1. Introduction

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TORSION NUMBERS OF AUGMENTED GROUPS WITH APPLICATIONS TO KNOTS AND LINKS

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Dedicated to the memory of Arnold E. Ross

ABSTRACT. Torsion and Betti numbers for knots are special cases of more general invariants b_r and β_r, respectively, associated to a finitely generated group G and epimorphism $\chi\colon G \to \mathbf{Z}$. The sequence of Betti numbers is always periodic; under mild hypotheses about (G,χ), the sequence b_r satisfies a linear homogeneous recurrence relation with constant coefficients. Generally, b_r exhibits exponential growth rate. However, again under mild hypotheses, the p-part of b_r has trivial growth for any prime p. Applications to branched cover homology for knots and links are presented.

1. INTRODUCTION

A *knot* is a simple closed curve in the 3-sphere S^3. Knots are *equivalent* if there is an orientation-preserving homeomorphism of S^3 that carries one into the other. Equivalent knots are regarded as the same. An *invariant* is a well-defined quantity that depends only on a knot equivalence class. Two knots for which some invariant differs are necessarily distinct.

Associated to any knot k and natural number r there is a compact, oriented 3-manifold M_r, the r-fold cyclic cover of S^3 branched over k. A precise definition can be found in [Li97] or [Ro76], for example. Topological invariants of M_r are invariants of k. Two such invariants, the first Betti number β_r and the order b_r of the torsion subgroup of $H_1(M_r;\mathbf{Z})$, were first considered by J. Alexander and G. Briggs [Al28], [AB27] and by O. Zariski [Za32]. The continuing interest in these invariants is witnessed by numerous papers (e.g., [Go72], [Me80], [We80], [Ri90] and [GS91]). We call b_r the r^{th} *torsion*

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number of k. We say that b_r is *pure* if the corresponding Betti number β_r vanishes (equivalently, $H_1(M_r;\mathbf{Z})$ is a pure torsion group).

Betti numbers are known to be periodic in r, and they are relatively easy to compute (see Proposition 2.2). A useful formula for pure torsion numbers was given by R. Fox in [Fo56]. Although the proof given by Fox was insufficient, a complete argument was given by C. Weber [We80]. Weber observed that the problem of computing non-pure torsion numbers is "*... une question plus difficile*".

Torsion and Betti numbers for knots are a special case of a more general, algebraic construction that depends only on an *augmented group*, consisting of a finitely generated group G and a surjection $\chi\colon G \to \mathbf{Z}$. We define torsion and Betti numbers in this general context. For a large class of augmented groups, including those that correspond to knots, we provide a formula for all torsion numbers, generalizing the formula of Fox. We prove that the sequence of torsion numbers satisfies a linear recurrence relation.

Torsion numbers tend to grow quickly as their index r becomes large. F. González-Acuña and H. Short [GS91] and independently R. Riley [Ri90] proved that the sequence of pure torsion numbers of any knot k has exponential growth rate equal to the Mahler measure of the Alexander polynomial of k. We improved upon this in [SW00] by showing that the entire sequence b_r grows at this rate and generalizing the result in a natural way for links. The proofs in [SW00] use a deep result about algebraic dynamical systems due to D. Lind, K. Schmidt and T. Ward (Theorem 21.1 of [Sc95]). Here we extend such results for torsion numbers b_r associated to many augmented groups. In contrast, we prove under suitable hypotheses that for any prime number p the *p-component* of b_r (i.e., the largest power of p that divides b_r) grows subexponentially. The proof relies on a p-adic version of Jensen's formula, proven by G.R. Everest and B. Ní Fhlathúin [EF96], [Ev99]. As a corollary we strengthen a theorem of C. Gordon [Go72] by proving that for any knot the sequence of torsion numbers either is periodic or else displays infinitely many prime numbers in the factorization of its terms.

In the final section we apply our techniques to the problem of computing homology groups of branched cyclic covering spaces associated to knots and links.

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