VIII. Applications and questions

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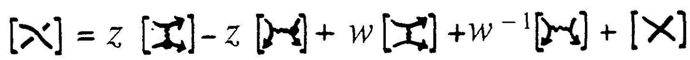
A particularly nice model occurs for index set in the form

$$\{-n, -n+2, \ldots, n-2, n\} \quad \text{with} \quad \lambda = w\,.$$

This gives a series of one-variable specializations of the Homfly polynomial. (See [40], [58], [93].) *Is there a Yang-Baxter model for the full Homfly polynomial?* This is an open question.

A similar approach works for the Dubrovnik form of the Kauffman polynomial. See [58], [93]. The expansion formula has the appearance.



(It is understood that reversing the orientation of a line is accompanied by the negation of its spin.) Once again, the dot on a line means that it has smaller spin.

VIII. Applications and questions

This section is devoted to a few applications of the skein and state models and related questions.

1. Let ∇_K denote the Conway polynomial. The skein model is embodied in the formula of section 6:

$$\nabla_K = \sum_{L,\,|L|=1} (-1)^{t-(L)} z^{t(L)}$$

from which we see easily that

$$\max \deg \nabla_K \leqslant V - S + 1 = \rho(K)$$

where V is the number of crossings in the diagram K, S is the number of Seifert circuits (the set of circuits obtained by splicing all crossings of K). On knows that $\rho(K) = \operatorname{rank}(H_1(F))$ where F is the Seifert spanning surface [42] corresponding to the diagram K. If K is an alternating link then $\max \deg \nabla_K = \rho(K)$ [76]. This is generalized to the class of alternative links in [42], using the FKT model. Is there a proof using the skein model? [1])

In the case where all the crossings are of positive type, we see from the skein model that all terms of ∇_K are positive, and it is then easy to see that the highest degree term is of degree $\rho(K)$.

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[1]) *Note added in proof:* A proof using the skein model for the theorem on alternative links has been found by John Mathias — University of Maryland, May 1989.

2. Similar remarks apply to the Homfly model of section 3. In the case of the Yang-Baxter model for the Homfly polynomial given in section 7, it is easy to see that the highest z-degree is $\rho(K)$ when K is positive — this time by constructing an appropriate spin state.

3. Thistlethwaite [89] proves that the writhe $w(K)$ is an ambient isotopy invariant for K alternating and reduced. It would be useful to see a proof of this result using the skein model for D_K (section 4).

4. The Alexander polynomial Δ_K is given by the formula

$$\begin{aligned}\Delta_K(t) &\doteq \nabla_K(\sqrt{t}-1/\sqrt{t}) \\ &= \sum_{|L|=1} (-1)^{t-(L)}(\sqrt{t}-1/\sqrt{t})^{t(L)}\end{aligned}$$

where $\doteq$ denotes equality up to sign and powers of t. One knows ([23]) that if K bounds a smooth disk in the upper 4-space $((x, y, z, t)$ with $t>0)$ then

$$\Delta_K(t) \doteq f(t)f(t^{-1})$$

for some polynomial $f(t)$. Can this fact be deduced directly from the skein model or from the FKT model? A solution should generalize to give new information about the full skein polynomial behaviours on slice links.

IX. Relations with mathematical physics

I have deliberately included a description of the Yang-Baxter models in this paper in order to raise the question of the relation of the skein models to mathematical physics. The Yang-Baxter models can be regarded as averages of scattering amplitudes over all possible spin states — hence as discrete Feynman integrals, or as partition functions for two-dimensional statistical mechanics models. The FKT model for the Conway polynomial can be seen [57] as the low temperature limit of a partition function of a generalized Potts model.

Meta-time

If we interpret the FKT model or the skein models in a particle interaction framework, then a curious and interesting issue arises:

Think of a particle moving forward and backward in "time" on a given universe. The "same" particle may traverse a given site (crossing) twice.