a normal subgroup of H then $D(\gamma)$ is in this subgroup (note that there are finitely many such normal subgroups).

Since $D(\gamma)$ has infinite order (if γ is non-trivial), $\langle D(\gamma)\rangle$ has positive dimension so that it contains a non-trivial one-parameter group. Hence every non trivial semi-simple element in $D(\Gamma)$ yields a one-parameter group contained in H_0. We now show that these one-parameter subgroups generate the connected component of the identity in H. Observe the following elementary fact: if a family of vectors spans the Lie algebra of a Lie group, then the one-parameter groups generated by these vectors generate the connected component of the identity. Therefore, we consider the linear span $\mathfrak{C}$ in the Lie algebra $\mathfrak{H}$ of H of the Lie algebras of all the subgroups $\langle D(\gamma)\rangle$ for γ semi-simple. It is enough to show that $\mathfrak{C} = \mathfrak{H}$. Note that $\mathfrak{C}$ is certainly non-trivial since semi-simple elements are Zariski dense in H. Note also that $\mathfrak{C}$ is invariant under the adjoint action of $D(\Gamma)$, hence under the adjoint action of H since $D(\Gamma)$ is Zariski dense in H. It follows that $\mathfrak{C}$ coincides with the product of some of the simple factors of $\mathfrak{H}$. The only possibility is that $\mathfrak{C} = \mathfrak{H}$ since otherwise, all the semi-simple $D(\gamma)$ would have some power contained in the same product of some but not all of the simple factors of H (note that the algebraic Abelian group $\langle D(\gamma)\rangle$ has a finite number of connected components). This implies that all semi-simple elements of $D(\Gamma)$ are contained in some non trivial normal subgroup of H. This is not possible by the following argument. In the algebraic group H, there is a non-empty open Zariski set consisting of semi-simple elements which are not contained in any non-trivial normal subgroup of H. Since $D(\Gamma)$ is Zariski dense in H, it intersects non-trivially this open set.

It follows that H_0 contains the connected component of the identity of H. Therefore H_0 is a semi-simple Lie group of finite index in H. By Kushnirenko's theorem, we can analytically linearize $\phi(H)$ (one also uses Remark 2.1) and in particular $\Phi(\Gamma)$.

Theorem 10.4 is proved.

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