have a common fixed point $\begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$ in $\mathbf{R}^2$ and the subgroup of $SL_2(\mathbf{Z})$ generated by

$$\begin{pmatrix} 13 & 22 \\ 10 & 17 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 17 & 10 \\ 22 & 13 \end{pmatrix}$$

is free of rank 2 and all its elements distinct from the identity are hyperbolic. See [K] and the following calculations:

$$\begin{pmatrix} 13 & 22 \\ 10 & 17 \end{pmatrix} = tu(tu^{-1})^3(tu)^2tu^{-1}tu\,,$$

$$\begin{pmatrix} 17 & 10 \\ 22 & 13 \end{pmatrix} = tu^{-1}(tu)^3(tu^{-1})^2tutu^{-1}\,,$$

where

$$t = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad u = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$$

with $\langle t, u\rangle / \{\pm 1\} \cong \mathbf{Z}_2 * \mathbf{Z}_3$.

The referee suggested to the author that the following could be shown (without the axiom of choice).

COROLLARY. *There exists a subset E_1 of $\mathbf{Z}^2$ such that, for every finite subset F of $\mathbf{Z}^2$, the symmetric difference of E_1 and F is congruent to E_1 relative to the group* $SA_2(\mathbf{Z})$.

Proof. This is a consequence of our main result and of Theorem 2 in [My] ($S = \mathbf{Z}^2$, $G = \langle \zeta, \eta \rangle$, $M = \{\zeta\eta, \zeta^2\eta^2, \zeta^3\eta^3, \ldots\}$, $\mathbf{F} = \{F \subseteq \mathbf{Z}^2 \mid F \text{ is finite}\}$). □

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