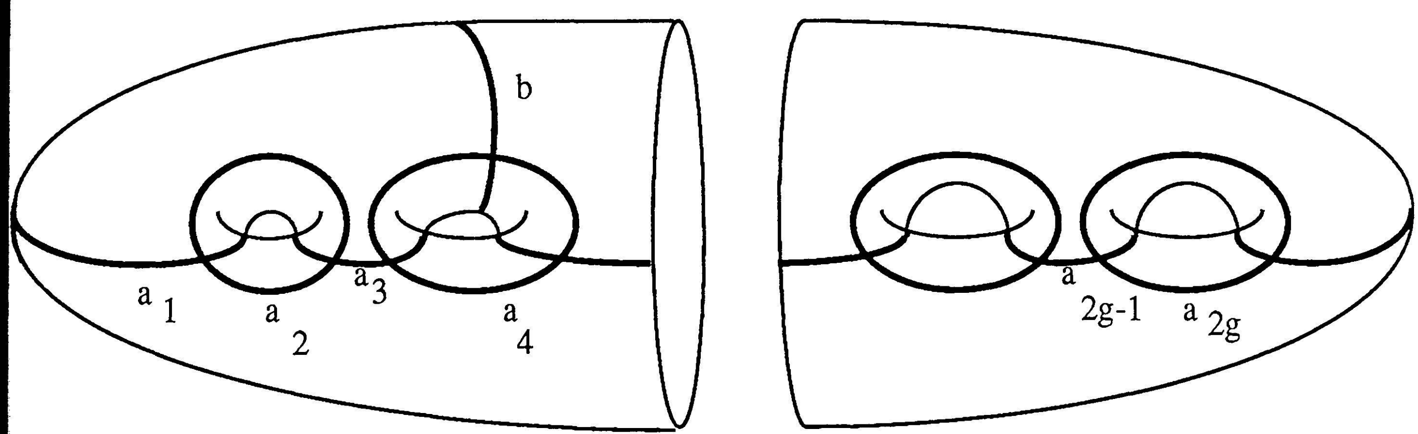

Figure 1.