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We can now obtain Corollary 1.4 stated in the Introduction.

**COROLLARY 5.4.** *Let  $G = \langle x_1, \dots, x_k \mid r_1, \dots, r_m \rangle$  be a nonelementary word-hyperbolic group and let  $H \leq G$  be a quasiconvex subgroup of infinite index. Let  $a_n$  be the number of freely reduced words in  $A = \{x_1, \dots, x_k\}^{\pm 1}$  of length  $n$  that represent elements of  $H$ . Let  $b_n$  be the number of all words in  $A$  of length  $n$  that represent elements of  $H$ . Then*

$$\limsup_{n \rightarrow \infty} \sqrt[n]{a_n} < 2k - 1$$

and

$$\limsup_{n \rightarrow \infty} \sqrt[n]{b_n} < 2k.$$

*Proof.* Note that  $k \geq 2$  since  $G$  is nonelementary. Put  $A = \{x_1, \dots, x_k\}$  and  $Y = \Gamma(G, H, A)$ . We choose  $x_0 := H1 \in VY$  as the base-vertex of  $Y$ . Note that  $Y$  is  $2k$ -regular by construction. Also, for any vertex  $x$  of  $Y$  and any word  $w$  in  $A \cup A^{-1}$  there is a unique path in  $Y$  with label  $w$  and origin  $x$ . The definition of Schreier coset graphs also implies that a word  $w$  represents an element of  $H$  if and only if the unique path in  $Y$  with origin  $x_0$  and label  $w$  terminates at  $x_0$ . Therefore  $a_n(Y)$  equals the number of freely reduced words in the alphabet  $A = \{x_1, \dots, x_k\}^{\pm 1}$  of length  $n$  that represent elements of  $H$ . Similarly,  $b_n(Y)$  equals the number of all words in  $A$  of length  $n$  representing elements of  $H$ . By Theorem 1.2,  $Y$  is nonamenable. Hence by Theorem 2.5,  $\alpha(Y) < 2k - 1$  and  $\beta(Y) < 2k$ , as required.

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