via the "asymptotic stretch" of $f_*: \pi_1 \to \pi_1$ with respect to the word metric in π_1 (see [Bow]). But this is not sharp even for linear automorphisms of tori T^n, where the entropy is expressed by the "k-dimensional stretch" on H_1 for some $k \leq n$ that equals the spectral radius of f_* on H_k. Such k-stretch can be defined, in general, in terms of $f_*: \pi_1(S) \to \pi_1(S)$ and the classifying map $S \to K(\pi_1, 1)$ (refining the spectral radius of f_* on H^k coming from $K(\pi_1, 1)$), but my obvious "proof" of the lower bound on the entropy by this k-stretch missed a hidden trap. This was also overlooked in [Ma$_3$] (for $f_*: H_1 \to H_1$, where a proper identification of the "k-stretch" with the spectral radius needs extra work), as was pointed out to me much later by David Fried. (The difficulty already appears for closed subsets S in the torus T^n invariant under linear automorphisms f of T^n, where one wishes to estimate the entropy of $f|S$ in terms of f_* acting on the *spectral* cohomology of S coming from T^n. On the other hand, the case of $T^n \to T^n$ is settled in [Mi-Pr].)

APPENDIX 6. Probably, the recent progress in Nielsen theory allows a description of the cases, where card(Fix$f$) is well controlled from below by some twisted Lefschetz number (see [Fel]).

APPENDIX 7. Nothing interesting to say.

APPENDIX 8. Minima of geometric functionals related to the logical complexity have been studied in depth by A. Nabutovski (see [Na] and references therein). Yet I do not feel ready yet to write this Appendix. For example, I do not see what is the actual influence of a suitable (?) complexity measure of $\pi_1(V)$ on the Plateau problem in V.

BIBLIOGRAPHY

[B-G-S] BALLMANN, W., M. GROMOV and V. SCHROEDER. *Manifolds of Non-Positive Curvature.* Birkhäuser, Boston, 1985.

[Bow] BOWEN, R. Entropy and the fundamental group. In: *The Structure of Attractors in Dynamical Systems. (Proc. Conf., North Dakota State Univ., Fargo, N.D. (1977))*, 21–29. Lecture Notes in Math., 668, Springer, Berlin, 1978.

[Br-Ha] BRIDSON, M. R. and A. HAEFLIGER. *Metric Spaces of Non-Positive Curvature.* Grundlehren der mathematischen Wissenschaften, Vol. 319. Springer, Berlin, 1999.

[Din] DINABURG, E. I. A connection between various entropy characterizations of dynamical systems (Russian). *Izv. Akad. Nauk SSSR Ser. Mat. 35* (1971), 324–366.

[Fa-Jo] FARRELL, F. T. and L. E. JONES. A topological analogue of Mostow's rigidity theorem. *J. Amer. Math. Soc. 2* (1989), 257–370.

[Fel] FEL′SHTYN, A. Dynamical zeta functions, Nielsen theory and Reidemeister torsion. *Mem. Amer. Math. Soc. 147* (2000), no. 699.

[Gro$_1$] GROMOV, M. Hyperbolic manifolds, groups and actions. In: *Riemann Surfaces and Related Topics (Proc. of the 1978 Stony Brook Conference), Ann. Math. Studies 97* (1981), 183–213, Princeton University Press, Princeton.

[Gro$_2$] —— Positive curvature, macroscopic dimension, spectral gaps and higher signatures. In: *Functional Analysis on the Eve of the 21st Century.* Vol. II (In honor of the eightieth birthday of I. M. Gel′fand. *Proc. conf. Rutgers Univ., 1993.*) 1–213, Birkhäuser, Basel, 1996.

[Ma$_1$] MANNING, A. Topological entropy and the first homology group. In: *Dynamical Systems – Warwick 1974* (Proc. Sympos. Appl. Topology and Dynamical Systems, Univ. Warwick, Coventry, 1973/1974; presented to E. C. Zeeman on his fiftieth birthday), 185–190. Lecture Notes in Math., 468, Springer, Berlin, 1975.

[Ma$_2$] —— Topological entropy for geodesic flows. *Ann. of Math. (2) 110* (1979), 567–573.

[Ma$_3$] —— Toral automorphisms, topological entropy and the fundamental group. In: *Dynamical Systems, Vol. II – Warsaw*, 273–281. Astérisque *50*, 1977.

[Mi-Pr] MISIUREWICZ, M. and F. PRZYTYCKI. Entropy conjecture for tori. *Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. 25* (1977), 575–578.

[Na] NABUTOVSKY, A. Disconnectedness of sublevel sets of some Riemannian functionals. *Geom. Funct. Anal. 6* (1996), 703–725.

Mikhaïl Gromov

Institut des Hautes Études Scientifiques
35, route de Chartres
F-91440 Bures-sur-Yvette
France